1. (10 pts.) Consider the following game: flip a fair coin, and bet $1 on H. (i.e. if it is a H, you win and get $1; if it is a T, you lose and pay $1). If you win, the game stops. Otherwise, you flip the coin again, this time doubling your bet. If you win, stop; else, repeat this game of flipping and doubling your bet each time, until you win and stop. For example, if you get 3 Tails in a row and then a Head, you lose $1 + 2 + 4 = 7 dollars on the first three throws, and win 8 dollars on the last throw, for a total win of $8 - 7 = 1 dollar.

(a). Suppose $f$ is the sum of all wins and losses at the end of the game. What is the distribution of $f$? Calculate its expectation and variance.

(b). Suppose $g$ is the maximum value you could lose before you finally win, i.e. the sum of all the losses right before a H is thrown. What is the distribution of $g$? Explain whether the expectation of $g$ is finite or infinite. If finite, calculate it.

2. (10 pts.) Suppose you are playing the same game, but now with an unfair coin with probability of H being $p$. What will be the distributions and expectations of $f$ and $g$? Explain whether for some values of $p$, expectation becomes infinite.

3. (10 pts.) Suppose you are playing the same game with a fair coin (as in Question 1), but this time you have only a finite pile of money and the game stops either when you lose all your money, or you throw a H and win. What will be the distributions and expectations of $f$ and $g$? You may assume that there are $2^n - 1$ dollars in your pile of money, for some fixed value of $n$. Assuming you only have a finite amount of money, is this game worth playing?