Recap of Lecture 6

- **Shared memory multiprocessors**
  - Caches may be either shared or distributed.
    - Multicore chips are likely to have shared caches
    - Cache hit performance is better if they are distributed (each cache is smaller/closer) but they must be kept coherent -- multiple cached copies of same location must be kept equal.
  - Requires clever hardware (see CS252, CS258).
  - Distant memory much more expensive to access.
  - Machines scale to 10s or 100s of processors.
- **Shared memory programming**
  - Starting, stopping threads.
  - Communication by reading/writing shared variables.
  - Synchronization with locks, barriers.

Outline

- **Distributed Memory Architectures**
  - Properties of communication networks
  - Topologies
  - Performance models
- **Programming Distributed Memory Machines using Message Passing**
  - Overview of MPI
  - Basic send/receive use
  - Non-blocking communication
  - Collectives

Architectures in Top 500, Nov 2014

Cluster
MPP
Constellations
SMP
SIMD
Single Processor
Historical Perspective

- Early distributed memory machines were:
  - Collection of microprocessors
  - Communication was performed using bi-directional queues between nearest neighbors.
- Messages were forwarded by processors on path.
  - "Store and forward" networking
- There was a strong emphasis on topology in algorithms, in order to minimize the number of hops = minimize time

Network Analogy

- To have a large number of different transfers occurring at once, you need a large number of distinct wires
  - Not just a bus, as in shared memory
- Networks are like streets:
  - Link = street.
  - Switch = intersection.
  - Distances (hops) = number of blocks traveled.
  - Routing algorithm = travel plan.
- Properties:
  - Latency: how long to get between nodes in the network.
    - Street: time for one car = dist (miles) / speed (miles/hr)
  - Bandwidth: how much data can be moved per unit time.
    - Street: cars/hour = density (cars/mile) * speed (miles/hr) * #lanes
    - Network bandwidth is limited by the bit rate per wire and #wires

Design Characteristics of a Network

- Topology (how things are connected)
  - Crossbar; ring; 2-D, 3-D, higher-D mesh or torus; hypercube; tree; butterfly; perfect shuffle, dragon fly, ...
- Routing algorithm:
  - Example in 2D torus: all east-west then all north-south (avoids deadlock).
- Switching strategy:
  - Circuit switching: full path reserved for entire message, like the telephone.
  - Packet switching: message broken into separately-routed packets, like the post office, or internet
- Flow control (what if there is congestion):
  - Stall, store data temporarily in buffers, re-route data to other nodes, tell source node to temporarily halt, discard, etc.

Performance Properties of a Network: Latency

- Diameter: the maximum (over all pairs of nodes) of the shortest path between a given pair of nodes.
- Latency: delay between send and receive times
  - Latency tends to vary widely across architectures
  - Vendors often report hardware latencies (wire time)
  - Application programmers care about software latencies (user program to user program)
- Observations:
  - Latencies differ by 1-2 orders across network designs
  - Software/hardware overhead at source/destination dominate cost (1s-10s usecs)
  - Hardware latency varies with distance (10s-100s nsec per hop) but is small compared to overheads
- Latency is key for programs with many small messages
Latency on Some Machines/Networks

<table>
<thead>
<tr>
<th>Machine</th>
<th>8-byte Roundtrip Latency (μs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elan3/Alpha</td>
<td>14.6</td>
</tr>
<tr>
<td>Elan4/IA64</td>
<td>6.6</td>
</tr>
<tr>
<td>Myrinet/ES</td>
<td>22.1</td>
</tr>
<tr>
<td>IB/G5</td>
<td>9.6</td>
</tr>
<tr>
<td>IB/Opteron</td>
<td>19.5</td>
</tr>
</tbody>
</table>

- Latencies shown are from a ping-pong test using MPI
- These are roundtrip numbers: many people use ½ of roundtrip time to approximate 1-way latency (which can’t easily be measured)

End to End Latency (1/2 roundtrip) Over Time

- Latency has not improved significantly, unlike Moore’s Law
- T3E (shm) was lowest point – in 1997

Performance Properties of a Network: Bandwidth

- The bandwidth of a link = # wires / time-per-bit
- Bandwidth typically in Gigabytes/sec (GB/s), i.e., \(8 \times 2^{20}\) bits per second
- Effective bandwidth is usually lower than physical link bandwidth due to packet overhead.

- Bandwidth is important for applications with mostly large messages

Bandwidth on Existing Networks

- Flood bandwidth (throughput of back-to-back 2MB messages)
Bandwidth Chart

Note: bandwidth depends on SW, not just HW

![Bandwidth Chart Image]

Performance Properties of a Network: Bisection Bandwidth

- **Bisection bandwidth**: bandwidth across smallest cut that divides network into two equal halves
- Bandwidth across “narrowest” part of the network

![Bisection Cut Diagram]

- Bisection bandwidth is important for algorithms in which all processors need to communicate with all others

Network Topology

- In the past, there was considerable research in network topology and in mapping algorithms to topology.
  - **Key cost to be minimized**: number of “hops” between nodes (e.g. “store and forward”)
  - Modern networks hide hop cost (i.e., “wormhole routing”), so topology less of a factor in performance of many algorithms
- Example: On IBM SP system, hardware latency varies from 0.5 us to 1.5 us, but user-level message passing latency is roughly 36 us.
- Need some background in network topology
  - Algorithms may have a communication topology
  - Example later of big performance impact

Linear and Ring Topologies

- **Linear array**
  - Diameter = n-1; average distance ~ n/3.
  - Bisection bandwidth = 1 (in units of link bandwidth).
- **Torus or Ring**
  - Diameter = n/2; average distance ~ n/4.
  - Bisection bandwidth = 2.
  - Natural for algorithms that work with 1D arrays.
Meshes and Tori – used in Hopper

- Two dimensional mesh
  - Diameter = \(2 \times \sqrt[n]{n} - 1\)
  - Bisection bandwidth = \(\sqrt{n}\)
- Two dimensional torus
  - Diameter = \(\sqrt{n}\)
  - Bisection bandwidth = \(2 \times \sqrt{n}\)
- Generalizes to higher dimensions
  - Cray XT (e.g., Hopper@NERSC) uses 3D Torus
  - Natural for algorithms that work with 2D and/or 3D arrays (matmul)

---

**Hypercubes**

- Number of nodes \(n = 2^d\) for dimension \(d\).
  - Diameter = \(d\).
  - Bisection bandwidth = \(n/2\).
  - \(0d\), 1d, 2d, 3d, 4d

- Popular in early machines (Intel iPSC, NCUBE).
  - Lots of clever algorithms.
  - See 1996 online CS267 notes.
  - Greycode addressing:
    - Each node connected to \(d\) others with 1 bit different.

---

**Trees**

- Diameter = \(\log n\).
- Bisection bandwidth = 1.
- Easy layout as planar graph.
- Many tree algorithms (e.g., summation).
- Fat trees avoid bisection bandwidth problem:
  - More (or wider) links near top.
  - Example: Thinking Machines CM-5.

---

**Butterflies**

- Diameter = \(\log n\).
- Bisection bandwidth = \(n\).
- Cost: lots of wires.
- Used in BBN Butterfly.
- Natural for FFT.

Ex: to get from proc 101 to 110,
Compare bit-by-bit and
Switch if they disagree, else not
Does Topology Matter?

1 MB multicast on BG/P, Cray XT5, and Cray XE6

Bandwidth (MB/sec)

02/09/2016

Evolution of Distributed Memory Machines

• Special queue connections are being replaced by direct memory access (DMA):
  • Network Interface (NI) processor packs or copies messages.
  • CPU initiates transfer, goes on computing.
• Wormhole routing in hardware:
  • NIs do not interrupt CPUs along path.
  • Long message sends are pipelined.
  • NIs don’t wait for complete message before forwarding
• Message passing libraries provide store-and-forward abstraction:
  • Can send/receive between any pair of nodes, not just along one wire.
  • Time depends on distance since each NI along path must participate.

See EECS Tech Report UCB/ECECS-2011-92, August 2011

02/09/2016

Dragonflies – used in Edison

• Motivation: Exploit gap in cost and performance between optical interconnects (which go between cabinets in a machine room) and electrical networks (inside cabinet)
  • Optical more expensive but higher bandwidth when long
  • Electrical networks cheaper, faster when short
• Combine in hierarchy
  • One-to-many via electrical networks inside cabinet
  • Just a few long optical interconnects between cabinets
• Clever routing algorithm to avoid bottlenecks:
  • Route from source to randomly chosen intermediate cabinet
  • Route from intermediate cabinet to destination
• Outcome: programmer can (usually) ignore topology, get good performance
  • Important in virtualized, dynamic environment
  • Programmer can still create serial bottlenecks
  • Drawback: variable performance
• Details in “Technology-Drive, Highly-Scalable Dragonfly Topology,” J. Kim, W. Dally, S. Scott, D. Abts, ISCA 2008

02/09/2016

Performance Models

02/09/2016
Shared Memory Performance Models

- Parallel Random Access Memory (PRAM)
- All memory access operations complete in one clock period — no concept of memory hierarchy ("too good to be true.")
  - OK for understanding whether an algorithm has enough parallelism at all (see CS273).
  - Parallel algorithm design strategy: first do a PRAM algorithm, then worry about memory/communication time (sometimes works)
- Slightly more realistic versions exist
  - E.g., Concurrent Read Exclusive Write (CREW) PRAM.
  - Still missing the memory hierarchy

Latency and Bandwidth Model

- Time to send message of length n is roughly
  \[ \text{Time} = \text{latency} + n \cdot \text{cost per word} \]
  \[ = \text{latency} + n / \text{bandwidth} \]
- Topology is assumed irrelevant.
- Often called “\( \alpha - \beta \) model” and written
  \[ \text{Time} = \alpha + n \beta \]
- Usually \( \alpha \gg \beta >> \) time per flop.
  - One long message is cheaper than many short ones.
  \[ \alpha + n \beta \ll n (\alpha + 1 \beta) \]
  - Can do hundreds or thousands of flops for cost of one message.
- Lesson: Need large computation-to-communication ratio to be efficient.
- LogP – more detailed model (Latency/overhead/gap/Proc.)

Alpha-Beta Parameters on Various Machines

- These numbers were obtained empirically

<table>
<thead>
<tr>
<th>machine</th>
<th>( \alpha )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T3E/Shm</td>
<td>1.2</td>
<td>0.003</td>
</tr>
<tr>
<td>T3E/MPI</td>
<td>6.7</td>
<td>0.003</td>
</tr>
<tr>
<td>IBM/LAPI</td>
<td>9.4</td>
<td>0.003</td>
</tr>
<tr>
<td>IBM/MPI</td>
<td>7.6</td>
<td>0.004</td>
</tr>
<tr>
<td>Quadrics/Get</td>
<td>3.267</td>
<td>0.00498</td>
</tr>
<tr>
<td>Quadrics/Shm</td>
<td>1.3</td>
<td>0.005</td>
</tr>
<tr>
<td>Quadrics/MPI</td>
<td>7.3</td>
<td>0.005</td>
</tr>
<tr>
<td>Myrinet/GM</td>
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</tr>
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<td>Myrinet/MPI</td>
<td>7.2</td>
<td>0.006</td>
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<td>Dolphin/MPI</td>
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<tr>
<td>GigE/VIPL</td>
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</tr>
<tr>
<td></td>
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<td>0.00872</td>
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</tbody>
</table>

\( \alpha \) is latency in usecs
\( \beta \) is BW in usecs per Byte

How well does the model predict actual performance?
Message Passing Libraries (1)

- Many "message passing libraries" were once available
  - Chameleon, from ANL.
  - CMMD, from Thinking Machines.
  - Express, commercial.
  - MPL, native library on IBM SP-2.
  - NX, native library on Intel Paragon.
  - Zipcode, from LLL.
  - PVM, Parallel Virtual Machine, public, from ORNL/UTK.
  - Others...
    - MPI, Message Passing Interface, now the industry standard.
- Need standards to write portable code.

Message Passing Libraries (2)

- All communication, synchronization require subroutine calls
  - No shared variables
  - Program run on a single processor just like any uniprocessor program, except for calls to message passing library
- Subroutines for
  - Communication
    - Pairwise or point-to-point: Send and Receive
    - Collectives all processor get together to
      - Move data: Broadcast, Scatter/gather
      - Compute and move: sum, product, max, prefix sum, …
      - of data on many processors
  - Synchronization
    - Barrier
    - No locks because there are no shared variables to protect
  - Enquiries
    - How many processes? Which one am I? Any messages waiting?
Novel Features of MPI

- **Communicators** encapsulate communication spaces for library safety
- **Datatypes** reduce copying costs and permit heterogeneity
- Multiple communication **modes** allow precise buffer management
- Extensive **collective operations** for scalable global communication
- **Process topologies** permit efficient process placement, user views of process layout
- **Profiling interface** encourages portable tools

MPI References

- The Standard itself:
  - at [http://www.mpi-forum.org](http://www.mpi-forum.org)
  - All MPI official releases, in both postscript and HTML
  - Latest version MPI 3.1, released June 2015
- Other information on Web:
  - pointers to lots of stuff, including other talks and tutorials, a FAQ, other MPI pages

Books on MPI

- Designing and Building Parallel Programs, by Ian Foster, Addison-Wesley, 1995.
- Parallel Programming with MPI, by Peter Pacheco, Morgan-Kaufmann, 1997.

Finding Out About the Environment

- Two important questions that arise early in a parallel program are:
  - **How many processes are participating in this computation?**
  - **Which one am I?**
- MPI provides functions to answer these questions:
  - **MPI_Comm_size** reports the number of processes.
  - **MPI_Comm_rank** reports the **rank**, a number between 0 and size-1, identifying the calling process
Hello (C)

```c
#include "mpi.h"
#include <stdio.h>

int main( int argc, char *argv[] )
{
    int rank, size;
    MPI_Init( &argc, &argv );
    MPI_Comm_rank( MPI_COMM_WORLD, &rank );
    MPI_Comm_size( MPI_COMM_WORLD, &size );
    printf( "I am %d of %d\n", rank, size );
    MPI_Finalize();
    return 0;
}
```

Hello (Fortran)

```fortran
program main
    include 'mpif.h'
    integer ierr, rank, size
    call MPI_INIT( ierr )
    call MPI_COMM_RANK( MPI_COMM_WORLD, rank, ierr )
    call MPI_COMM_SIZE( MPI_COMM_WORLD, size, ierr )
    print *, 'I am ', rank, ' of ', size
    call MPI_FINALIZE( ierr )
end
```

Hello (C++)

```cpp
#include "mpi.h"
#include <iostream>

int main( int argc, char *argv[] )
{
    int rank, size;
    MPI::Init(argc, argv);
    rank = MPI::COMM_WORLD.Get_rank();
    size = MPI::COMM_WORLD.Get_size();
    std::cout << "I am " << rank << " of " << size << "\n";
    MPI::Finalize();
    return 0;
}
```

Notes on Hello World

- All MPI programs begin with MPI_Init and end with MPI_Finalize
- MPI_COMM_WORLD is defined by mpi.h (in C) or mpif.h (in Fortran) and designates all processes in the MPI “job”
- Each statement executes independently in each process
  - including the printf/print statements
- The MPI-1 Standard does not specify how to run an MPI program, but many implementations provide
  `mpirun -np 4 a.out`
### MPI Basic Send/Receive

- We need to fill in the details in

```
Process 0                Process 1
  Send(data)              Receive(data)
```

- Things that need specifying:
  - How will “data” be described?
  - How will processes be identified?
  - How will the receiver recognize/screen messages?
  - What will it mean for these operations to complete?

### Some Basic Concepts

- Processes can be collected into **groups**
- Each message is sent in a **context**, and must be received in the same context
  - Provides necessary support for libraries
- A group and context together form a **communicator**
- A process is identified by its **rank** in the group associated with a communicator
- There is a default communicator whose group contains all initial processes, called **MPI_COMM_WORLD**

### MPI Datatypes

- The data in a message to send or receive is described by a triple (address, count, datatype), where
- An MPI datatype is recursively defined as:
  - predefined, corresponding to a data type from the language (e.g., MPI_INT, MPI_DOUBLE)
  - a contiguous array of MPI datatypes
  - a strided block of datatypes
  - an indexed array of blocks of datatypes
  - an arbitrary structure of datatypes
- There are MPI functions to construct custom datatypes, in particular ones for subarrays
- May hurt performance if datatypes are complex

### MPI Tags

- Messages are sent with an accompanying user-defined integer tag, to assist the receiving process in identifying the message
- Messages can be screened at the receiving end by specifying a specific tag, or not screened by specifying MPI_ANY_TAG as the tag in a receive
- Some non-MPI message-passing systems have called tags “message types”. MPI calls them tags to avoid confusion with datatypes
MPI Basic (Blocking) Send

MPI_SEND(start, count, datatype, dest, tag, comm)

- The message buffer is described by (start, count, datatype).
- The target process is specified by dest, which is the rank of the target process in the communicator specified by comm.
- When this function returns, the data has been delivered to the system and the buffer can be reused. The message may not have been received by the target process.

A Simple MPI Program

```c
#include "mpi.h"
#include <stdio.h>
int main( int argc, char *argv[])
{
    int rank, buf;
    MPI_Status status;
    MPI_Init(&argv, &argc);
    MPI_Comm_rank( MPI_COMM_WORLD, &rank );
    /* Process 0 sends and Process 1 receives */
    if (rank == 0) {
        buf = 123456;
        MPI_Send( &buf, 1, MPI_INT, 1, 0, MPI_COMM_WORLD);
    }
    else if (rank == 1) {
        MPI_Recv( &buf, 1, MPI_INT, 0, 0, MPI_COMM_WORLD, &status );
        printf("Received %d\n", buf);
    }
    MPI_Finalize();
    return 0;
}
```

MPI Basic (Blocking) Receive

MPI_RECV(start, count, datatype, source, tag, comm, status)

- Waits until a matching (both source and tag) message is received from the system, and the buffer can be used
- source is rank in communicator specified by comm, or MPI_ANY_SOURCE
- tag is a tag to be matched or MPI_ANY_TAG
- receiving fewer than count occurrences of datatype is OK, but receiving more is an error
- status contains further information (e.g. size of message)

A Simple MPI Program (Fortran)

```fortran
program main
    include 'mpif.h'
    integer rank, buf, ierr, status(MPI_STATUS_SIZE)
    call MPI_Init(ierr)
    call MPI_Comm_rank( MPI_COMM_WORLD, rank, ierr )
    C Process 0 sends and Process 1 receives
    if (rank .eq. 0) then
        buf = 123456
        call MPI_Send( buf, 1, MPI_INTEGER, 1, 0, MPI_COMM_WORLD, ierr )
    else if (rank .eq. 1) then
        call MPI_Recv( buf, 1, MPI_INTEGER, 0, 0, MPI_COMM_WORLD, status, ierr )
        print *, "Received " , buf
    endif
    call MPI_Finalize(ierr)
end
```

Slide source: Bill Gropp, ANL
A Simple MPI Program (C++)

```cpp
#include "mpi.h"
#include <iostream>
int main( int argc, char *argv[])
{
    int rank, buf;
    MPI::Init(argc, argv);
    rank = MPI::COMM_WORLD.Get_rank();
    // Process 0 sends and Process 1 receives
    if (rank == 0) {
        buf = 123456;
        MPI::COMM_WORLD.Send( &buf, 1, MPI::INT, 1, 0 );
    } else if (rank == 1) {
        MPI::COMM_WORLD.Recv( &buf, 1, MPI::INT, 0, 0 );
        std::cout << "Received " << buf << \n;    }
    MPI::Finalize();
    return 0;
}
```

Retrieving Further Information

- **Status** is a data structure allocated in the user's program.
  - In C:
    ```c
    int recvd_tag, recvd_from, recvd_count;
    MPI_Status status;
    MPI_Recv(..., MPI_ANY_SOURCE, MPI_ANY_TAG, ..., &status )
    recvd_tag  = status.MPI_TAG;
    recvd_from = status.MPI_SOURCE;
    MPI_Get_count( &status, datatype, &recvd_count );
    ```

MPI is Simple

- Many parallel programs can be written using just these six functions, only two of which are non-trivial:
  - MPI_INIT
  - MPI_FINALIZE
  - MPI_COMM_SIZE
  - MPI_COMM_RANK
  - MPI_SEND
  - MPI_RECV

Another Approach to Parallelism

- **Collective** routines provide a higher-level way to organize a parallel program
- Each process executes the same communication operations
- MPI provides a rich set of collective operations...
Collective Operations in MPI

• Collective operations are called by all processes in a communicator
• **MPI_BCAST** distributes data from one process (the root) to all others in a communicator
• **MPI_REDUCE** combines data from all processes in communicator and returns it to one process
• In many numerical algorithms, **SEND/RECEIVE** can be replaced by **BCAST/REDUCE**, improving both simplicity and efficiency

Example: Calculating Pi

E.g., in a 4-process run, each process gets every 4th interval. Process 0 slices are in red.

• Simple program written in a data parallel style in MPI
  • E.g., for a reduction (recall “tricks with trees” lecture), each process will first reduce (sum) its own values, then call a collective to combine them
• Estimates pi by approximating the area of the quadrant of a unit circle
• Each process gets 1/p of the intervals (mapped round robin, i.e., a cyclic mapping)

MPI can be simple

• Claim: most MPI applications can be written with only 6 functions (although which 6 may differ)

Using point-to-point:
  • **MPI_INIT**
  • **MPI_FINALIZE**
  • **MPI_COMM_SIZE**
  • **MPI_COMM_RANK**
  • **MPI_SEND**
  • **MPI_RECEIVE**

Using collectives:
  • **MPI_INIT**
  • **MPI_FINALIZE**
  • **MPI_COMM_SIZE**
  • **MPI_COMM_RANK**
  • **MPI_BCAST**
  • **MPI_REDUCE**

• You may use more for convenience or performance

Example: PI in C – 1/2

```c
#include "mpi.h"
#include <math.h>
#include <stdio.h>
int main(int argc, char *argv[])
{
  int done = 0, n, myid, numprocs, i, rc;
  double PI25DT = 3.141592653589793238462643;
  double mypi, pi, h, sum, x, a;
  MPI_Init(&argc,&argv);
  MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
  MPI_Comm_rank(MPI_COMM_WORLD,&myid);
  while (!done)  {
    if (myid == 0) {
      printf("Enter the number of intervals: (0 quits) ");
      scanf("%d", &n);
      double PI25DT = 3.141592653589793238462643;
      double mypi, pi, h, sum, x, a;
      MPI_Init(&argc,&argv);
      MPI_Comm_size(MPI_COMM_WORLD,&numprocs);
      MPI_Comm_rank(MPI_COMM_WORLD,&myid);
      while (!done) {
        if (myid == 0) {
          printf("Enter the number of intervals: (0 quits) ");
          scanf("%d", &n);
        }
        MPI_Bcast(&n, 1, MPI_INT, 0, MPI_COMM_WORLD);
        if (n == 0) break;
```
Example: PI in C – 2/2

```c
h = 1.0 / (double) n;
sum = 0.0;
for (i = myid + 1; i <= n; i += numprocs) {
    x = h * ((double)i - 0.5);
    sum += 4.0 * sqrt(1.0 - x*x);
}
mypi = h * sum;
MPI_Reduce(&mypi, &pi, 1, MPI_DOUBLE, MPI_SUM, 0,
            MPI_COMM_WORLD);
if (myid == 0)
    printf("pi is approximately %.16f, Error is .16f
", pi, fabs(pi - PI25DT));
}
```

Example: PI in Fortran – 1/2

```fortran
program main
  include 'mpif.h'
  integer done, n, myid, numprocs, i, rc
  double PI25DT, mypi, pi, h, sum, x, z
  data done/.false./
  data PI25DT/3.141592653589793238462643/
  call MPI_Init(ierr)
  call MPI_Comm_size(MPI_COMM_WORLD,numprocs, ierr)
  call MPI_Comm_rank(MPI_COMM_WORLD,myid, ierr)
do while (.not. done)
  if (myid .eq. 0) then
    print *,
    "Enter the number of intervals: (0 quits)"
    read *, n
  endif
  call MPI_Bcast(n, 1, MPI_INTEGER, 0,
                MPI_COMM_WORLD, ierr)
  if (n .eq. 0) goto 10
```

Example: PI in Fortran – 2/2

```fortran
h   = 1.0 / n
sum = 0.0
do i=myid+1,n,numprocs
    x = h * (i - 0.5)
    sum += 4.0 / (1.0 + x*x)
enddo
mypi = h * sum
call MPI_Reduce(mypi, pi, 1, MPI_DOUBLE_PRECISION,
                MPI_SUM, 0, MPI_COMM_WORLD, ierr)
if (myid .eq. 0) then
    print *, "pi is approximately ".16f", pi,
    "Error is ", abs(pi - PI25DT)
enddo
10 continue
```

Example: PI in C++ - 1/2

```cpp
#include "mpi.h"
#include <math.h>
#include <iostream>
int main(int argc, char *argv[])
{
    int done = 0, n, myid, numprocs, i, rc;
    double PI25DT = 3.141592653589793238462643;
    double mypi, pi, h, sum, x, a;
    MPI::Init(argc, argv);
    numprocs = MPI::COMM_WORLD.Get_size();
    myid     = MPI::COMM_WORLD.Get_rank();
    while (!done) {
        if (myid .eq. 0) {
            std::cout << "Enter the number of intervals: (0 quits) ";
            std::cin >> n;
            if (n == 0) break;
```
Example: PI in C++ - 2/2

```c
h = 1.0 / (double) n;
sum = 0.0;
for (i = myid + 1; i <= n; i += numprocs) {
    x = h * ((double)i - 0.5);
    sum += 4.0 / (1.0 + x*x);
}
mpi = h * sum;
MPI::COMM_WORLD.Reduce(&mpi, &pi, 1, MPI::DOUBLE,
    MPI::SUM, 0);
if (myid == 0)
    std::cout << "pi is approximately 
        " << pi << 
    " Error is " << fabs(pi - PI25DT) << "\n";
MPI::Finalize();
return 0;
```

Synchronization

- **MPI_Barrier( comm )**
  - Blocks until all processes in the group of the communicator `comm` call it.
- Almost never required in a parallel program
  - Occasionally useful in measuring performance and load balancing

Synchronization (Fortran)

- **MPI_Barrier( comm, ierr )**
  - Blocks until all processes in the group of the communicator `comm` call it.

Synchronization (C++)

- **comm.Barrier();**
  - Blocks until all processes in the group of the communicator `comm` call it.
Collective Data Movement

- Broadcast
  - P0
  - P1
  - P2
  - P3

- Scatter
  - P0
  - P1
  - P2
  - P3

- Gather

Comments on Broadcast, other Collectives

- All collective operations must be called by all processes in the communicator.
- MPI_Bcast is called by both the sender (called the root process) and the processes that are to receive the broadcast.
  - "root" argument is the rank of the sender; this tells MPI which process originates the broadcast and which receive.

More Collective Data Movement

- Allgather
  - P0
  - P1
  - P2
  - P3

- Alltoall
  - P0
  - P1
  - P2
  - P3

Collective Computation

- Reduce
  - P0
  - P1
  - P2
  - P3

- Scan
  - P0
  - P1
  - P2
  - P3
**MPI Collective Routines**

- Many Routines: Allgather, Allgatherv, Allreduce, Alltoall, Alltoallv, Bcast, Gather, Gatherv, Reduce, Reduce_scatter, Scan, Scatter, Scatterv
- **All** versions deliver results to all participating processes, not just root.
- **V** versions allow the chunks to have variable sizes.
- Allreduce, Reduce, Reduce_scatter, and Scan take both built-in and user-defined combiner functions.
- MPI-2 adds Alltoallw, Exscan, intercommunicator versions of most routines

**MPI Built-in Collective Computation Operations**

- MPI_MAX: Maximum
- MPI_MIN: Minimum
- MPI_PROD: Product
- MPI_SUM: Sum
- MPI_LAND: Logical and
- MPI_LOR: Logical or
- MPI_LXOR: Logical exclusive or
- MPI_BAND: Binary and
- MPI_BOR: Binary or
- MPI_BXOR: Binary exclusive or
- MPI_MAXLOC: Maximum and location
- MPI_MINLOC: Minimum and location