MATH 16 A-LECTURE.  SEPTEMBER 4, 2008.

PROFESSOR: WELCOME BACK.  HELLO.  SO I DON'T KNOW WHAT'S WRONG WITH THE AIR CONDITIONING.  SO HOPEFULLY IT WILL GET COOLER BY THE ENDS OF LECTURE.  SO I HAVE A FEW ANNOUNCEMENTS TODAY.  SO SEVERAL PEOPLE NOTICED THAT THEIR GSI WAS NOT IN THE ROOM THAT THEY SHOWED UP TO ON TUESDAY.  THAT'S BECAUSE TELEBEARS CHANGED THE ROOM AT THE LAST SECOND AND DIDN'T NOTIFY ANYBODY.  MAKE UP NEXT WEEK BY GIVING YOU A QUIZ OR THE MATERIAL COVERED THIS WEEK.  SO THAT IS, SEE WEB PAGE FOR NEW ROOM ASSIGNMENTS.  SO EXPECT AND I'LL JUST SAY OBVIOUSLY IF YOU COULDN'T TURN IN YOUR HOME WORK, WE'LL DEAL WITH IT.  BECAUSE HE WASN'T THERE TO TAKE IT.  SO LAST TIME DAN-COHEN GAVE YOU A LITTLE TALK.  HE WAS THE HEAD TA.  THINGS HAVE CHANGED.  IF YOU HAVE A QUESTION NOW, THE DEPARTMENT SECRETARY BARBARA PEAVY IS IN CHARGE OF IT ALL.  SO PLEASE GO TO HER OFFICE.  AND EVERYTHING IS POINTED TO HER E-MAIL, WHATEVER IS ON THE WEB PAGE.  HOMEWORK IS POSTED FOR NEXT WEEK.  AND I WILL FINISH ALL OF CHAPTER ZERO TODAY AND START CHAPTER ONE.  SO THE HOMEWORK WILL INDEED, IT'S A SUBSET OF ALL THE HOMEWORK.  IT'S A SUBSET FROM CHAPTER ZERO.  YOU DON'T HAVE TO DO ALL OF IT, DON'T WORRY.

THE OTHER THING IS THAT SEVERAL PEOPLE ASKED ME ABOUT THOSE EXAMPLES I GAVE.  AND THE QUESTION WAS, HAS ANYTHING BAD OF REALLY HAPPENED BECAUSE OF BAD ARITHMETIC.  REMEMBER I WAS TELLING YOU COMPUTERS LIE?  HAS ANYTHING REALLY BAD GONE WRONG, ONE DAY IN 1990S SIX THE EUROPEAN SPACE AGENCY LAUNCH DOLLARS ITS BIGGEST ROCKET OF.  PUT UP SATELLITES FOR -- THEY LAUNCHED, AFTER 40 SECONDS UP IN THE SKY IT BLUE UP.  $30,070,000,000 DOWN THE TUBES BECAUSE OF ARITHMETIC ROMPLE NOBODY WAS ON BOARD.  IT WAS JUST SATELLITE STUFF.  AND I HAVE A POINTER ON THE WEB PAGE IF YOU WANT IT READ MORE ABOUT THE EMBARRASSING MISTAKES THEY MADE.

SO THAT'S ALL OF ANNOUNCEMENT FROM LAST TIME.  SO LET'S KEEP GOING.  TALKS ABOUT ZEROS OF FUNCTIONS.
AND SOMETIMES I USE THIS LANGUAGE ROOTS OF, $f(x) = 0$. SO WHERE DOES $x$ SATISFY THAT EQUATION. AND LAST TIME WE TALKED ABOUT THE QUADRATIC FORMULA. LET'S WRITE THAT DOWN AGAIN. (ON BOARD). OKAY. WE GOT THAT FAR LAST TIME. AND I DREW PICTURES TO INDICATE THAT THERE COULD BE EITHER TWO ROOTS, 0, 10, OR NO ZEROS. DEPENDING ON WHETHER THIS PARABOLA, BECAUSE THAT'S A PARABOLA WHETHER IT INTERSECTS THE X-AXIS TWO TIMES, ONE TIME OR NO TIMES. THEY'RE ALL POSSIBLE. THE FORMULA TELLS YOU EVERYTHING. SO JUST A TYPE LITTLE EXAMPLE HERE. IF I DO $x^2 - 3x + 2 = 0$ AND YOU PLUG IN, LET'S ME JUST DO IT, NEGATIVE, NEGATIVE THREE IS THREE, PLUS MINUS NEGATIVE THREE SQUARED MINUS FOUR TIME TWO TIMES ONE ALL MINES (ON BOARD). AND SO THAT'S THREE PLUS OR MINUS THE SQUARE ROOT OF NINE MINUS EIGHT IS ONE. SO IT'S TWO AND ONE. GOOD I GET THAT RIGHT? SO WE CAN ALL PLUG THAT IN. THE REASON I GAVE THAT EXAMPLE ASK THAT DOING THAT IS THE SAME PROBLEM AS IF I ASKED YOU TO DO THIS OTHER THING. SO FIND THE ZEROS OF, THAT'S THE PROBLEM I JUST SOLVED, IT'S EXACTLY THE SAME QUESTION, TO FACTOR $x^2 - 3x + 2$. WHAT DOES IT MEAN IT FACTOR? THAT MEANS IT WRITE THIS THING WAS PRODUCT OF $x - a$ SOME CONSTANT YOU HAVE TO FIGURE OUT, TIMES $x - b$ SOME OTHER CONSTANT YOU HAVE TO FIGURE OUT CHT THAT'S WHAT FACTING IS. SO I CLAIM IT'S THE SAME PROBLEM. SO LET'S JUST ASK WHY. WHERE IS IS THIS POLYNOMIAL ZERO. HOW COULD THIS POLYNOMIAL WRITTEN THIS WAY POSSIBLY BE ZERO? EITHER THAT FACTOR IS ZERO ON THE OTHER ONE IS ZERO THAT'S ONLY TWO POSSIBILITY. HOW CAN THAT BE ZERO? $x$ HAS GOT TO EQUAL $a$. THAT ONE, $x$ HAS GOT TO EQUAL $b$. IMPLIES $a$ AND $b$ ARE THE ZEROS, WE JUST FIGURED OUT FROM THE QUADRATIC FORMULA. $x - a$ TIMES $x - b$. SO THOSE ARE THE SAME PROBLEMS. SO LET'S ME DO ONE MORE EXAMPLE. HOW ABOUT FIND ZEROS $x^4 - 6x^2 + 8$. THAT'S THE SAME PROBLEM. SO IT DOESN'T LOOK, LOOK THERE'S A FOUR UP THERE. WHY IS THAT THE SAME PROBLEM. THIS IS REALLY QUADRATIC, IF I FIGURE OUT HOW TO DO RIGHT SUBSTITUTION. I'M GOING TO DO SUBSTITUTION,
I'm going to write substitute, take $x^2$ and replace with the letter $s$. I can do that. So what does $x$-to the fourth become? It becomes $s$-to what power? $s^2$ minus six $x$ plus eight, there we have a quadratic. I can use QAT Rad I can formula now. And if I do it, I will get that $s$-equals two and four. Those are the two possible roots for this one. That gives you $s$. But what about $x$? I have to go back it $x$. That means $x^2$ equals two or $x^2$ equals four. So what are the roots of the original. I get those two, they're still quadratic. I get those two, they're still quadratic.

Buzz we can do them in our heads. So what are the two possibility if $x$-equals two. (On board) or minus the square root of two. In $x^2$-equals four I can have $x$-equals two or minus two. So all together, I just use the quadratic formula twice and I can solve this slightly more complicated problem. In fact you problem I had to do this on your first homework?

A so I hope that's not too surprising. If I draw a pictures of this now, so here's a function, polynomial that has four different roots. minus two, minus square root of two, plus square root of two and plus two. Let's me put them all here in the axis. minus two, minus square root of two, plus square root of two and plus two. So what does this thing look like? Let's just plot it. and knowing where the zeros are is the most important part of plotting or graphing because you know that's where it has to cross the $x$-axis. So what happens when $x$-gets really big. This function here when $x$-can really big going to be positive or negative? When $x$-is really a big number then $x$-before is positive. So start up here somewhere. Come down and cross zero exactly there. Then I'll go negative. And then I have \( \text{crossed,} \) crosses zero there again. So you know your only choice to go back up. like that. Cross again. There's next zero. No choice you have
TO TURN AROUND AND CROSS AGAIN. AND THERE'S NEXT
ZERO YOU HAVE NO CHOICE BUT IT TURN AROUND AND CROSS
AGAIN. AND SO THAT'S YOUR BEST GUESS AND IT'S
CORRECT OF WHAT THE PLOT, THE GRAVITY OF THE
FUNCTION LOOKS LIKE. SO KNOWING THE ZERO TELLS YOU
QUITE A BIT ABOUT WHAT THE GRAPH HAS TO LOOK LIKE.

SO ARE THERE ANY QUESTIONS ABOUT THAT.

STUDENT: DO YOU THINK YOU COULD SWITCH THE BOTTOM BOARD UP.

PROFESSOR: YES. WE TRIED TO MOVE THIS AND IT'S WIRED DOWN. SO
SORRY. I'LL TRY NOT TO USE THE BOTTOM OF THE BOARD SO MUCH ON
THAT SIDE. SO LET ME TELL YOU, SEE IF YOU SAW A PATTERN. A
QUADRATIC. SO THAT'S A POLYNOMIAL, OF DEGREE TWO HAS TWO ZEROS.
NOW THAT EXAMPLE UP THERE THAT WAS DEGREE FOUR, THAT'S CALL A
QUART I CAN JUST FOR FUN. A POLYNOMIAL OF DEGREE FOUR, HOW MANY
ZERO GET HAVE IN FOUR. OKAY. AND IF I HAD A CUBIC, POLYNOMIAL
OF DEGREE THREE HAS THREE ZEROS, UP TO THREE ZEROS BECAUSE YOU
DON'T HAVE TO HAVE ALL OF THEM. AND WE SHOULD NOT BE SURPRISED
THAT THE GENERAL FACT IS A POLYNOMIAL OF DEGREE N-HAS AT MOST
N-ZEROS. OKAY. AND SINCE POLYNOMIALS ARE SO IMPORTANT PEOPLE
HAVE BEEN WORK FOR MANY YEARS TO TRY AND FIND BETTER WAY TO TRY
AND FIND ROOTS. SO HERE WE HAVE QUADRATIC FORMULA. THAT'S BEEN
KNOWN FOR A COUPLE OF YEARS. THERE'S ALSO A FORMULA SO FAR
CUBIC. BUT WE NEVER BOTHERED (INAUDIBLE). THIS IS A FORMULA.
EVEN HAIR YOUR FOR QUART I CAN. BUT FOR CENTURIES, NOBODY OF
FOUND A FORMULA FOR DEGREE ANY BIGGER, GREARNLGT FIEPLET PEOPLE

LOOKED FOR CENTURIES AM FINAL LIR, FINALLY THERE'S A PROO.

SOMEONE NAMED G-A-L O-I-S-PROVED THERE IS NO FORMULA. NO MATTER
WHAT YOU WRITE DOWN FOR DEGREE GREATER THAN OR EQUAL TO FIVE.
AND THAT'S ONE OF THE BIGGEST RESULTS OF MATHEMATICS. I JUST
THOUGHT I'D PUT IT IN. IT'S INTERESTING WHY WE NECESSITY THIS.
WE ALMOST DIN'T KNOW THIS. HERE'S WHAT HAPPENED. GAL WITH A
CHALD CHALLENGED SOMEONE TO A DUAL. HE HENT WRITTEN DOWN ANY OF
These imagine yet. He went home and spent all night writing it down in a letter he sends to a friend. The next day he went to a dual at age 20. This happened a long time ago. It was ohm because he had a passion not just for the dual but in mathematics that we know this stuff today. So that's, there used to be a wall painting of gal with a flat math department because he was also involved in French revolutionary politics. Which may have had something to do with the dual too. Wall painting in evan hall. I just put on on math web page if you're interested.

Let me go over here. Gal with a g-a-l o i-s.

So let me give you one more example from a business point of view. So again there's cost and revenues. So the cost of having a certain number of customers is some function, c-of s. And suppose that's equal to 195 plus 12 s. And suppose the revenue, so obviously this says that there's a base cost, pay the rent, whatever and then each customer cost you some amount it work on but you also make money, that's why you're in business. So the revenue having s-customers is revenue is, let's me get this right, 32 s-you make a lot of money. But if you have too many, extra costs add up. Excuse me. There's some extra costs so eventually the revenue goes down and it's .21 s-squared. So we do the usual thing. Want it figure out the profit of having s-customers, p-of s-and that's going to be the revenues minus the cost. And that I've chosen to be a quadratic. Okay. And so you subtract 12 from 32 is plus 20 s. So the natural question you might, so let's plot that. The parabola opens up downward because there's a negative number in front of s-squared so eventually it has to be down. So the natural question is for which values of s-the number of customers do I make a profit? Okay. So pick picture wise you're asking if this is p-of s-and that's s-where is the profit positive. And it's going to be positive in that gal. So you have to find at that root and that root am call them a-and b. So s-has to be between a-and b-where a-and b-are the zeros. And I won't do the arithmetic. But it's
USING THE QUADRATIC FORMULA TO FIGURE OUT HOW MANY CUSTOMERS UNIT TO HAVE. BUT IN YOU'RE IN BUSINESS YOU DON'T WANT TO JUST HAVE A POSITIVE PROFIT. YOU WANT TO MAXIMUM THE PROFIT. SO WHERE'S THE MAX? THAT'S THE ONE. AND SO WHAT YOU'LL REALLY LIKE KNOW FOR WHICH MAGIC VALUE OF S-DO THE MAXIMUM PROFIT AND FOR THAT I WILL NEED CALCULUS. THAT'S CHAPTER ONE. SO THE GOAL IS TO MAXIMIZE P-OF S-AND FOR THAT I NEED DERIVATIVES WHICH I'LL GET TO SHORTLY. OKAY. SO THAT'S A LITTLE REVIEW OF FINDING ZEROS.

SO LET ME GO ON AND QUICKLY GO ON TO SECTION 0.5. WHICH IS REVIEW OF LAW OF EXPONENTS. AND POWER FUNCTION. AND SO WITH THIS, IF THAT IS AN INTEGER AND N-IS GREATER THAN ZERO THEN OF COURSE WE KNOW THAT D-B, B-TIMES -- NEGATIVE ONE UP THERE THAT MEANS TAKE THE RECIPROCAL. B-TO THE ZERO IS ONE, NOW WE HAVE TO BE A LITTLE BIT CAREFUL ABOUT THAT. WHAT IF B-EQUALS ZERO? THAT'S, WE DON'T DO THAT, OKAY. SO YOU HAVE TO AVOID THAT FORMULA, ZERO TO ZERO DOES NOT MAKE SENSE ANYMORE THAN ZERO DIVIDED BY ZERO DOES. AND HERE B-EQUALS ZERO, ALSO DON'T DO IT. DON'T DIVIDE BY ZERO. SO THESE ARE, I'M GOING DO GIVE YOU THE RULES OF EXPONENTS BUT YOU HAVE TO BE CAREFUL THAT THE NAME IS RIGHT. YOU DON'T DO SOMETHING THAT GIVES YOU NONSENSE. SO THE OTHER THING THAT CAN HAPPEN IS, WELL WHAT B-EQUALS ONE OVER N.

WELL THIS IS THE SAME AS SAYING B-EQUALS R-TO THE N-OR R-IS JUST THE N-TH ROOTS OF B-. THAT'S WHAT IT MEANS. JUST ANOTHER NOTATION FOR TAKING ROOTS. AND YOU HAVE TO BE A LITTLE BIT CAREFUL HERE AM SO WHAT IF B-IS POSITIVE THIS MAKE PERFECT SENSE. YOU CAN TAKE THE ROOTS OF ANY POSITIVE NUMBER. WHAT IF B-'S NEGATIVE? AM I ALWAYS ALLOWED TO DO THIS. WHAT IF B-IS NEGATIVE? ARE THERE ANY RESTRICTIONS ON THE VALUES OF N-I'M ALLOWED TO USE? CAN I GET THE SQUARE ROOT OF MINUS ONE, THAT WOULD BE N-EQUALS TWO AND B-EQUALS MINUS ONE. SO I'M NOT ARE A LOUD TO TAKE SQUARE ROOTS OR ANY OTHER EVEN ROOTS, SO I CAN'T TAKE THE SQUARE ROOT OF A NEGATIVE NUMBER. TAKE THAT, N-HAS TO BE ODD. OTHERWISE IT DOESN'T MAKE SENSE.

STUDENT: B-HAS (INAUDIBLE).
PROFESSOR: SO B-IS LESS THAN ZERO, SO SAY B-IS NEGATIVE ONE AND I KNOW IT TAKE A ROOT OF IT, AM I ALLOWED, I CAN CERTAINLY TAKE THE SQUARE ROOTS. WILL BE SQUARE ROOT OF NECK ONE AND ONLY DOING REAL NUMBER IN CLASS. IF TURN THE CUBED ROOT YOU CAN DO IT. WHAT IS THE CUBED ROOTS OF MINUS ONE? MINUS ONE. WHAT ABOUT THE FOURTH ROOT? I CAN'T DO THAT CAN I? IF THERE WERE NUMBER R-THAT EQUALS NEGATIVE ONE TO THE QUARTER, THEN R-SQUARED WOULD BE, WELL THIS IS THE LAW OF EXPONENTS, R-SQUARED WOULD BE THE SQUARE ROOT OF MINUS ONE BUT I DON'T HAVE THAT. I CAN'T DO IT. SO THIS IS THE LAW OF EXPONENTS WHICH I'LL WRITE DOWN IN A SECOND.

IS THAT TOO FAR DOWN AT THE BOTTOM OF BOARD?

SO AND NOW WHAT IF WE TAKE B-TO A POWER, WHICH IS SOME RATIONAL NUMBER. M DIVIDED BY N. ANY OLD RATIONAL NUMBER AND AND THE WAY WE'RE GOING IT FIND THAT IS FIRST YOU TAKE THE N-ROOT AND MULTIPLY IT BY ITSELF M TIMES. AND THIS IS ONE OF THE LAWS OF EXPONENTS BUT THIS IS THE WAY WE'RE GOING TO DO IT. SO LET'S JUST, IF B-OF ONE OVER N-IS DEFINED, IT HAS TO BE DEFINE THE WAY IT WAS BEFORE. SO LET'S GO AN EXAMPLE. EIGHT TO THE 53RD, THAT MEANS FIRST I TAKE THE CUBED ROOT AND THEN I TAKE THE FIFTH POWER. SO WHAT'S THE CUBED ROOT OF EIGHT? TWO TO THE FIFTH AND TWO TO THE FIFTH IS 32. SO THAT'S THE WAY YOU WOULD USE THOSE SORTS OF RULES. WHAT ABOUT EIGHT TO THE MINUS FIVE THIRDS. THE WAY WE FOUND THAT IS THAT'S JUST ONE OVER EIGHT TO THE FIVE THIRDS BECAUSE WE JUST FIGURED OUT AND THAT'S ONE OVER 32. AND

FINALLY LET ME JUST DO MINUS EIGHT TO THE MINUS FIVE THIRDS.

OKAY. SO THE RULE SAYS YOU PUT THE ONE THIRD ON THE SNIDE AND THEN YOU NEED MINUS FIVE OUT AS THE EXPENT. SO WHAT'S THE CUBE ROOT OF MINUS EIGHT? THAT'S OKAY BECAUSE THREE IS ODD. SO I'M ALLOWED TO DO THAT. SO WHAT'S THE CUBE ROOT OF MINUS EIGHT? MINUS TWO. OKAY. SO NOW I'M GOING TO USE THIS RULE AGAIN. SO THAT'S THE RECIPROCAL OF MINUS TWO TO THE FIFTH POWER. BECAUSE
THAT'S WHEN I HAVE A MINUS SIGN UP THERE I JUST TAKE THE RECIPROCAL. MINUS TWO TO THE FIFTH POWER. IT'S EITHER PLUS THURSD TWO OR MINUS 32. WHICH IS IT? MINUS 32 E-OH, OKAY. OKAY FINE. THAT'S JUST USING ALL THE LAW OF EXPONENTS TO FIND YOUR WAY THROUGH THESE PROBLEMS. I KEEP TALKING ABOUT THESE LAWS OF EXPONENTS. I SHOULD WRITE THEM DOWN.

SO I'VE USED THIS ONE SEVERAL TIMES SO LET ME DO IT. D-B-TO THE R-TIMES B-TO THE S-THAT'S THE SAME THING AS B-TO THE R-PLUS S. SO EXPONENTS. SO WE JUST DO AN EXAMPLE. TWO CUBED TIME TWO TO THE MINUS SEVENTH EQUALS TWO TO THE THREE MINUS SEVEN EQUALS TWO TO THE MINUS FOUR WHICH IS ONE OVER TWO TO THE FOURTH. THAT'S THE NEXT RULE BUT I'LL USE IT ANYWAY. AND THAT'S ONE OVER 16. OKAY. SO THAT'S RULE ONE. SECOND RULE OF EXPONENTS, IF I TAKE NEGATIVE EXPONENT I JUST USE RECIPROCAL. AND I HAVE TO BE A LITTLE CAREFUL. NOT ALLOWED TO DIVIDE BY ZERO. LET ME JUST BE CAREFUL ABOUT THAT. SO FOR EXAMPLE, SEVEN TO THE MINUS TWO IS ONE OVER SEVEN TO THE TWO WHICH IS ONE, 49TH. THIS NEXT ONE IS A CONSEQUENCE OF THE LAST ONE. IF I TAKE B-TO THE R-DIVIDED BY B-TO THE S, THAT'S THE SAME THING AS B-TO THE R-TIMES B-TO THE NEGATIVE S. I DID SUBSTITUTION IN THERE. AND NOW I CAN USE THE FIRST RULE AND JUST SUBTRACT THE EXPONENTS. SO THAT'S ANOTHER ONE. SO IF I DO THREE SQUARED DIVIDED BY THREE CUBED, THAT'S THE SAME THING AS THREE TO THE TWO MINUS THREE, EQUALS THREE TO THE MINUS ONE IS ONE THIRD. OKAY. SO WE'VE GOT A FEW MORE OF THESE RULES.

B-TO THE R-TO THE POWER OF S. B-TO THE R-TIMES S. SO EXAMPLE HERE WOULD BE TWO CUBED TO THE FIFTH POWER EQUALS TWO TO THE THREE TIMES FIVE EQUALS TWO TO THE 15 EQUALS WHATEVER THAT IS, 32,768. IF YOU'RE A COMPUTER SCIENCE YOU KNOW ALL THESE, RIGHT? DON'T WORRY. I'M NOT ASKING YOU TO KNOW ALL OF THESE. WE DO POWERS OF TEN. SO THESE ARE EASY. OKAY. HOW ABOUT A-TIMES B-TO THE POWER OF R, THAT'S THE SAME THING AS A-TO THE R-TIMES B-TO THE R. WE'RE ALLOWED TO DO THAT. SO SIX FACTORS INTO TWO TIME THREE, SO I CAN WRITE THAT AS TWO TO THE R-TIMES THREE TO
THE R. SIX TO THE R. AND NOW I’LL JUST DO THAT SAME RECIPROCAL TRICK. IF I DO A-DIVIDED BY B-THAT’S THE SAME A-TO THE R-OVER B-TO THE R. OKAY. SO THOSE ARE ALL THE RULES WE GET TO USE. AND USE THEM FREELY.

TO NOW EVERYTHING I’VE SAID HERE, TO DEFINED EVERYTHING CAREFULLY. SO FAR, I TOLD YOU EVERYTHING YOU COULD POSSIBLY NEED TO KNOW WHEN YOUR EXPONENT IS IS A RATIONAL NUMBER MUCH EVERYTHING I TALKED ABOUT IS B-TO SOME INTEGER TIMES -- IT ALL MAKES SENSE. BUT YOU COULD ALSO MATHWRITING DOWN THE FOLLOWING. B-TO THE IRRATIONAL NUMBER AM SQUARE ROOT OF TWO. WHAT DOES THAT MEAN? IT HAS A PERFECTLY GOOD MEANING BUT YOU NEED LOGARITHMS AND EXPONENTS. AND EXPONENTIALS TO DO IT. AND WE’LL GET THERE IN CHAPTER FOUR. YOU CAN TAKE BECOME TO ANY REAL POW \YOU WANT\UNIT \{^\}.(-\}{\{-\}) \YOU CAN’T DID HE DPIEN IT BY THIS NICE SIMPLE WAY BY TACK TAKING ROOTS AND STUFF. BUT ALL THESE RULES THAT I WROTE DOWN ARE ARE GOOD FOR RATIONAL OR IRRATIONAL. SAME RULES.

OKAY. JUST TO GET YOUR ATTENTION AGAIN I’M TALK ABOUT MONEY. COMPOUND TRMPLET THIS IS AN EXPONENTIAL FUNCTION IF RAW OUR DAILY LIVES. TALK ABOUT THE NATIONAL DEBT BUT THIS IS MORE PLEASANT. SO SUPPOSE YOU PUT X-DOLLARS INTO A SAVINGS ACCOUNT. OR CD OR SOMETHING LIKE THAT. WITH AN ANNUAL INTEREST RATE OF I-PERCENT. THAT’S A GOOD PERCENTAGE. AND YOU LET’S SAY, I-COULD BE 5 PERCENT FOR EXAMPLE WHICH WOULD MEAN.'05, ACTUALLY THAT NUMBER. SO AND YOU KEEP IT THERE. FOR LET’S SAY N-YEARS. HOW MUCH MONEY DO YOU HAVE? AND TO KEEP IT SIMPLE LET’S ASSUME THAT THE BANK DOESN’T FAIL AND NEED TO BE BAILED OUT AND ALL AT THAT KIND OF STUFF. SO START WITH X-DOLLARS AND AFTER ONE YEAR HOW MUCH ARE YOU GOING DO GET. MULTIPLY BY SOME FACTOR. AFTER ONE YEAR HOW MUCH MONEY DO YOU HAVE. ONE PLUS. ONE POINT ’05, WHAT ABOUT AFTER TEN YEARS? YOU JUST KEEP, EVERY YEAR YOU GET A FACTOR OF THAT. SO THAT’S A FAMILIAR FUNCTION. SO LET’S MAKE IT A LITTLE BIT MORE INTERESTING. SO THE QUESTION THAT YOU CAN ASK IS, SUPPOSE THE INTEREST IS COMPOUNDED MORE THAN ONCE A YEAR. SEW
SUPPOSE IN OTHER WORDS THEY COMPOUND, THIS IS A BANK BEING TERM, QUARTER EARLY, FOUR TIME A YEAR. SO HOW MUCH MONEY DO YOU GET. THAT MEANS AFTER ONE QUARTER, EVERY THREE MONTHS, YOU ONLY GET'S QUARTER OF THE INTEREST. BUT YOU DO THAT FOUR TIME A YEAR. SO AFTER ONE YEAR, YOU'VE DONE AT THAT FOUR TIMES. SO THAT'S HOW MUCH YOU HAVE. AND IF YOU DO IT N-YEARS, YOU DO THAT TO THE N. SO THAT WOULD BE COMPOUNDED QUARTER EARLY AM HOW ABOUT COMPOUNDED DAILY? NOW WE GET X-WHO WANT TO TELL ME? I-DIVIDED BY 365. 365 N. AND HOW ABOUT COMPOUNDED EVERY HOUR? YOU DON'T HAVE TO DO THE ARITHMETIC. BUT WHAT WOULD I BUTT IN THERE? 365 DAYS TIMES 24 HOURS A DAY. OKAY. OKAY. THAT'S. AND SO ON. WE COULD ALSO TALK ABOUT DOING IT EVERY SECOND. SO LET'S SUPPOSE THAT I HAVE A THOUSAND DOLLARS. AND I DO ALL THIS STUFF FOR ONE YEAR. LET'S THINK ABOUT HOW MUCH MONEY I GET. SO LET'S MET WRITE DOWN THE FORMULAS. SO I HAVE QUARTER EARLY. HERE YOU HAVE QUARTERLY AND I'M GOING TO DO ALL FOR ONE YEAR AM THIS IS WHAT THE NUMBERS ARE. THE NEXT IS A THOUSAND BUCKS. SO IF I DO IT QUARTERLY I GET A THOUSAND $50, $15.95 SENT. IF I DO IT THIS OMPG I GET $1,051.27 SENT. LET'S DO IT EVERY HOUR AM I GET $1,051.271 SENT. I DON'T WIN VERY MUCH. LET'S SUPPOSE I DO IT INSTANTANEOUSLY, EVERY MICRO SECOND OR FASTER. IT'S GOING TO APPROACH A LIMIT. AND IT'S NOT GOING TO BE MUCH BIGGER AM IT'S GOING TO BE ABOUT THIS. SO IT TURNS OUT THAT THIS THING APPROACHES A FORMULA, WHICH WE'LL LEARN ABOUT IN CHAPTER FOUR. IT'S GOING TO BE X-TIMES A FUNCTION WHICH IS CALLED EX-AND OF ONE PLUS ONE. X-OF I. SO THERE'S THIS FUNCTION CALLED THE EXPONTENTIAL IN CHAPTER FOUR AND IT TURNS OUT THAT AFTER AWHILE IT DOESN'T MATTER HOW MUCH THEY COMPOUND IT. IT JUST GROWS UNTIL IT STOPS AND THAT'S HOW BIG IT GETS. EXPONENTIAL FUNCTION. COMPOUND INTEREST IS, HAS THIS PROPERTY. ARE THERE ANY QUESTIONS ABOUT THAT?

SO LET ME SAY A LITTLE BIT. SO THAT'S THE ENDS OF SECTION 0.5 IN THE BOOK. SO LET ME JUST SAY A LITTLE BIT MORE ABOUT

A TWO PI R. AND THIS THING IS PI R-SQUARED.


A CYLINDER. I'LL NEED THAT. SO MY CYLINDER IS GOING TO BE
X-from top to bottom and the circle will have radius $R$. So what about its area? So I have three pieces, there's a top circle, bottom circle and then I have flat side. What about the two circles, what's their surface area? Use that from over there. So that's the area of one skill. I need two circles am two. Okay plus how about the (inaudible). So imagination you sort of cut it and laid it out flat. What would it look like? It would be a rectangle, if I just cut it and opened is it. So all I inside to do is think of sides as being a rectangle, lay it out flat. So all I have to know is the length and height of that rectangle. So this distance is $x$. And what's the other distance? Two $\pi R$. And finally the volume? So the general rule is if it look the same no matter where you slice it you take the area the slice and multiply by height. That's how we did it here. So the volume of this thing, it's $x$-by $z$. $x$-times $z$-and then how high is it, it's $y$-. So I multiply all thee together the so take the area of the base and multiply by how high it is. Okay. So there's the kind of formulas we get.

And I suppose I ought to do a sphere. But run out of space. I'll do it right here. A sphere the radius $R$. Who remembers? The surface area? The surface area is four $\pi R$-squared and the volume is four thirds $\pi R$-cubed. For those, so this is a sneak preview for chapter one and for those of you who remember the definition of a derivative. What happens when you differentiate that? You get this. What happens when you differentiate that? You get that. That's not an accident. Okay. So they're connected. But we'll come back to that after we define derivatives.

Okay. So a little word problem to make sure we understand this. So I want to build a box. And there's my box and. Same one as bro. X-in that direction. Y-up and z-in the distance going in and out of board am and build it and I want the bottom to be made out of something strong. Aluminum. At five bucks a square foot. That's a squared foot. And I want to make the
THIS IS PERU VIEW. I INSIDE TO FIND DERIVATIVES IN ORDER TO
SOLVE THAT AT LEAST FOR NATURAL QUESTION.

ONE LAST KIND OF EXAMPLE TO CONNECT WORDS AND PICTURES. SO
SUPPOSE YOU SOMEHOW MANAGE IN YOUR, IS THIS HIGH ENOUGH TO SEE?
MANAGE TO MAKE A PLOT FROM YOUR BUSINESS WHICH SAYS THIS IS THE
NUMBER OF CUSTOMERS, AND HERE YOU CAN ACTUALLY FIGURE OUT WHAT IS
THE PROFIT AS A FUNCTION OF NUMBER OF CUSTOMERS. AND SUPPOSE IT
LOOKS LIKE, LET'S JUST SUPPOSE. SO THE QUESTION I'M GOING TO
ASK NOW IS IF YOU HAVE THIS INFORMATION HOW DO YOU ANSWER CERTAIN
NATURAL BUSINESS QUESTIONS ABOUT IT. IF YOU HAD THIS PICTURE.
SO LET ME ASK A QUESTION. WHAT ARE YOUR FIX COSTS YOU HAVE TO
PAY ANYWAY EVEN IF YOU HAVE NO CUSTOMERS AT ALL. WHERE DO I
LOOK ON THIS PICTURE. I LOOK RIGHT THERE. AND THOSE ARE THE FIX
COST. GOT TO PAY THAT NO MATTER WHAT. THE OTHER NATURAL
QUESTION SUPPOSE I HAVE THIRD CUSTOMERS. WHAT'S MY PROFIT,
SIMPLE QUESTION. I LOOK AT .30, AND I GO UP HERE AND ASK WHAT
IS P-OF 30, THAT'S A PRETTY OBVIOUS THING. SO THE NEXT QUESTION I
COULD ASK IS I ABSOLUTE NEED TO HAVE A PROFIT OF A THOUSAND. HOW

MANY CUSTOMERS DO I NEED? WHAT WOULD I DO WITH THIS GRAPH? I
WOULD DRAW A HORIZONTAL LINE RIGHT AT A THOUSAND. AND I LOOK
WHERE IS X. HIT THAT POINT AND THAT POINT. AND WHATEVER THIS
IS, X-ONE, X-TWO I KNOW THAT P-OF S-TWO IS A THOUSAND. AND I
KNOW P-OF S-ONE IS A THOUSAND. THESE ARE NATURAL QUESTIONS TO
ASK ABOUT IT. AND IF I WANT TO MAKE A PROFIT OF AT LEAST A
THOUSAND, HOW MANY CUSHION MERS DO I NEED? WELL THAT'S OBVIOUSLY
GOING TO BE THIS GRID RIGHT HERE, WHERE THE FUNCTION IS BIGGER
THAN A THOUSAND. SO IT WILL BE BETWEEN S-ONE AND S-TWO. AND THE
MOST NATURAL QUESTION OF ALL IS, HOW DO I MAXIMIZE MY PROFIT.
WHERE IS THAT ON THE PICTURE? HERE IT IS THE MAXIMUM PROFIT AND
THAT'S WHATSOEVER IT IS OVER HERE AND THERE'S THE MAGIC S-THAT YOU
WANT TO FIND TO MAXIMIZE PROFIT. HOW DO WE FIND THAT POINT? THE
ANSWER IS CALCULUS. DERIVATIVES WHICH WE'RE ABOUT TO GET TO. I
JUST WANT IT DRAW PICTURES AND CONNECT IT TO SOMETHING.

AND THAT'S INDEED ALL I HAD TO SAY ABOUT CHAPTER ZERO. ANY
QUESTIONS BEFORE I GO ON?

OKAY. LET ME TELL YOU THE PLAN HERE. THE PLAN IS TO MORAL FOLLOW BOOK. SO RECALL SOME INTERESTING PROPERTIES AND A COUPLE OF NEWS ONES OF THE SLOPE OF A LINE. BECAUSE WE NEED IT UNDERSTAND SLOPE OF A LINE IN ORDER TO UNDERSTAND THE SLOPE OF A CURVE AT A POINT. BECAUSE THAT INDEED, IF WE HAVE THE SLOPE OF A CURVE WHICH IS GEOMETRY THAT IS GOING TO LEAD US TO FIND THE DERIVATIVE. IN SECTION 1.3. AND I'LL DO IT KIND OF BY PICTURES. AND THEN I'LL DO IT AGAIN, THE DERIVATIVE DEFINED CAREFULLY,

USING LIMITS. AND THAT'S THE IDEA WE'RE GOING TO USE TO DEFINE THE SLOPE OF A LINE VERY CAREFULLY. NOW NOT EVERY FUNCTION HAS A DERIVATIVE. SO I NEED TO TALK ABOUT WHEN YOU'RE ACTUALLY ALLOWED TO DO THIS. SO THAT'S THE QUESTION WHEN A FUNCTION IS CONTINUOUS, SORT OF SMOOTH, AND WHEN IT'S DIFFERENTIABLE. WHEN I'M ALLOWED TO COMPUTE A DERIVATIVE. WHEN IT'S REALLY SMOOTH. SO I HAVE SOME RULES ABOUT WHAT I'M ALLOWED TO DO THAT. SPEAKING OF RULES I'LL FINALLY GET RULE FOR COMPUTING DERIVATIVES. THERE ARE A BUNCH OF THEM. NOT ALL IN CHAPTER ONE. MORE IN CHEARP TWO. AND FINALLY THE REST OF THE CHAPTER, 1.7, 1.8 ARE APPLICATIONS OF DERIVATIVES IN VARIOUS PLACES SO. THAT'S GOING TO BE THE GAME PLAN FOR THE NEXT WEEK. WEEK AND A HALF.

SO I WROTE IS THIS DOWN BEFORE. JUST GOING DO WRITE DOWN THE DEFINITION AGAIN AND HOPEFULLY TELL YOU SOME NEW FACTS ABOUT IT. SO THERE'S ANY FAVORITE EQUATION OF A LINE. THAT'S CALL THE SLOPE. THAT'S CALL THE Y-INTERCEPT. AND IF I DRAW THIS PICTURE HERE, THERE'S THE POINTS ZERO COMMA B-THAT'S WHY IT INSECTS THE Y-AXIS. AND IF I GO UP HERE, THE .1 COMMA SOMETHING, SO I'M JUST THINKING X-EQUALS ONE, SO STICK IN X-EQUALS ONE I GET N-PLUS B. SO IT'S PRETTY OBVIOUS NOW IF I DRAW THAT LITTLE TRIANGLE, THIS POINT HERE, THAT DISTANCE, I MOVE OVER BY ONE. AND HOW FAR HAVE I MOVED UP. I'VE GONE FROM B-TO, -- SO THE FIRST PROPERTY, AND I COULD DO THAT ANYWHERE ACTUALLY. SO HERE'S A POINTS. X-COMMA Y. AND M X-PLUS B-SO, THERE'S ANY OLD POINT THAT COULD BE ON THE LINE. I GO UP HERE AND MOVE OVER BY ONE. SO I GO TO
THE POINTS X-PLUS ONE M TIMES X-PLUS ONE, PLUS B. THAT'S THE
SAME AS X-PLUS ONE M X-PLUS B-(ON BOARD). THAT SO IT'S A BLANCH
ONE. AND HOW FAR UP HAVE I MOVED? I'VE GONE FROM M X-PLUS B-TO
M X-PLUS B-PLUS -- THIS IS A REALLY SIMPLE PROPERTY. WHICH IS IT
SAYS IF YOU MOVE, IF YOU INCREASE X-BY ONE, THEN Y-INCREASES BY
M. THAT'S WHAT IT SAYS. IF M IS NEGATIVE IT MEANS WE GO DOWN
OF COURSE. SO IF I HAVE HERE, HERE'S A NICE STRAIGHT LINE?

A Y-EQUALS MINUS TWO X-PLUS ONE. SO HERE'S
THE POINTS ZERO COMMA ONE. AND IF I DRAW THIS
PARTICULAR, LET'S SEE, SO HERE IF I DRAW A, LET ME
TWO X-PLUS, GIVE MYSELF SOME MORE SPACE HERE AM TWO
X-PLUS TWO, SO HERE'S THE .0 COMMA TWO IF I MOVE
OVER BY ONE, THEN I GET TO HERE. ONE COMMA ZERO, IF
I MOVED OVER BY ONE AND MOVED DOWN BY NEGATIVE TWO.
SO IT'S SORT THE SAME IDEA EXCEPT I GO DOWN.

I'LL SAY DECREASES F-M IS LESS THAN ZERO. SO THAT'S A SIMPLE
PROPERTY. (ON BOARD). SO LET ME JUST DRAW ALMOST THE SAME
PICTURES. SO HERE'S GOING TO BE A POINT X-ONE COMMA Y-ONE WHICH
IS RIGHT THERE. AND BECAUSE IT'S THE SAME EQUATION THAT'S GOING
to BE M X-ONE PLUS B. AND LET ME TAKE ANY OTHER OLD POINT WHICH
I'LL CALL X. AND Y. WHICH IS GOING TO BE X-COMMA M X-PLUS B.
THEN LET ME WRITE DOWN THESE TWO EQUATIONS. I WANT TO FIND A
DIFFERENT WAY OF WRITING DOWN AN EQUIVALENT EQUATION OF A LINE.
JUST GOING TO REORGANIZE THIS THING IN A SLIGHTLY DIFFERENT WAY.
WRITE DOWN Y-EQUALS M X-PLUS B. AND Y-ONE EQUALS M X-ONE PLUS B.

SO TWO TRUE EQUATIONS FOR EACH POINT ON AND THE LINE. AND NOW I
WANT IT TAKE THESE TWO EQUATIONS AND SUBTRACT ONE FROM THE OTHER.
ANYONE REMEMBER HOW TO DO THAT. BY TWO EQUATIONS I CAN ADD THEM
AND GET A NEW EQUATION OR SUBTRACT THEM AND GET A NEW EQUATION.
SO IF I SUBTRACT THIS ONE FROM THAT WHAT DO I GET. SUBTRACT THIS
EQUATION FROM THAT EQUATION. SO I GET Y-MINUS Y-ONE OVER THERE.
AND HERE I GET THE TOP MINUS THE BOTTOM. THAT'S STILL AN
EQUATION. BUT I CAN SIMPLIFY THAT A LITTLE BIT. THE B’S CANCEL. AND THE M FACTORS OUT. AND I GET THIS. FORMULA WHICH IS ALSO TRUE OF THE LINE. AND THIS HAS A NAME. THIS IS CALLED THE POINT SLOPE EQUATION OF A LINE. WHY IS THAT? BECAUSE TO BE ABLE IT WRITE THIS DOWN YOU HAVE TO KNOW THE SLOPE OBVIOUSLY. SO IN I KNOW THAT AND THE SLOPE I CAN WRITE DOWN THAT EQUATION. JUST A DIFFERENT WAY OF WRITING DOWN THE SAME EQUATION. (ON BOARD). AND FOR SOME REASON THE BOOK, THIS IS CALLED PROPERTY THREE. I’D LIKE TO DO THREE BEFORE TWO FOR THIS. JUST A DIFFERENT WAY OF THE WRITING DOWN THE EQUATION OF A LINE.

NOW IF I GET THAT EQUATION, (ON BOARD) AND I DIVIDE BOTH SIDES BY X-MINUS X-ONE, I’LL DO THAT, I GET THAT. THAT’S PERFECTLY EQUIVALENT. AND IF I LOOK AND SEE WHAT THAT MEANS, LET ME AGAIN DRAW MY LINE. SO HERE’S MY POINT, X-ONE COMMA Y-ONE?

A HERE’S MY OTHER POINTS, X-COMMA Y-. LET ME DRAW THE SAME TRIANGLE I’VE BEEN DRAWING ALL ALONG. X-MINUS X-ONE, THAT’S HOW LONG THE BASE IS. AND HOW LONG IS THIS SIDE? Y-MINUS Y-ONE. SO WHAT THIS IS SAYING IS THAT THE RATIO OF THAT SIDE TO THAT SIDE IS THE SLOPE M. SAME AS IT HAS BEEN BEFORE. THIS DOESN’T HAVE TO BE ONE. YOU ALWAYS GET THE SAME RATIO. AND THE BOOK THINKS WE CALL THAT PROPERTY TWO. OKAY.

NEXT QUESTION TO ASK IS SUPPOSE I HAVE TWO LINES. Y-EQUALS M ONE X-PLUS B-ONE. AND Y-EQUALS M TWO X-PLUS B2. TWO NATURAL QUESTIONS TO ASK ABOUT TWO LINES ARE WHEN ARE THEY PARALLEL. AND WHEN ARE THEY PERPENDICULAR. TWO PERFECTLY REASONABLE QUESTIONS. TWO LINES, LET’S DO THE PARALLEL ONE FIRST. IF AND ONLY IF, THAT MEANS IT HAPPENS EXACTLY WHEN SOME VERY SIMPLE PROPERTY IS TRUE. WHAT HAS TO BE TRUE ABOUT, THEY HAVE TO HAVE THE SAME SLOPE. SO THERE’S ONE LINE. THERE’S ANOTHER LINE. IF THEY’RE GOING TO BE PARALLEL, I THINK YOUR GEOMETRIC INTUITION SAYS YOU BETTER HAVE THE SAME SLOPE OTHERWISE THEY’RE GOING TO INTERSECT SOME DAY. ACTUALLY I WILL, WHY NOT, I’LL PROVE IT. TAKE TWO
LINES. SO LET ME JUST PROVE THAT PROPERTY. SO I'LL SORT OF DO IT, SO WHEN SOMEBODY WRITE, WHEN A MATHEMATICIAN WRITES IF AND ONLY IF THAT MEANS YOU HAVE TO DO TWO THING. IF THEY'RE PARALLEL THEN, M ONE EQUALS M TWO. BUT YOU ALSO HAVE TO SHOW IF M ONE EQUALS M TWO, THAT THEY'RE PARALLEL. THAT MEANS THEIR EQUIVALENT. I'M GOING TO SHOW THAT IF THEY'RE PARALLEL, TAKE PARALLEL LINES AND I'M GOING TO SHOW THAT M ONE EQUALS M TWO. SO IF THEY'RE PARALLEL, IF THOSE TWO LINES ARE PARALLEL, THAT MEANS THAT THEY, WHAT DON'T THEY DO? THEY NEVER INSECT. SO THAT'S YOUR GEOMETRY. SO NOW WE HAVE TO MOVE TO ALGEBRA WHAT CAN YOU SAY ABOUT THOSE TWO EQUATIONS? THAT MEANS THAT I CAN'T SOLVE FOR FOR A PLACE WHERE THEY INTERSECT THE SO WHAT DOES THAT MEAN? I CAN'T FIND A PLACE WHERE THIS ONE EQUALS THE OTHER ONE. THAT'S WHAT IT MEANS IF THEY DON'T INTERSECT, I CAN'T GET THEM TO YOS. IN THEY CROSS THERE WOULD BE SOMEPLACE THIS HAPPENS. SO THAT MEANS THAT I CAN'T SOLVE, LET ME DO IT M ONE X-MINUS M TWO X-EQUALS B TWO B-ONE. SO THAT MEANS I CAN'T SOLVE X-TIMES M ONE MINUS M TWO EQUALS B TWO MINUS B-ONE. I CAN'T SOLVE THAT ARE TO X-. HOW CAN YOU POSSIBLY NOT BE ABLE IT SOLVE THAT FOR X. WHAT'S WRONG WHERE JUST DIVIDING. WHAT'S WRONG WITH. SO I SOLVED FOR IT? CAN'T DO THAT. WHAT'S THE ONLY WAY -- WHAT AM I NOT ALLOWED TO DIVIDE ABOUT? ZERO. OKAY. SO I CAN'T DIVIDE BY THAT SO THAT MEANS M ONE EQUALS M TWO. ADD M TWO TO BOTH SIDES. THAT'S MOVING FROM RIGHT SIMPLE GEOMETRIC BACK THROUGHOUT ALGEBRA. AND ONE LAST PROPERTY. I THINK THE BOOK CHOSE TO CALL THIS PROPERTY FOUR. IF YOU WANT TO FOLLOW IT.

HERE ONE MORE QUESTION I SAID IS NATURAL IT ASK ABOUT TWO LINE. OF THE FIRST ONE IS WHETHER THEY'RE PARALLEL AND THE SECOND ONE IS WHEN THEY'RE PERPENDICULAR. (ON BOARD). AGAIN THIS IS, HAPPENS EXACTLY WHEN, IF AND ONLY IF SOME SIMPLE PROPERTY AGAIN IS TRUE OF THE SLOPES. SO LET ME WRITE IT THIS ONE, M ONE TIMES M TWO IS EQUAL TO MINUS ONE. (ON BOARD). THAT MEANS ONE IS NEGATIVE RECIPROCAL. MAYBE SEE IT THIS WAY. (ON BOARD). EITHER WAY. SO PICTURE, SO HERE IS Y-EQUALS TWO X. AND
SO THIS WOULD HAVE TO BE Y-EQUALS MINUS ONE-HALF X-PLUS, THAT
WOULD BE A RIGHT ANGLE. NINETY DEGREES, RIGHT ANGLE.

90 DEGREES. SO HOW MANY PEOPLE KNOW HOW TO PROVE THIS ONE? SO
I'M JUST WONG DECKER IF YOU LOOK. I THINK PEOPLE LEARN THIS IN
HIGH SCHOOL. SO DID YOU LEARN WHY IT WAS TRUE IN HIGH SCHOOL? I
DIDN'T THINK SO. LET ME TAKE A FEW LINES AND SHOW YOU WHY THIS
IS TRUE. BUT I NEED ONE MORE FACT FROM HIGH SCHOOL TO DO THAT.
WE HAVE IT REMEMBER WHAT SIMILAR TRIANGLES ARE.

SO RECALL SIMILAR TRIANGLES SO. LET ME DRAW TWO SOMewhat
SIMILAR TRIANGLES. AND TRY IT MAKE THEM LOOK SIMILAR AM WHAT
DOES IT MEAN FOR TWO TRIANGLES TO BE SIMILAR? WHAT HAS TO BE
TRUE BEFORE ALL THESE ANGLES. THEY'RE ALL THE SAY. THETA ONE
THETA TWO THETA THREE. ALL THE SAME. THAT'S HAD A A SIMILAR
TRIANGLE IS. SO LET'S SEE, SUPPOSE THAT THE LENGTHS OF THE
SIDES, I'LL CALL IT L ONE. AND CALL THIS L TWO BECAUSE IT DROPS
AT AN ASSISTANCE. AND LABEL SIDES HERE S-ONE S-TWO S-THREE. SO
WHAT DO WE, SUPPOSE THESE TWO TRIANGLES RADIO SIMILAR. SO ALL
THESE THREE ANGLES MAX. SOMETHING HAS TO BE TRUEM THEY HAVE TO
BE PROPORTIONAL TO ONE ANOTHER. SO THAT MEANS IF L THREE IS
TWICE AS BIG AS S-THREE, THEN THAT HAS TO BE TWICE AS BIG AS
THAT. AND THAT HAS TO BE TWICE AS BIG. THAT'S WHAT SIMILAR
TRIANGLES ARE. IF I TAKE THIGHS RATIOS THEY ALL HAVE TO BE THE
SAME. SO THAT'S THE GEOMETRY I NEED FROM HIGH SCHOOL TOO PROVE
THIS FACTOR. SO LET JUST TRY IT NOW.

SO THERE ARE A LOT OF DIFFERENT PROOFS FOR THIS. SO HERE I'M

GOING TO DRAW MY TWO LINES. AND I'M GOING IT ASSUME THEY'RE
PERPENDICULAR. THAT MEANS '93 ANGLE. SHOW THAT THE SLOAPS
HAVE THAT PROPERTY. SO HERE'S HOW IT GOES. LET ME DRAW A
HORIZONTAL LINE. RIGHT THROUGH. AND LET ME NOW GIVE THIS ANGLE
A NAME. I'M GOING TO CALL IT THETA ONE. AND I'LL GIVE THIS
ANGLE A NAME. CALL IT THETA TWO. I DON'T KNOW WHAT THEY ARE.
AND NOW LET ME DRAW A TRIANGLE. A RIGHT TRIANGLE. AND I'LL MAKE
This length, let me, let’s say this is line Y. And here’s line two. So that’s $y = m_2 x - (on board)$. So now I’m going to make that length one. What’s this length? If this is line one? That divided by that is ... so one of my property one of a line, if I draw this triangle here and this is a line slope $m_1$ and I go over one in that direction how far up do I get in this direction. $m_1$. Let me do the same thing over here. Go over one. How far is that? $m_2$. $m_2$ is negative. Sloping down. Going down it goes down by $m_2$. So here I have two triangles. I claim that those are similar triangles. Let’s see if we can figure that out. I claim. What do you have to do to prove they’re similar? I have to prove all three angles are auto the same. So here’s the first trick. So this is a straight line. Here’s are three angles. $\theta_1$, 90 middle. What do those have to add up to. 180. Now I don’t know what angle this is. I’ll just call it $\theta_1$. What do these three angles add up to. All the angle of a triangle always add up to 180. So I get $\theta_1$. This angle plus 90, plus this angle equals 180.

That, so this comes from straight line. This comes from adding up the three angles in the triangle. So can you tell me anything about $\theta_1$ and $\theta_2$? In $\theta_2$ satisfies this and these are the same except $\theta_2$, so what has to be true about $\theta_1$ and $\theta_2$? They’re equal. Only possibility. Let me call that $\theta_2$. Let me do the same trick on this triangle. I’ll just call it $\theta_1$. And that’s a 90-degree angle?

A. Add up those three things. I get $\theta_1$, plus 90, plus $\theta_2$ equals, that’s a triangle, add up to 180. That plus that plus that. So let’s look at those two equations. What does this theta have to be? Has to be both, theta equals theta one. Maybe I should call this one, let me call it theta hat. So this guy here is theta one. Theta one, theta two ‘93, theta one theta 290 degrees. Those are two similar triangles. So these are similar.
TWO SIMILAR TRIANGLES. SO NOW WHAT DO I KNOW ABOUT SIMILAR TRIANGLES? THEIR SIDES ARE PROPORTIONAL TO ONE another. SO TWO TRIANGLES ARE SIMILAR. SO LET’S JUST TAKE THE RATIO. SO WHAT DO I HAVE TO DO? LET ME TAKE THE RATIO OF THE SIDE OPPOSITE THETA ONE. SO THE SIDE OPPOSITE THETA ONE HERE AND THE SIDE OPPOSITE THETA ONE THERE. TAKE THEIR RATIO. SO THE RATIO OF SIDES OPPOSITE THETA ONE, THAT’S M ONE DIVIDED BY ONE. M ONE IS THE SIDE OPPOSITE OF


SO THIS ANGLE IS THETA ONE. HOW BIG IS THAT ANGLE? FROM THERE TO THERE? IT’S THETA ONE PLUS 90. ALL RIGHT. SO THAT’S, AND SO NOW IF I TAKE M ONE DIVIDED BY ONE, THAT’S THE OPPOSITE SIDE FROM THIS ANGLE DIVIDED BY THE ADJACENT SIDE. HOW DO I WRITE THAT AS TRIGONOMETRIC FUNCTION OF A ANGLE. IT’S AN N-.
OPPOSITE OVER ADJACENT, THAT'S TANGENT. NOW THIS GUY OVER HERE,
THE OTHER SLOPE IS A TANGENT OF THETA ONE PLUS 90 DEGREES. THAT
GETS ME THE OTHER ONE. THAT SAYS THAT M TWO IS A TANGENT OF
THETA ONE PLUS 90. SO NOW WHAT DO I HAVE TO DO? I WANT TO SHOW
THAT M ONE TIMES M TWO IS MINUS ONE. SO PLUG IN I NEED TO SHOW
THE TANGENT OF THETA TIMES THE TANGENT OF THETA ONE PLUS 90 IS
MINUS ONE. THAT'S THE TRIGONOMETRIC IDENTITY FROM HIGH SCHOOL.
THIS IS IT. SO THIS IS TRUE, IS A TRIG IDENTITY. WHICH I WON'T
ASSUME YOU KNOW BUT THAT'S THE OTHER WAY TO DO IT IN ONE LINE.
SEE YOU NEXT WEEK.