
PROFESSOR: WOULD YOU LIKE TO HEAR ABOUT THE MIDTERM? I'M READY TO TALK ABOUT IT IF YOU ARE. SO WHAT I WANT TO DO TODAY IS TALK ABOUT THE MID-TERM. FOR SOME OF YOU IT MIGHT BE THE FIRST MIDTERM OF. IT'S AN INTERESTING EXPERIENCE. TALK ABOUT RULES AND THEN ACTUALLY REVIEW FOR IT. BUT THE FIRST RULE IS BRING A PHOTO ID. BECAUSE TO TURN YOUR EXAM IN YOUR TEACHING ASSISTANT WILL EITHER RECOGNIZE YOU OR ASK FOR THIS. ONE OR THE OTHER. BUT BRING YOUR PHOTO ID IN. I WANT TO TALK ABOUT THE OTHER RULES. AND JUST TWO MINUTES ON THIS. YOU'LL BE SURPRISED WHEN YOU GET HERE ON THURSDAY. AND THERE ARE ALL ON THE WEB PAGE TOO IN SOME DETAIL. SO CLOSED BOOK. AND I'LL WRITE IT THIS WAY. CLOSED ALMOST EVERYTHING. CLOSED BOOK.

NOTES, COMPUTER. NETWORK. PHONE, ET CETERA BUT IT'S OPEN BRAIN OF COURSE. SO THERE WILL BE THREE VERSIONS OF THE TEST. SO
ALWAYS BE SITTING NEXT TO SOMEBODY WITH A DIFFERENT VERSION.

YOU GET FOUR POINTS FOR THE FOLLOWING. MAKE SURE YOU DO IT.

YOUR NAME. YOUR TA'S NAME. THE NEIGHBOR TO YOUR LEFT. AND

THE NEIGHBOR TO YOUR RIGHT. SO WHEN YOU SIT DOWN YOU'LL GET THE

FRONT PAGE THAT LOOKS LIKE THIS. AND YOU'LL FILL IN ALL

STUFF JUST TO MAKE SURE YOU DO A ARE ALL THAT. AND IF YOU

HAPPEN TO BE SITTING NEXT TO THE I'LL OR THE WALL, THAT'S OKAY. BUT

OTHERWISE THE NAME OF THE PERSON. IF YOU WANT TO GET

CREDIT, PARTIAL OR FULL CREDIT, PLEASE SHOW YOUR WORK. AND AN

EXAMPLE OF HOW MUCH WORK YOU HAVE TO SHOW, WELL, I'LL DO SOME OF THOSE

PROBLEMS TODAY FOR YOU BUT THERE ARE ALL IN THE IS SAMPLE

MIDTERMS ON THE WEB PAGE. IF YOU LOOK AT THAT, THAT'S HOW

WORK WE EXPECT YOU TO SHOW. TRUE, FALSE IS JUST TRUE, FALSE.

MAY ONLY USE TECHNIQUES THAT WE'VE TAUGHT. SO JUST THREE

SECTION 1.6. THAT'S WHAT I MIDTERM COVERS. SOME OF YOU HAVE HAD

CALCULUS BEFORE. YOU'RE FREE TO USE THOSE OTHER IDEAS TO
CHECK YOUR ANSWERS BUT YOU ACTUALLY HAVE TO SOLVE THEM USING THE TECHNIQUE WE'VE TAUGHT SO EVERYONE'S ON THE SAME PLAYING FIELD.

OKAY. IS THAT, ANYBODY HAVE ANY QUESTIONS. ALL THIS WILL BE WRITTEN DOWN. YOU GET TO HEAR IT AGAIN ON THURSDAY. I'D TELL YOU SO NO SURPRISES. AGAIN, PLEASE BRING YOUR ID.

SO THE OTHER INTERESTING NEWS THAT THERE ARE NOW COMPLETE TRANSCRIPT OF EVERY WORD I'VE SAID, EVERY BAD JOKE ON THE WEB PAGE FOR EVERY LECTURE. NOW, AND WE CAN THANK RHONDA RYAN SITTING IN THE FRONT ROW. NOW, WHAT ARE THERE IS EVERY WORD I'VE SAID MORE OR LESS BUT NOT EVERYTHING THAT I'VE NECESSARILY WRITTEN ON THE BOARD OR EVERY PICTURE I'VE DRAWN. SO YOU'RE NOT LEFT OFF THE HOOK FROM TAKING NOTES. BUT THESE MIGHT BE USEFUL. BUT HOPEFULLY IN THE FUTURE NOW THAT WE REALIZE THIS RESOURCE IS AVAILABLE I MIGHT POST THEM -- ON THE WEB PAGE. IT'S LOOK FOR ANNOUNCEMENTS. YOU'LL SEE IT THERE. SO ALL THESE RULES ARE THERE. EVERYTHING IS ON THE WEB PAGE. I TRY TO
KEEP

THAT UP-TO-DATE. WHAT ELSE DID I WANT TO SAY? SO HOW TO
PREPARE. OKAY. LET'S GET TO THE EXAM.

3

AND THIS IS GENERIC ADVICE THAT YOU PROBABLY HEARD MANY
TIME
BEFORE IN MANY OTHER CLASSES. BUT I WILL WRITE IT DOWN
AGAIN.

SO THE FIRST ONE IS, WRITE A SHORT, MAYBE ONE, TWO PAGE
SUMMARY
OF THE MOST IMPORTANT IDEAS, DEFINITIONS, THEOREMS,
EXAMPLES,
WHATEVER IT IS THAT YOU THINK IS IMPORTANT FOR TO YOU
UNDERSTAND
IT, RULES, ... WHATEVER YOU CALL THEM TO BE COVERED. BY THE
ACT OF
DECIDING WHAT'S IMPORTANT AND WRITING THEM DOWN, ON A LITTLE
BIT
OF PAPER, MEANS YOU'LL REMEMBER IT. SO THAT'S A PRETTY
OBVIOUS
THING TO DO. AND SECOND PART IS DUE PRACTICE TESTS. ON THE
WEB
PAGE. SO ANY QUESTIONS? WHAT I'M GOING TO DO NOW IS GIVE
YOU MY
VERSION OF, MAYBE NOT ONE OR TWO PAGE, THE MOST IMPORTANT
FROM THE BEGINNING OF CLASS. I’LL SKIP CHAPTER ZERO BUT

MOST IMPORTANT IDEAS FROM CHAPTER ONE BEING SECTIONS 1.13,

1.6.

STUDENT: ARE THE PRACTICES TESTS ON-LINE AN ACCURATE

REPRESENTATION OF THE LENGTH AND FORMAT.

PROFESSOR: YES. SO THE MOST RECENT ONE I'M, WHICH WAS

GIVEN

LAST YEAR I'M QUITE SURE WAS 80 MINUTES AND WE HAVE 80

MINUTES.

THE EARLIER ONES I'M NOT QUITE SURE. BUT THE MOST RECENT

ONE

WAS.

STUDENT: (INAUDIBLE).

PROFESSOR: THERE ARE USING THE RULES, SO IF YOU LOOK AT THE

PROBLEM

MIDTERM THAT'S POSTED ON THE WEB PAGE IS SAYS SOLVE THIS

RULE.

USING RULINGS THAT YOU KNOW. GO STEP-BY-STEP USING THE

THINK OF THAT AS A PROOF, IT'S A PROOF. THINK OF THE RULES 4

STEP-BY-STEP.

STUDENT: SCRATCH PAPER.

PROFESSOR: IF YOU NEED SCRATCH PAPER, ASK US, WE HAVE IT.

PLUS

A STAPLER, ET CETERA. AND ONE MORE THING, WHEN YOU'RE
FINISHED I WILL ASK YOU TO LINE UP AND HANDS IN YOUR EXAM TO YOUR TA.

TO PREVENT TRAFFIC CHAOS I WILL DRAW A LITTLE PICTURE ON THE

BOARDS THAT SAY DANIEL'S STUDENTS SIT HERE ET CETERA. (INAUDIBLE)

QUESTION. AS LONG AS IT'S LEGIBLE. NO BLUE BOOKS. YOU'LL DO

ALL YOUR WRITING ON THE PIECES OF PAPER WE HANDOUT.

STUDENT: CAN YOU USE YOUR OWN SCRATCH PAPER.

PROFESSOR: NO WE WILL SUPPLY SCRATCH PAPER. JUST --

ANYTHING ELSE?

ALL RIGHT. SO HERE'S MY PERSONAL SUMMARY OF THE MOST

IMPORTANT IDEAS. BUT AS I SAID IT'S YOU GOING THROUGH THE

BOOK AND WRITING DOWN YOUR OWN VERSIONS THAT WILL MAKE YOU LEARN.

I'M GOING TO ASSUME CHAPTER ZERO IS TAKEN FOR GRANTED. YOU

KNOW ALL THOSE ALGEBRAIC RULES. SO HERE'S SECTION 1.1, IN MY MINDS

STRAIGHT THINGS THAT ARE IMPORTANT IN THAT SECTION. SLOPES OF

LINE. REVIEWING WHAT IT IS, STRAIGHT LINE. SO HERE'S A

PICTURE OF A STRAIGHT LINE. AND THERE'S B. AND I'LL DRAW A

LITTLE TRIANGLE THERE WITH BASIC ONE AND TYPE M. HIT ANY

POINTS ON IT WHICH I'LL CALL X-ONE COMMA Y-ONE. SO HOW
MANY DIFFERENT WAY, SO B-IS CALLED Y-INTERCEPT. M IS THE SLOPE.

HOW MANY WAYS CAN YOU WRITING DOWN THE EQUATION OF A LINE.

THERE'S THAT WAY AND ALSO Y-MINUS Y-ONE EQUALS M (ON BOARD). AND OR

IF YOU LIKE, M IS Y-MINUS Y-ONE OVER X-MINUS X ONE. SO ALL THESE WE'VE GONE ON SO I DON'T WANT TO TEACH IT AGAIN. SO I'M JUST SUMMARIZING WHAT I THINK IS MOST IMPORTANT. AND WHAT ELSE?

TWO LINES WITH SLOPES M ONE, AND M TWO ARE PARALLEL IF THEIR SLOPES ARE EQUAL, AND THEY'RE PERPENDICULAR IF M ONE TIMES M TWO IS NEGATIVE ONE. AND THAT'S ALMOST EVERYTHING FROM SECTION 1.1. EXCEPT A BIT OF TERMINOLOGY WHICH IS IT THAT THE WORD SLOPE IS ALSO SYNONOMOUS WITH RATE OF CHANGE. THERE'S NOT, SO FOR WORD PROBLEMS THAT'S IMPORTANT, MATHEMATICAL IDEAS. IMPORTANT FOR UNDERSTANDING WORD. THAT'S SECTION 1.1.

QUESTIONS? HOW ABOUT SLOPES OF CURVES. DOES THAT GO
TRANSCRIPT TOO. SO WE'VE DRAWN A PICTURE OF A CURVE GIVEN BY
Y-EQUALS F-OF X. AND I'VE DRAWN THE TANGENT LINE. AND THIS PARTICULAR POINT THERE I'LL CALL IT X-COMMA Y-. MAYBE I'LL GIVE IT A NAME P-. TANGENT LINE AT POINTS P. SO THERE'S
THE PICTURE. AND SO WHAT IS A PROPERTY OF A TANGENT LINE. IT INTERSECTS THE CURVE EXACTLY ONCE NEAR THE POINT AND THERE'S ONLY ONE WAY TO DO IT. ONLY ONE. AND THAT MEANS THAT THERE'S NO CORNERS. AND THE PICTURE, IF YOU HAVE, THERE'S A CURVE THE ABSOLUTE VALUE FUNCTION. THERE'S MORE THAN ONE WAY TO DRAW A LINE THROUGH ONE POINT THAT ONLY AND INTERSECTS ONCE. SINCE THERE MORE THAN ONE THERE'S NO TANGENT THERE. THERE'S EXACTLY ONE WAY TO DO IT. THE WAY IT IS THERE. SO THEN THE OTHER IDEAS WAS THAT THE SLOPE OF THE CURVE AT THE POINTS P-WAS DEFINED TO BE THE SLOPE OF ITS TANGENT LINE. SO IF THIS HAS SLOPE M THEN THE SLOPE OF CURVE IS 6 M. AND THAT'S PRETTY MUCH THE WHOLE SUMMARY OF SECTION 1.2.

SO THE OTHER THING, SO COMBINING WITH A WE KNOW ABOUT 1.1
AND 1.2 WE CAN WRITE DOWN THE TANGENT LINE TO THE CURVE Y-
EQUALS
F-OF X-AT A PARTICULAR POINT, LET'S CALL IT A-COMMA F-OF A,
IS SO
WE KNOW IT GOES TO THAT POINT, AND I KNOW THE SLOPE, SO
THERE IS
THE EQUATION OF THE TANGENT LINE. AND IT'S EXACTLY THIS ONE
SLOPE
RIGHT THERE. EXCEPT X-ONE IS A. Y-ONE IS F-OF A-AND THE
IS F-PRIME. SO THAT GIVES YOU THE EQUATION. AND AND THE
OTHER
IMPORTANT THING IS THAT THE WAY WE CAN COMPUTE THE
DERIVATIVE,
THE SO THERE'S THE POINT AT WHICH I WANT THE SLOPE OF
TANGENT
7
LINE. I PICK ANOTHER POINT. DRAW A STRAIGHT LINE THROUGH
THEM. THIS IS CALLED THE SECANT LINE. SO THIS POINT HERE,
PLUS H.
THERE'S X, THERE I'VE MOVED OVER A LITTLE BIT, CALL IT X-
IS IS
SO THAT DISTANCE FROM THERE TO THERE IS H. THIS POINT HERE
THIS
F-OF X. THAT HEIGHT UP THERE, THAT'S F-OF X-PLUS H. SO
TAKING
DISTANCE IS F-OF X-PLUS H-MINUS F-OF X. AND AS I KEEP
TO
THIS POINT AND SLIDE IT DOWN THE CURVE UNTIL IT GETS CLOSE
That then secant line approach tangent line. -- As this point slides down the curve of that slope of the secant line, that's another way of defining the derivative. That is the careful definition. So in this thing here is the slope of secant line. And so what does the limit mean? It means that this expression gets arbitrarily close to whatever it's supposed to approach which is f-prime of x-if h-gets arbitrarily small but is never actually gets to zero. So that was the definition. Because if h-were zero I won't is have two points. I couldn't draw a secant line. Arbitrarily close. So that's section 1.3. I hope this is review.

Okay so 1.4. That's where we did limits a little bit more carefully. Up there I wrote it down in English but in this section talk about it a little bit more. So we talked about what does it mean when we write down something like this. That moons, I'll just write it down again. The book repeats itself so this is going to repeat., g-of x-gets as close as you like
ARBITRARILY
CLOSE TO THE VALUE $L$, THE NUMBER $L$, AS $X$-GETS ARBITRARILY
CLOSE

TO A-BUT IT NEVER ACTUALLY GETS THERE. $X$-DOES NOT EQUAL $A$.

SO

THIS, I THINK IS THE FIRST, IS A PLACE WHERE I HAVE SOME
PICTURES, SOME EXAMPLES THAT MAKE IT EASY TO UNDERSTAND WHEN
TO
ELIMINATE THIS. SOMETIMES PICTURE HELP YOU REMEMBER MORE
THAN
ANYTHING ELSE. SO THE STANDARD PICTURE WOULD BE, HERE A
NICE
SMOOTH CURVE. THERE'S $A$. THERE'S $L$. AND THAT'S WHAT THE
GRAPH
LOOKS LIKE, THEN AS YOU GET CLOSER TO $A$-THE VALUE GETS
CLOSER TO
$L$. SO THAT'S THE STANDARD PICTURE. SO THERE THE LIMIT
EXISTS.

BUT HERE'S SOME OTHER EXAMPLES. THERE'S $A$. THERE $L$. SO

THE
FUNCTION HAS A DIFFERENT VALUE RIGHT AT THAT POINT. THAT'S

OKAY.

AS LONG AS YOU GET ARBITRARILY CLOSE YOU'RE STILL CLOSE TO

AND HERE'S ONE MORE EXAMPLE. THE LIMIT EXIST AGAIN, EQUALS

$L$ AND

THE FUNCTION ISN'T EVEN DEFINED. HAS NO VALUE WHATSOEVER.
WE

DON'T CARE BECAUSE X DOES NOT EQUAL A IN THE DEFINITION LIMIT.

SO THESE THREE PICTURES HELP YOU REMEMBER WHAT THIS DEFINITION IS.

IT ALSO HELPS TO HAVE A FEW PICTURES WHERE THE LIMIT DOES NOT EXIST. SO HERE'S ONE. HERE'S THE POINT A. AND THIS, THE LIMIT DOES EXIST BECAUSE COME FROM THE LEFT YOU GET CLOSE TO ANOTHER. COME FROM THE RIGHT YOU GET CLOSE TO ANOTHER ONE. THERE'S NO ONE NUMBER YOU GET CLOSE TO. HERE'S ANOTHER EXAMPLE.

SOMETHING LIKE A HYPERBOLA. YOU DON'T GET CLOSE TO ANY FINITE NUMBER. ONE SIDE YOU GET PLUS INFINITY. OTHER SIDE NEGATIVE -- AND HERE'S ONE MORE EXAMPLE WHICH IS HARD TO DRAW BUT, WIGGLES UP AND DOWN. INFINITELY OFTEN AS YOU GET CLOSE TO A. IN NONE OF THESE CASES THE LIMIT DOES NOT EXIST. SO THESE ARE THE THREE PICTURES TO KEEP IN MIND.
The other thing this section had was rules you can so use.

Theorem. You have limit it evaluate. And what rules do you get it use to evaluate them? Well here they are. Suppose think two limits exist. Then these rules are a going did you can bill more complicated functions that exist. So here's they are. The first rule says you take something you know and multiply by a constant. Factor out constant. Multiply by the limit. Next one says take two functions and add them our subtract or multiply them. Your choice. And take the limit.

And all you have to do is take the limit separately and then add, subtract or multiply. Okay. And I won't write it down, division work too if you don't divide by zero. So any of those four operations are perfectly fine. And that's the ends of section 1.4.

So I wrote these out a little bit longer hands. Just writing these out shorthand. When I talked about them first they were longer.
NEXT SECTION, 1.5 IS HOW CAN YOU TELL IF SOMEBODY HANDS YOU A FUNCTION IF YOU CAN DIFFERENTIATE IT. THAT'S CALL DIFFERENTIABILITY. AND A LATER IDEA, CALLED CONTINUITY.

SO DEFINITION. \( f(x) \) IS CONTINUOUS AT A POINT \( a \) IF YOU TAKE THE LIMIT AS \( x \) APPROACHES \( a \) OF \( f(x) \) AND THAT EXIST, FURTHERMORE IT JUST EQUALS \( f(a) \). AND THE PICTURE IS THE MOST SIMPLE WAY.

HAVE A NICE SMOOTH CURVE THAT YOU CAN DRAW WITHOUT LIFTING YOUR CHALK FROM THE BOARD. THAT'S THE SIMPLEST. NO HOLD. SO THAT'S THE DEFINITION. SO THIS IS CONTINUOUS. THERE'S A PICTURE. AND HERE'S SOME EXAMPLES THAT ARE NOT CONTINUOUS OR DISCONTINUOUS. AND THEY'RE BASICALLY THE SAME THREE OVER THERE.

SO THIS IS NOT CONTINUOUS, THE SAME THREE PICTURE WHICH I WON'T WRITE DOWN AGAIN.

STUDENT: WHAT ABOUT THE TOP TWO.

PROFESSOR: SO THESE, OKAY, THESE ARE ALSO DISCONTINUOUS.

SAME THREE AS BEFORE AS WELL AS THESE THREE. RIGHT BECAUSE HERE
A-DOESN'T EVEN EXIST. SO (ON BOARD). AND HERE F-OF A-IS, (INAUDIBLE) VALUE.

STUDENT: CAN YOU RAISE THE BOARD.

PROFESSOR: SURE. LET'S ME JUST SAY ONE MORE THING HERE.

AND I'LL FINISH SECTION 1.5. SO WHY DO I CARE ABOUT BEING CONTINUOUS? FOR F-OF X-TO BE DIFFERENTIABLE, IT MUST BE CONTINUOUS. BUT NOT THE OTHER WAY AROUND. FOR F-OF X-TO BE DIFFERENTIABLE IT HAS TO BE CONTINUOUS. IT HAS TO BE SMOOTH.

THERE'S NO WAY YOU CAN HAVE A TANGENT LINE IF ARE FOR THIS POINT THAT'S TANGENT TO THAT. THAT DOESN'T WORK AT ALL. AND THESE, THIS DOES NOT HAVE A TANGENT LINE AND THIS ONE DOESN'T AND ONE OBVIOUSLY DOESN'T SYSTEM SO YOU NEED TO HAVE FOR A FUNCTION F-TO BE DIFFERENTIABLE, IT MUST BE CONTINUOUS. BUT ALONE IS NOT ENOUGH TO BE DIFFERENTIABLE. AND THE EXAMPLE FUNCTION WITH CORNERS. THE ABSOLUTE VALUE. FUNCTION WITH CORNERS IS CONTINUOUS, BUT IT'S NOT DIFFERENTIABLE.
STUDENT: CAN YOU TAKE THE LIMIT HERE.

PROFESSOR: SO THIS FUNCTION IS CONTINUOUS BECAUSE AS YOU GET CLOSER TO ZERO FROM EITHER THE RIGHT OR LEFT THE ABSOLUTE VALUE GOES TO ZERO. SO IT'S A CONTINUOUS FUNCTION, BUT SINCE IT HAS A CORNER I CAN'T DRAW A UNIQUE TANGENT LINE THROUGH THAT POINT. THIS LIMIT EXISTS. AND IT EQUALS ZERO BUT THE DERIVATIVE DOESN'T. BECAUSE THERE'S MORE THAN ONE TANGENT LINE. WHICH WE CAN DRAW.

OKAY. AND I'M ALMOST DONE. AND MAYBE I WILL LEAVE SOME OF THESE Rules UP AND JUST WRITE THE REST HERE. AM GOING TO DO NOW SECTION 1.6 WHICH HAS MORE Rules. SECTION 1.6, MORE Rules. LEAVE THE TWO Rules I HAD OUT BEFORE. AND ONE OF Them SAID, IF YOU HAVE A FUNCTION THAT YOU KNOW HOW TO DIFFERENTIATE AND MULTIPy BY CONSTANT, JUST FACTOR OUT THE CONSTANT. THE OTHER One SAID IF I HAVE A FUNCTION, TWO FUNCTIONS, AND I WANT TO DIFFERENTIATE THEIR SUM, I JUST ADD THEIR DerIVATIVES.

AND THEN WE ALSO HAD THE GENERALIZED VERSION OF THIS WHICH
you take function, take to some power, so r-is a non zero constant, then it looks very much like the power rule, it's times the function of r-minus one but you still have to multiply by the derivative of the function of so those were all the rules you were given for computing derivatives and these are all the ones you need or are allowed to use on the midterm. i'm not sure that would fit on two pages. maybe on notebook paper. it's supposed to be a short summary.

student: if a function is differentiable, does it always mean that the limit exists.

professor: yes. so if the function is differentiable then it will be continuous and for t-to be differentiable it must be continuous. the limit must exist. this limit must exist equal the value of function. it has to be a nice smooth-looking thing like this. but just because that limit exist does not
MEAN

IT'S DIFFERENTIABLE BECAUSE IF THERE'S A CORNER THE LIMIT EXISTS, ZERO HERE, BUT THERE'S NO TANGENT, NO DERIVATIVE. SO IT ONLY GOES ONE WAY. SO DIFFERENTIABILITY IMPLIES CONTINUITY.

BUT CONTINUITY IS NOT IMPLIED FOR DIFFERENTIABILITY.

STUDENT: (INAUDIBLE).

PROFESSOR: IT'S IN CLASS. SOMEBODY ELSE IS GOING TO WALK IN THE DOOR WHEN CLASS IS OVER AND THAT WILL BE THE END. ANY QUESTIONS ABOUT IT.

STUDENT: WORDS PROBLEM (INAUDIBLE). DO YOU THINK WE SHOULD FOCUS ON ALL THE ONES THAT ARE GIVEN IF THE BOOK OR --

PROFESSOR: IF YOU LOOK AT THE EXAMPLE MIDTERMS THEN YOU WILL SEE THERE ARE NOT REALLY SO MANY WORD PROBLEMS, LONG WORDY WORD PROBLEMS LIKE IN THE BOOK. OF COURSE YOU HAVE TO FOLLOW THE INSTRUCTIONS IN THE ASSIGNMENTS BUT NOT LONG WORD PROBLEMS TRANSLATING TO THE ECONOMIC REALM TO THE CALCULUS REALM.

STUDENT: CAN YOU GO OVER A QUESTION (INAUDIBLE).

PROFESSOR: THAT'S MY NEXT STEP. IS EVERYONE HAPPY WITH THIS?
OKAY. I THOUGHT I'D TAKE ONE OF ONES FOR WHICH THE ANSWERS WERE NOT POSTED.

SO LET ME JUST DO SOME TRUE FALSE ONES. (ON BOARD) OKAY.

THESE TWO LINES, SO EQUATIONS OF STRAIGHT LINE ARE PERPENDICULAR.

STUDENT: CAN YOU RAISE THAT BOARD.

PROFESSOR: SO TRUE FALSE. SO I NOW OF COURSE, THIS IS THE ANSWERS IN SECTION ONE POINT B-RULES THE IDEA IS IN SECTION 1.1 THIS YOU NEED TO KNOW. SO HOW CAN YOU TELL THAT TWO LINES ARE PERPENDICULAR? LOOK AT THEIR SLOPES AND THEIR PRODUCT HAS NEGATIVE ONE. SO I NEED IT CHANGE THIS NO AN EQUIVALENT ONE WHERE THE SLOPE IS SITTING THERE. SO I JUST HAVE TO DO A LITTLE BIT OF ALGEBRA TO FIGURE OUT WHAT THE TWO SLOPES ARE. AND I DO THAT WHAT DOES THIS TURN INTO? AND I DON'T CARE WHAT IS. IT'S IRRELEVANT. I DON'T HAVE TO SPEND TIME FIGURING OUT. AND WHAT DOES THIS LOOK LIKE? AND THE ANSWER IS TRUE. (ON BOARD). BECAUSE TWO, NEGATIVE TWO TIMES ONE-HALF IS EQUAL
MINUS ONE. DID I DO THAT RIGHT? NO. I DID THE ALGEBRA
THAT'S ALL RIGHT. BUT I, YOU GET CREDIT FOR WRITING T.
JUST CIRCLE IT. THIS IS CHAPTER ZERO. TRUE OR FALSE? FALSE.
WHAT SHOULD IT BE? (ON BOARD). MINUS X. JUST SO YOU KNOW THE
EXPONENTS. (INAUDIBLE). HOW ABOUT D-D-X-OF TWO TO THE
EQUALS FOUR TIMES TWO CUBED? I HEAR DISAGREEMENT. HOW MANY
PEOPLE SAY FALSE? I THINK THE FALSES HAVE IT. SO WHAT'S
TRUE IS D-D-X-OF X-TO THE FOURTH EQUALS FOUR TIMES X-CUBED AND THEN
IF YOU PLUG IN TWO THIS WOULD EQUAL THAT. BUT IN THIS THING
HERE,
14 THERE'S NO X-IN IT, IT'S A CONSTANTS. SO WHEN YOU
DIFFERENTIATE IT YOU GET ZERO. THIS ACTUALLY EQUALS ZERO, SO THIS WAS
DESIGNED TO FOOL YOU. THAT'S WHY IT'S A GOOD TRUE FALSE QUESTION.
IS THAT CLEAR?
OKAY. NEXT TRUE FALSE. THE LAST TIME THAT HAPPENED,
MIND.  (LAUGHTER).  OKAY.  LET'S TRY THIS ONE.  SO NEED IT

CHANGE

THIS INTO A FOR WHERE WE HAVE A RULE THAT APPLIES.  CHANGE

THAT

INTO X-TO THE WHAT POWER?  THIS IS X-TO THE ONE TIMES X-TO

THE

HALF.  SO IT'S ACTUALLY THREE HALVES AND NOW I CAN USE THE

POWER

RULE AND I GET THAT EQUALS (ON BOARD).  IS IT TRUE?  TRUE.

OKAY.

SO THREE HALVES MINUS ONE IS EQUAL TO THE ONE-HALF.  AND TWO

(ON

TIMES THREE HALVES IS THREE.  SO YES.  SO THAT ONE WORKS.

BOARD).

ANY QUESTIONS ABOUT THE TRUE FALSE ONES BEFORE I GO

ONTO THE

NEXT ONE?  IS THERE -- NO?  OKAY.  SO LET'S ME ALSO SAY THAT

YOU

SHOULD KNOW THAT YOU SHOULD, YOU ARE PERFECTLY WELCOME TO GO

TO

ANY TA'S OFFICE HOURS AND ANY TA SECTION TO LISTEN TO

DISCUSSION,

NOT ONLY YOUR OWN.  SO IF THERE'S ROOM IN THE BACK IT STAND

AND

THERE.  IT'S OKAY.  YOU'RE NOT RESTRICTED TO LISTENING ONLY

ASKING QUESTIONS ONLY OF YOUR TA.  HERE'S THE NEXT QUESTION.

AND

HERE'S A PICTURE.  OF A FUNCTION.  AND WITH A TANGENT.

AND

I'LL GIVE YOU A LITTLE BIT OF INFORMATION ABOUT THIS
That point is three. That points over there is six. And this point is four. Where the tangent is. And so the question is, find f-of four f-prime of four, and find approximate f-of 3.9.

So that's the question. So this is all you're given. So what are we going to do? So we're going to approximate this function by a straight line. That's the theme of much of chapter one. And straight line is there. So let's write down the equation for a straight line. So we know it goes through, we take a particular point it goes through. So it goes through the 0, comma six. So there's the equation if I could only figure out the slope. So what's the slope of that line? Say again. Negative one-half. So if, to make sure you get credit, you might do the math wrong you could say it's minus three divided by six because it's that divided by that. And then you can simplify it and say it's minus one-half x plus three. A
LITTLE BIT EASIER. SO TAKE THAT INTERMEDIATE STEP. MAKE
SURE YOU DID IT RIGHT. HOW DO I FIGURE OUT F-OF FOUR. WHAT DO I
DO, WITH THIS EQUATION WITH THE TANGENT LINE? I PLUG IN FOUR.
IT'S MINUS ONE-HALF TIMES FOUR PLUS THREE OTHERWISE KNOWN AS
ONE.
OKAY. SO BECAUSE I KNOW THE EQUATION OF THAT LINE I CAN
JUST PLUG IN ONE. SO I KNOW WHAT THAT IS. F-PRIME OF FOUR,
THAT'S EVEN EASIER. THAT'S JUST THE SLOPE OF LINE. THAT'S MINUS
ONE-HALF. BECAUSE THAT'S WHAT THE TANGENT LINE. SLOPE OF
THIS FUNCTION AT THAT POINT IS EXACTLY THE SLOPE OF TANGENT LINE,
WE FIGURED OUT, IT'S MINUS A HALF. AND THE LAST QUESTION
OUT, HOW DO I APPROXIMATE THE WIGGLELY LINE. LET ME JUST FIGURE
LET ME JUST GO EVALUATE STRAIGHT LINE AT 3.9. THAT'S CLOSE
ENOUGH. THAT'S WHAT IT MEANS IT APPROXIMATE MAY. SO JUST
16 THAT. AND PLUG IN MINUS ONE-HALF TIME 3.9 PLUS THREE AND
WHATEVER THE HECK THAT IS, I'LL COPY IT, 1.05. DO
ARITHMETIC IN REAL TIME IS --

STUDENT: DO WE HAVE TO SIMPLIFY.

PROFESSOR: SO THE QUESTION IS WORTH 14 POINTS YOU MIGHT GET 13 POINTS FOR NOT DOING IT ALL. SOMETHING LIKE THAT. SO E-WAS, SORRY FORGOT THAT ONE. SO I SKIPPED ONE TRUE FALSE QUESTION.

BY THE WAY DON'T GUESS ON TRUE FALSE BECAUSE WE'LL TAKE OFF TWO POINTS IF YOU GET THE WRONG ANSWER. I'LL LEAVE THE RULES UP HERE.

OKAY SO. HERE'S THE LAST TRUE FALSE QUESTION. WHAT KIND OF CURVE IS THAT? IT'S A PARABOLA. IT HAS A TANGENT LINE PARALLEL TO TWO X-PLUS Y-EQUALS ONE. OKAY. SO THAT MEANS THERE'S SOME POINT ON THIS PARABOLA WHERE THE SLOPE OF A TANGENT LINE IS EQUAL TO THE SLOPE OF THIS ONE, THAT'S WHAT PARALLEL MEANS, SAME SLOPES. SO WHAT IS THE SLOPE OF THIS ONE?

I'LL WRITE IT THAT WAY. SO THE SLOPE HAS TO EQUAL MINUS TWO. SO DOES (ON BOARD) HAVE A DERIVATIVE EQUAL TO MINUS TWO SOMEWHERE. WHAT IS THE DERIVATIVE, I-E, SO WHAT'S THE DERIVATIVE OF THAT, SEVEN X-SQUARED, DOES 14 X-EQUAL MINUS TWO
SOMEWHERE?

YES OR NO. CAN I SOLVE THIS FOR X. (ON BOARD). SO YES,

TRUE.

X-EQUALS MINUS ONE OVER SEVEN.

STUDENT: DON'T GUESS ON THE TRUE FALSE. ISN'T THAT, IF

YOU

TAKE AWAY TWO AND DON'T GIVE US TWO, ISN'T THAT THE SAME

THING.

PROFESSOR: YOU GET IT, IF YOU WERE TO GUESS RANDOMLY ON

EVERY

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QUESTION THEN THE EXPECT THE OUTCOME WOULD BE HALF RIGHT

HALF

WRONG YOU'D GET ZERO. SO WE WANT, BUT IF YOU DIDN'T DO, IF

YOU

GOT TWO POINTS FOR GETTING IT RIGHT AND NO POINTS FOR

GETTING IT

WRONG AND YOU JUST GUESSED THEN YOUR EXPECT THE OUTCOME

WOULD BE

HALF FULL CREDIT OR FIVE POINTS MUCH WE WANT IT TO BE ZERO

ON

AVERAGE. IT'S A STANDARD PROSE YOU ARE FOLLOWED IN MANY

COURSES

OF

IN TRUE FALSE TEST. MATHEMATICAL REASON FOR IT. SO F-PRIME

OF

THAT

F-PRIME FOUR IS A SLOPE OF A CURVE AT THAT POINT AND THE SLOPE OF

CURVE IS DEFINED FOR THE SLOPE OF A TANGENT LINE. SO WE
FIGURED OUT THE SLOPE OF A TANGENT LINE WAS MINUS THREE OR SIX OR
MINUS ONE-HALF. IS THAT OKAY?

STUDENT: (INAUDIBLE).

PROFESSOR: IF YOU LEAVE IT BLANK IT'S ZERO. LEAVE IT BLANK IF
YOU DON'T KNOW. IF YOU WERE REASONABLY SURE THAT IT'S IN
YOUR INTEREST TO WRITE DOWN WHAT I THINK IT IS IF YOU'RE PRETTY
SURE.

STUDENT: CAN YOU RAISE THAT BOARD.

PROFESSOR: I NEED A BUTTON TO LOWER THE PODIUM. OKAY

SHOULD WE GET GOING? LIMIT, LOTS OF LIMITS.

STUDENT: CAN YOU GO OVER THE -- THERE WAS ONE ON MIDTERM
ONE, GIVES YOU NEGATIVE FOUR COMMA TWO. NUMBER FIVE.

PROFESSOR: OKAY. I CAN DO THEM (INAUDIBLE). SO WHICH
VERSION OF, WHICH MIDTERM.

STUDENT: FROM MIDTERM ONE.

PROFESSOR: I BELIEVE THIS IS THE FEBRUARY 16, 2007 MIDTERM.

ONE WITH THE ANSWERS ON THE WEB PAGE. FINE, NO PROBLEM.
FIND THE EQUATIONS OF THE TANGENT LINES THAT GO THROUGH THE POINT MINUS FOUR COMMA TWO AND ARE TANGENT TO THE GRAPH OF Y-EQUALS ONE OVER X. SO LET ME DRAW A PICTURE HERE. SO LET ME DRAW Y-(INAUDIBLE). SO THAT'S JUST HYPERBOLA. SO WHAT THIS IS ASKING FOR IS, CAN YOU DRAW A STRAIGHT LINE THROUGH THIS TANGENT TO THIS FUNCTION. WELL, INTUITIVELY, HERE'S ONE, RIGHT THERE AS TANGENT LINE ANDS THERE'S SOME POINT WHERE THIS INTERSECTS. BUT THERE'S ANOTHER ONE. RIGHT? THERE ARE TWO POINTS THAT YOU CAN IMAGINE DRAWING THERE. SO JUST IMAGINE THIS IS A PIVOT. AND I CAN TILT IT UP UNTIL IT HIT THE HYPERBOLA THERE. THERE ARE TWO POINTS. FIND THEM. SO FIND THESE TWO EQUATIONS. SO THERE'S AN EQUATION FOR THIS LINE AND ONE FOR THAT ONE. AND IF I CAN ONLY FIND THAT POINT. X-ONE COMMA Y-(ON BOARD). IF I HAVE TWO POINTS I KNOW THE EQUATION OF A STRAIGHT LINE. SO I HAVE TO FIGURE OUT WHERE THAT INTERSECTS AND I CAN WRITE DOWN THE TWO EQUATIONS. SO LET'S, LET ME CALL THIS POINT X-COMMA Y. SO WHAT'S THE SLOPE
OF TANGENT RIGHT THERE? THIS IS Y-EQUALS ONE OVER X. SO
Y-PRIME? POWER LAW, MINUS ONE OVER X-SQUARED. SO THIS
THAT THE SLOPE HERE IS MINUS ONE OVER X-SQUARED. SO THAT'S
THE SLOPE RIGHT THERE. IS THERE ANOTHER WAY I CAN WRITE DOWN
THE SLOPE? I HAVE TWO POINTS ON A LINE. CAN I WRITE DOWN THE
IF I KNOW TWO POINTS ON THE LINE? I TAKE THE DIFFERENCES IN
THE Y-DIVIDED BY THE DIFFERENCE IN THE X-S. SO HERE, LET ME
WRITE THIS DOWN. SO I HAVE THIS EQUATION. (ON BOARD). WHICH I'D
19 TO SOLVE FOR X-AND Y. BUT I HAVE, THIS IS, I HAVE TWO
VARIABLES. I CAN'T SOLVE THIS YET. HOW DO I GET DOWN TO ONE VARIABLE?
WHAT ELSE DO I SUBSTITUTE IN Y-EQUALS ONE OVER X. SO I GET
OVER X-MINUS TWO DIVIDED BY X-PLUS FOUR EQUALS MINUS ONE
OVER X-SQUARED. (ON BOARD). OKAY. NOW I HAVE AN EQUATION WITH
ONE VARIABLE X. AND WE SOLVE FOR IT. SO IT'S A LITTLE
MESSY, IF
I simplify it, it will just be a quadratic equation. So multiply both sides by $x^2$. And bother sides by $x + 4$. Multiply by $x - 4$ plus gets rid of that. So now that's the equation. And I multiply, so multiply by $x^2$ and get $x - 2x^2$ equals $-x - 4$. That really is a quadratic equation. So right this way. Two $x^2$ minus two $x - 4$ equals zero. Or $x^2$ minus $x - 2$ equals zero. I think I'm doing this right. And used quadratic formula if I wanted. We can just factor it. How do I factor this guy? How about board bore. $x - 2$ times $x + 1$. I think that works. So $x$ equals either two or minus one. So this here is two, and this over here is minus one. So we're almost there. So if $x$ is two what's $y$ right there? Sitting on the hyperbola. So it's a half. So there's that point. And over here what is this points? If I'm sitting at minus one. $y$ is the reciprocal of minus one. So it's also minus one. So there are two points. And now I can write down two points.
ON A

STRAIGHT LINE.

STUDENT: HOW DID YOU GET THE ONE.

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PROFESSOR: SO THIS POINT IS ON HYPERBOLA. SO $Y = \frac{1}{X-}$

STUDENT: OKAY.

PROFESSOR: $Y = \frac{1}{X}$. $Y = \frac{1}{X-1}$. OKAY. IS THAT OKAY?

STUDENT: I CAN'T REALLY SEE WHAT YOU'RE DOING.

PROFESSOR: SORRY. I WAS DOING THIS QUICKLY AS THOUGH I

WERE

TAKING AN EXAM. SO LET'S SEE, SHOULD I START UP HERE AND

WALK

THROUGH IT AGAIN.

STUDENT: NO.

STUDENT: CAN YOU EXPLAIN HOW YOU PUT THE POINT IN. I KNOW

WHERE

THE GOT THE X-S BUT WHY DID YOU GET THE Y-S FROM.

PROFESSOR: SO $Y = \frac{1}{X-}$

STUDENT: SO YOU JUST PUT IT IN THERE.

PROFESSOR: YES.
STUDENT: HOW DID YOU GET THE EQUATIONS FOR THE TANGENT LINES.

STUDENT: CAN WE LEAVE IT IN THAT FORM.

PROFESSOR: IF YOU'VE GOTTEN THIS FAR, YOU'LL GET PROBABLY ALL THE CREDIT. IF YOU GET THIS FAR AND DON'T FINISH DOING THE ARITHMETIC -- WOULD BE NICE IF YOU DO IT BECAUSE IT WOULD BE EASIER FOR US TO GRADE I'M PUTTING IN ALL THE INTERMEDIATE STUFF.

JUST WRITE DOWN THE WORK AS YOU DO IT YOURSELF AND IF YOU PUT IT ALL DOWN AND MAKE AN ALGEBRA MISTAKE ALONG THE WAY, YOU'LL GET MOST OF THE CREDIT. IF YOU DON'T PUT IN ANY INTERMEDIATE WORK AND -- HOW CAN WE GIVE YOU ANY PARTIAL CREDIT. IF YOU DON'T PUT DOWN THE WORK. ANY OTHER QUESTIONS.

STUDENT: SHOULDN'T THE SLOPE BE NEGATIVE 1/16, NOT NEGATIVE ONE FOURTH?

PROFESSOR: SO THE SLOPE IS NEGATIVE ONE OVER X-SQUARED. X-IS TWO, RIGHT? DID I DO THAT WRONG? I THINK I DID THAT RIGHT. YOU AGREE THAT I GOT THAT RIGHT?

STUDENT: YEAH.

PROFESSOR: AND THEN THE SLOPE Y-PRIME IS NEGATIVE ONE OVER X-SQUARED. YOU COULD HAVE ALSO GOTTEN THE SLOPE BY TAKING DIFFERENCE IN Y-S DIVIDED BY DIFFERENCE IN X-S BECAUSE I
KNOW TWO

POINTS ON THE LINE YOU CAN ALWAYS COMPUTE THE SLOPE THAT WAY.

IT WAS EASIER TO DO IT THAT WAY BECAUSE IT'S JUST SITTING THERE.

STUDENT: HOW DID YOU GET $x - 2x^2 = -4$.

PROFESSOR: I GOT THIS BY, SO WE'RE GOOD TO HERE, SO THEN I JUST TRY TO GET RID OF THE DENOMINATOR. THAT GAVE ME THIS.

ANOTHER WAY TO DO IT. THIS TIMES THIS EQUALS THAT TIME THAT'S ANOTHER WAY TO DO IT. THIS TIMES THIS EQUALS THAT TIME.

LOTS OF WAYS TO DO IT YOU GET THE SAME ANSWER. AND MULTIPLY THROUGH TO GET THIS. AND TURNED IT INTO A STANDARD OLD QUADRATIC. OKAY.

GLAD YOU ASKED ABOUT THAT ONE. IT'S MY FAVORITE QUESTION ON THE EXAM. HINT, HINT.

SO LET'S DO SOME OTHER QUESTIONS. LIMITS. LIMITS.

FIRST ATTEMPT SHOULD BE TO SAY MET MET PLUG IN TWO AND SEE.
WHAT HAPPENS, AND YOU GET ZERO OVER ZERO IF YOU JUST PLUG IN
TWO.

SO OTHERWISE IT WON'T BE TOO EASY. YOU HAVE TO FACTOR OUT
COMMON

FACTOR. THAT SHOULD BE YOUR MOTIVATION HERE. LET ME TRY TO
TWO. FACTOR IT OUT. I KNOW WHAT ONE OF FACTOR IS. BECAUSE IT'S
ZEROS. WHEN X-EQUALS TWO. SO THE FACTOR HAS TO BE MINUS

SO THAT'S JUST SITTING THERE, THE ONLY QUESTION IS WHAT DO I
HAS STICK HERE? AND ARE WHATEVER IT IS THAT TIMES NEGATIVE TWO
TIMES TO EQUAL MINUS SIX. WHAT IS IT? AND WHATEVER THIS IS THAT
EQUAL NEGATIVE TWO HAS TO EQUAL NEGATIVE EIGHT. SO THAT BETTER

PLUS FOUR. THOSE TWO FACTORS CANCEL. AND NOW I HAVE NO
TROUBLE STICKING IN TWO AND I GET FIVE, SIXTHS. OKAY. WE CAN MAKE
UP

LOTS OF EQUATIONS THAT ARE VARIATIONS ON THIS. EXCEPT WE

TRY IT

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DISGUISE IT A LITTLE BIT. HERE'S A VARIATION. YOU PLUG IN
FOUR

YOU GET ZERO FOR ZERO THERE'S COMMON FACTOR. SO HOW DO I
FIGURE OUT THE COMMON FACTOR. IF I WRITE SQUARE ROOT OF X-IS THE
So we can see this as a common factor. One of the factors will be the square root of $x - 2$. The other is the square root of $x + 2$. And this almost cancels. Except I'm left with a minus sign. So left with (on board) is minus one quarter. Now there are a couple of ways to do this. One way, well, let's see, $t$ goes to zero. One way to do it (on board), is to multiply it by one in disguise. Try to get rid of that square root. That certainly multiplying by one. And the denominator is going to look like this. (on board). What happens when I multiply this, these two things squared, divided, the fours cancel. The $t$ - $s$ cancel. And I'm left with one divided by something that doesn't go to zero. So I can just plug in one divided by the square root of four plus two equals one (on board). That's one way to do it. There's another way which...
REQUIRES NO WORK AT ALL. ALMOST, IT REQUIRES (INAUDIBLE).

NOT MUCH.

STUDENT: WHAT WAS THAT SECOND TO LAST.

PROFESSOR: YOU'VE DONE ALL THIS CANCELLATION STUFF. AT THAT POINT NUMERATOR ALL I'VE LEFT WITH IS ONE. NOW I PLUG IN T-EQUALS DO. SO I GET THE SQUARE ROOT OF FOUR PLUS -- IS ONE QUARTER.

I HOPE YOUR HANDWRITING ON THE MIDTERM IS NEATER THAN MINE.

STUDENT: (INAUDIBLE).

PROFESSOR: I WILL DO THAT ONE NEXT. LET ME JUST DO THIS ONE, ONE MORE WAY. BECAUSE IT USES AN IDEA THAT WAS DUE ON TODAY'S HOMEWORK.

SO HERE'S THE OTHER WAY TO DO IT. (ON BOARD). IS TO RECOGNIZE THAT THIS THING IS COMPUTING A DERIVATIVE OF A FUNCTION AT A PARTICULAR POINT. SO LET'S JUST CHANGE THIS. SO COMPUTING A DERIVATIVE. SO JUST TO MAKE IT EASY TO SEE, CHANGE T-TO H-
CERTAINLY DOESN'T CHANGE ANYTHING. I DIDN'T CHANGE VERY MUCH.

T-TO H. AND I CHANGED TWO TO THE SQUARE ROOT OF FOUR. AND I

CHANGED, WELL, OKAY. SO DOES THIS LOOK LIKE I'M DIFFERENTIATING

A FUNCTION? IS THIS GOING TO BE F-OF FOUR PLUS H-MINUS F-OF FOUR

DIVIDED BY H? FOR SOME PARTICULAR FUNCTION F? WHAT

(INAUDIBLE). IT IS THE SQUARE ROOT IF F-OF X-EQUALS THE SQUARE

ROOT OF X. SO WHAT IS THIS? THIS IS JUST THE DERIVATIVE AT WHICH POINT? F-PRIME AT FOUR. OOPS I'M GIVING AWAY HOMEWORK ANSWER. OH, WELL, WHICH IS F-PRIME IF F-PRIME IS X-TO THE ONE-HALF? IT'S ONE-HALF TIMES X-TO THE MINUS ONE-HALF, THAT'S THE DERIVATIVE, AT X-EQUALS FOUR. SEE IF I GET THE SAME ANSWER.

FOUR TO THE MINUS ONE-HALF, WHAT IS FOUR TO THE MINUS ONE-HALF? SO IT ACTUALLY WORKS. YES, FOUR TO THE ONE-HALF IS TWO, FOUR TO THE QUARTER AND THE MINUS ONE-HALF IS A HALF. HALF TIMES A HALF IS A QUARTER AND I GET THE SAME ANSWER, I GOT BEFORE. IT'S TECHNIQUE FROM SECTION 1.6. THERE WAS A HOMEWORK QUESTION ON TODAY'S A HOMEWORK
INVOLVED RECOGNIZING THIS. EITHER WAY. I DON'T CARE.

THERE'S

MORE THAN ONE WAY TO DO IT.

STUDENT: CHANGE OF H-

PROFESSOR: I DID THAT SO KEEP WRITING DOWN DERIVATIVES WITH

H-GOING TO ZERO FOR SO LONG I DECIDED TO DO THAT. YOU

DIDN'T HAVE TO. I WAS JUST DOING IT BECAUSE THIS SHOULD LEAP OUT

AT YOU AS DERIVATIVE OF A FUNCTION. BUT T, H, THEY'RE ALL EQUALLY

GOOD.

SOMEBODY ASKED ABOUT LIMITS AS T-GOES TO INFINITY. SO

HERE THE QUESTION. NOW IF YOU PLUG IN INFINITY WHATEVER THAT

MEANS YOU GET INFINITY DIVIDED BY INFINITY. THAT DOESN'T MAKE

SENSE.

SO I HAVE TO CHANGE IT. SO WHAT I'M GOING TO DO IS TAKE THE

SQUARED.

SO I'M GOING DO GET A CONSTANT PLUS STUFF AT THAT GETS

SMALLER.

THAT WILL MAKE IT EASIER TO UNDERSTAND. SO IF I TAKE EACH

STEP AND DIVIDE BY T-SQUARED I GET TWO PLUS, WHAT DO I GET

WHETHER I
DIVIDE BY T-SQUARED? T-TO SOME POWER. SO THIS IS T-TO THE
FIRST
POWER TIME T-TO THE ONE-HALF. SO ALL TOGETHER THIS IS T-TO
WHAT
POWER? ONE PLUS ONE-HALF IS THREE HALVES, SO THIS IS ALL
BY
TOGETHER THE SAME AS T-TO THE THREE HALVES AND THEN DIVIDE
THAT
T-SQUARED. SO IT'S T-TO THE MINUS ONE-HALF OR ONE OVER THE
OVER
SQUARE ROOT OF T. AND THEN I GET PLUS ONE OVER T-SQUARED.
DENOMINATOR
IS THE NUMERATOR. THE DENOMINATOR IS MINUS ONE PLUS THREE
LARGER
T-SQUARED. RIGHT. SO I JUST TOOK THE NUMERATOR AND
DO
AND DIVIDED BY T-SQUARED AM SO NOW WHAT HAPPENS AS T-GETS
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AND LARGER NOTICE NUMERATOR AM IT GOES TO ZERO AND IT GOES
ZERO. SO NOW THIS IS TWO DIVIDED BY MINUS ONE EQUALS MINUS
TWO.
JUST START LETTING T-GET BIGGER AND BIGGER THE NUMERATOR
GETS TO
INFINITY. THE DENOMINATOR GOES TO NEGATIVE INFINITY. AND
INFINITY DIVIDED BY INFINITY MAKE NO SENSE. --
STUDENT: IF YOU GIVE HALF CONSTANT, LIKE TWO NEGATIVE ONE
WITH
The derivative would the limit not make sense because they all go to zero?

Professor: If this is constant seven and constant throw.

The only thing that matters is the constant in fronts of highest power t. All these other guys go to zero. There's the constant's one. I'm not sure what you mean. In that case constant is one. Here the constant was minus one in the original problem. Now we have time to do the fourth to last question.

I'll just do one.

So this rules of derivatives which I managed to are a void erasing, here they are. All you're allowed to use. Do this following derivative. There it is. (On board) So all I'm going to do is apply those rules. So just write down all the steps.

Now I'm going to use the generalized power rule by changing notation. (On board). So this is some function. So this is going to be f-of x. To the one-half power. So this is going to be two
A half times f-of x-to the a half minus one times d-d-f-(on board). Going to give myself more room. So two times a half with one. So f-of x-is f-plus two (on board). To the power a 27 half minus one, that is minus a half. Times d-d-x-of this stuff. (on board). So far so good? So I still have to do this. Keep applying the rules until I get down to the end. X-plus -- (on board) okay so it's going to be d-d-x-of x-plus d-d-x-of two x-squared plus one cubed. Equals -- my hands's tired. Let me do ditto rules. This is the generalized power rule again. Some function. G-of x-cubed. So this is going to be three times g-of x-to the power three minus one times d-d-x-g-of x. That's the generalized power rule. So it's this mess again. One plus three times two x-squared plus one. (on board) and finally I differentiate that. So maybe I'll just, abbreviate again. Four x. I guess I'll put it all together here. I'll just do it
WAY. DIVIDED BY X-PLUS TWO X-SQUARED PLUS ONE CUBED TO THE POWER, TO THE SQUARE ROOT. THERE WE GO. (ON BOARD). LOTS OF NOTATIONS FOR THE SAME THING. MINUS ONE HALF MEANS THE SQUARE ROOT OF THE DENOMINATOR.

STUDENT: WE LEAVE IT LIKE THAT.

PROFESSOR: THIS IS THE ANSWER. IT DOESN’T GET ANY SIMPLER. NO MORE CANCELLATION YOU CAN DO AT THIS POINT.

STUDENT: GO FOUR X---

PROFESSOR: YEAH. THIS IS FINE. THIS IS FINE. YOU'VE GOTTEN RID OF ALL THE DERIVATIVES. I'VE HAD ENOUGH. IS THAT OKAY?

GOOD LUCK ON THURSDAY.