MATH 16 A-LECTURE SEPTEMBER 18, 2008.

PROFESSOR: SO WHILE I'M WAITING FOR SOMEONE TO SHOW UP WITH THE NEW BATTERY FOR THE MICROPHONE, I WILL TRY TO SPEAK LOUDLY. BUT IT MIGHT BE A LITTLE HARD TO HEAR MY AT FIRST FOR A FEW MINUTES UNTIL A NEW BATTERY SHOWS UP.


SO MY VARIABLE, I'LL USE S, KINDS OF TRADITIONAL FOR THE DISTANCE TRAVELED, STARTING AT ZERO BY TIME T. SO BY, SO FOR EXAMPLE, BY A CAR ON A HIGHWAY, OR A BALL THROWN INTO THE AIR, AND SO WE'RE GOING TO WRITE DOWN SOME FORMULAS AND DERIVATIVES
FOR THESE KINDS OF THINGS TO TRY TO FIGURE IT OUT. BUT THE
COMMON FACT, LET'S ME WRITE DOWN THAT AVERAGE RATE OF CHANGE I
HAD OVER THERE FOR THAT FUNCTION. SO IF PHYSICAL TERMS THAT'S
THE DISTANCE TRAVELED FROM TIME T-TO T-PLUS H-DIVIDED BY THE TIME
AND THAT'S THE AVERAGE VELOCITY. DURING THAT TIME IS INTEGRAL. AND
OF COURSE IF I LET H-GET SMALLER AND SMALLER, EVENTUALLY THAT
APPROACHES SOMETHING ELSE. NOW MY VOICE WILL LAST THE WHOLE
LECTURE. LET ME FINISH WRITING. I WASN'T QUITE DONE WITH THAT
BOARD. I SHOULD SAY THAT ALONG TIME AGO SOMEBODY NAMED NEWTON
INVENTED GNAWED THE TO UNDERSTAND THESE PROBLEMS. SO IT'S AN
OLD EXAMPLE. OKAY. SO AS H-GETS SMALLER AND SPLAWRL IT
APPROACHES OF DERIVATIVE JUST AS IT DID OVER THERE. WE GIVE THAT
FUNCTION A NAME, V OF T-WHICH IS THE VELOCITY AT EXACTLY T.
OKAY. SO THAT'S JUST NOTATION. RIGHT, SO S-IS THE DISTANCE,
S-PRIME IS NORTH FUNCTION. CALL IT V OF T, V FOR VELOCITY. AND
THERE'S ONE OTHER IMPORTANT FUNCTION. I CAN DO THE SAME TRICK
WITH THE VELOCITY. AND SO WHAT'S THAT? THAT'S THE CHANGE IN
VELOCITY FROM T-TO T-PLUS H-DIVIDED BY THE CHANGE IN TIME H. AND
AS H-GETS SMALLER AND SMALLER THAT APPROACHES THE FUNCTION WHICH
IS DERIVATIVE D-PRIME OF T-AND GIVE THAT A NAME CALLED
ACCELERATION. SO THEY'RE JUST DERIVATIVES. AND SINCE THE
VELOCITY IS THE FIRST DERIVATIVE, THIS IS THE SECOND DERIVATIVE.
LET ME USE A NOTATION I USED LAST TIME. IT'S THE SECOND
DERIVATIVE OF POSITION WITH RESPECT TO TIME. SO THAT'S THE
NOTATION FROM LAST TIME. START WITH, SORRY, SO WHAT I'M WRITING
HERE, PUT IT OVER HERE, SO THE VELOCITY IS THE DERIVATIVE WITH
RESPECT TO TIME. AND THIS IS THE SECOND DERIVATIVE. WITH RESPECT TO TIME OF THE POSITION X. OKAY. SO THOSE ARE JUST NAMES FOR FUNCTIONS WHICH WE LEARN HOW TO DEFINE LAST TIME. BUT WHAT I WANT TO PAY ATTENTION TO HERE, WE'RE GOING TO SOLVE PROBLEMS, MAKE IT'S EASY TO MAKE MISTAKES UNLESS YOU PAY ATTENTION TO THE UNITS. SO LET ME WRITE DOWN THE UNITS FOR THESE. SO POSITION, MEASURE IS IN FEET OR METERS OR WHATEVER. AND VELOCITY YOU CAN MEASURE IN DIFFER WAYS. UNTIL YOU GET ALL THE THE \YOU WANT\UNIT RIGHT YOU'LL GET NONSENSE ANSWERS. SO WRITE THIS DOWN. S-OF T, LET ME USE SQUARE BRACKETS TO INDICATE CHAT -- SO IF YOU WANT TO BE SPECIFIC YOU PICK ONE. SO LET'S JUST SAY FEET FOR THE SAKE OF ARGUMENT. SO VELOCITY, IF YOU LOOK AT IT UP THERE, IT'S DISTANCE DIVIDED BY TIME, IT'S A LIMIT AND THE NUMERATOR IS A CHANGE, IS A DISTANCE AND THE DENOMINATOR IS TIME SO. THAT'S HOW WE'RE GOING TO MEASURE VELOCITY. AND SO JUST FOR BEING CONCRETE I'LL SAY FEET PER SECOND. THE HOURS, MINUTES, SECONDS BUT YOU GET THE WRONG ANSWER UNTIL WE'RE CONSISTENT. AND FINALLY, SO WHAT ARE THE UNITS OF ACCELERATION? SO IT'S VELOCITY DIVIDED BY TIME. SO IT'S THE UNITS OF VELOCITY WHICH I'LL WRITE THAT DOWN IN A SECOND, DIVIDED BY TIME. SO HOW DO WE MEASURE VELOCITY. WE JUST DECIDED IT WAS FEET PER SECOND. AND WE JUST DECIDED TIME WAS SECOND. SO ANOTHER WAY TO SAY THAT IS FEET DIVIDED BY SECONDS, TIME SECONDS OR SECOND SQUARED. SO WE'RE GOING DID CHECK THAT WHEN WE DO OUR ALGEBRA BECAUSE WE HAVE TO GET THE UNITS RIGHT. OKAY. SO LET'S JUST DO AN EXAMPLE NOW. PUT IT ALL TOGETHER.
SOMEONE'S GOING TO THROW A BALL STRAIGHT UP IN THE AIR. AND THEY'RE GOING TO LET GO WHEN, SO THE BALL, WHEN THE BALL IS EXACTLY 6 FEET OFF THE GROUND. SO 6 FEET ABOVE THE GROUND. SO GOING UP. SO THAT'S HAD HANDS LET'S GO. AND WHEN THEY LET GO, IT'S GOING TO BE GOING AT CERTAIN IS SPEED WITH INITIAL VELOCITY, IS GOING TO BE 128 FEET PER SECOND STRAIGHT UP. SO THAT'S A PERFECTLY WELL DEFINED PHYSICAL SITUATION. THROW THE BALL, YOU KNOW WHERE IT IS. 6 FEET OFF THE GROUND AND YOU KNOW HOW FAST IT'S GOING. AND YOU WANT TO KNOW WHAT HAPPENS TO THE BALL. SO LET ME ASK TWO SPECIFIC QUESTIONS. WHEN DOES IT HIT THE GROUND? SO YOU ASSUME YOU LET GO AT TIME ZERO AM SOMETIME LATER, AT T-TIME EQUALS SOMETHING IT HITS THE GROUND. SO THIS IS, WE'RE GOING TO HAVE TO SOLVE FOR T-. WHAT IS T-WHEN, HOW DO WE KNOW WHEN IT HITS THE GROUNDS? WHEN THE DISTANCE FROM THE GROUNDS ZERO. SO THEN NAME THAT FUNCTION WE NEED IT FIGURE OUT THIS FUNCTION. FROM S-TO T. THAT WOULD BE THE DISTANCE FROM THE GROUNDS EQUALS ZERO. SO HERE'S THE GROUND. HERE'S THE PERSON THROWING IT. HERE'S THE BALL. IT GOES UP. IT COME STRAIGHT BACK DOWN AGAIN. AND SO THIS DISTANCE HERE IF THE BALL IS THERE. THAT'S HOW FAR OFF THE GROUNDS YOU ARE AT TIME T-. GOING TO GO UP AND COME DOWN AM AT SOME POINT THIS WILL BE ZERO. WHEN DO WE GET PERFECTLY REASONABLE QUESTION TO ASK. AND THE OTHER QUESTION I WANT IT ASK IS HOW FAST IS IT GOING WHEN IT HITS? HOW FAST DOES IT HIT THE GROUND? OKAY. SO HOW FAST IS THE QUESTION ABOUT THE VELOCITY. SO CALLING V OF T. THAT IS THE VELOCITY WHETHER IT HITS. OKAY. SO LET ME WRITE DOWN THE ANSWER. AND THEN LET'S JUST LOOK AT IT. AND MAKE SURE IT MAKE SOME SENSE. SO I WILL
EXPLAIN WHY IT IS RIGHT. SO HERE'S THE FUNCTION. AND WE WILL
CHECK IT. NICE SIMPLE POLYNOMIAL. AND IT'S MEASURED IN FEET OFF
THE GROUND. FEET UP. OKAY. SO I CLAIM THAT'S THE FUNCTION.
BUT LET'S TRY IT USE CALCULUS TO SEE IF WE BELIEVE THAT IT SHOULD
DO THE RIGHT THING. SO THERE'S THE ANSWER.

SO LET ME WRITE THE FUNCTION HERE. AND SO NOW LET'S COMPUTE
V OF T. GAUZE WE HAVE TO CHECK THAT MA MAKE SENSE TOO. WHO CAN
TELL ME WHAT IS F-OF V, THE DERIVATIVE THIS THING. MINUS 32 T,
128, OKAY. AND I ALSO NEEDS TO KNOW THE ACCELERATION. SO I HAVE
TO DIFFERENTIATE THAT ONE MORE TIME. I GETS MINUS 32, A
CONFLICT. SO THIS IS IN UNITS, THIS IS IN FEET. THIS IS FEET,
WHAT? PER SECOND. AND THIS IS FEET PER SECOND PER SECOND.
SECOND SQUARED. OKAY ALL THOSE UNITS ARE NICE AND CONSISTENT.
SO WHAT DO WE WANT TO CHECK? WHERE DO WE START, THAT'S, WE KNOW
WHEN IT GUY LET'S GO. S-OF ZERO IS SIX. SO IT'S 6 FEET OFF THE
GROUND. THAT'S OKAY BECAUSE THAT'S WHEN THE PERSON LET'S GO.
SO FAR SO GOOD. THE OTHER THING IN THE PROBLEM WAS HOW FAST IT
WENT WHEN GOT STARTED SO PLUG ZERO IN THIS YOU GET ONE 28 FEET
PER SECOND GOING UP. AND THAT WAS PART OF PROBLEM. SO THAT'S
OKAY. AND FINALLY WHAT'S A-OF T. IT'S A CONSTANT. FEET PER
SECOND PER SECOND. AND IF THERE WAS PHYSICS CLASS I'D SAY YOU'RE

DONE. THAT'S THE ACCELERATION OF GRAVITY. THAT'S THE RIGHT
THING. SO THAT'S, THIS IS WHAT GRAVITY DOES. ALWAYS PULLS DOWN
EQUALY HARD NO MATTER WHERE YOU ARE. SO YOU JUST TAKE IT ON AT
THIS POINT FACE VALUE THAT I'VE GOTTEN EQUATION RIGHT. I'M NOT,
PHYSICS IS NOT A PREREQUISITE. THIS IS WHY I WROTE IT DOWN AS A
STARTING POINT. OKAY. SO LET'S SEE IF WE CAN ANSWER THE QUESTION. THIS IS THE FUNCTION. LET'S GO ANSWER THE QUESTIONS. THAT WE POSED.

SO WHAT WAS THE FIRST QUESTION? WHEN DOES IT HIT THE GROUPED? AND SO THAT'S WHAT WE GET BEFORE I SOLVE S-AND T-EQUALS ZERO. SO I LOOK AT FUNCTION AND IT'S A QUADRATIC. SO USED QUADRATIC FORMULA. AND SO THERE ARE TWO POSSIBLE ANSWERS. AND I'LL JUST FILL IF THE QUADRATIC FORMULA. IT'S A PLUS OR MINUS SIGN. OKAY. SO THERE'S THE QUAD RAT FIGURE FORMULA. AND TWO POSSIBLE ANSWERS ARE (ON BOARD). PLUS AND MINUS. OKAY. WHICH ONE IS RIGHT? SO NOW WE HAVE TO USE PHYSICAL INTUITION TO GET THE RIGHT ANSWER. THERE ARE TWO ANSWERS AND I HEARD SOME PEOPLE ARE SUGGESTING THIS IS THE RIGHT ONE. SO THIS IS BEFORE THE PERSON LET GO. SO WHAT DOES THAT MEAN? WHAT WOULD HAVE HAPPENED, IT'S NEGATIVE TIMES BEFORE THEY STARTED. SO THIS IS THE PAST. THIS IS THE FUTURE. AND THAT'S (INAUDIBLE). SO THE NEXT THING WE HAVE TO ASK IS, HOW FAST IS IT GOING WHEN IT HITS THE GROUNDS. SO I HAVE TO PLUG IN V OF EIGHT-POINT OWE 47. WHICH IS ABOUT MINUS 129 AND A HALF AND THE UNITS ARE FEET PER SECOND. SO LET'S JUST SEE IF THIS MAKE SENSE. LET'S USE A LITTLE COMMON SENSE TO SAY HOW WE GOTTEN THE RIGHT ANSWER. SO THE PERSON WHEN IT LET'S GO IS GOING UP AT 128 FEET PER SECOND. AND THEN START COMING BAG DO YOU THINK AND HOW FAST WOULD IT BE RIGHT HERE. WHEN 60 FEET ABOVE THE GROUNDS.

STUDENT: (INAUDIBLE).

PROFESSOR: IT SHOULD BE GOING EXACTLY AS FAST BACK DOWN. AND THAT INTUITION IS CALLED THE CONSERVATION OF ENERGY AM SO AT THIS
POINT IT'S GOING JUST AS FAST BACK DOWN AS WHETHER IT STARTED.  
SO HERE IT FALLS A LITTLE FARTHER. -- IT'S NEGATIVE, IT'S GOING 
DOWN. SO ALL INTUITION JIVES WITH THIS. ONE MORE INTERESTING 
QUESTION. PERFECTLY NATURAL. HOW HIGH DOES IT GET. LET'S DO 
THAT. OKAY SO THAT'S THE ENGLISH LANGUAGE ONE. HOW HIGH DOES 
IT GET? HOW DO I, WHAT IS SPECIAL ABOUT THAT POINT? 
(INAUDIBLE). SO I HAVE TO SOLVE FOR V OF T-EQUALS ZERO. AND 
THAT'S NOT VERY HARD BECAUSE, SO DIVIDE 128 BY 32. AND SO THE 
TIME IS FOUR, THAT'S WHEN IT REACHES THE STOP. AND WHEN YOU PLUG 
THAT IN YOU GET 262 FEET, THAT'S HOW FAR OFF THE GROUND. SO WHAT 
I INSIDE TO DO, SO THE VERY TOP, WHAT HAPPENS THE BALL STOPS FOR 
A INSTANT. IT'S VELOCITY IS ZERO. IT'S NOT GOING UP OR DOWN. 
AND SO I TAKE ANY EQUATION FOR V. V OF T, MINUS 32, SET AT THAT 
EQUAL TO ZERO AND SOLVE FOR T. AND THAT, AND SO YOU JUST SAW 
THAT LINEAR EQUATION FOR T-AND THE ANSWER IS FOUR -- FOUR TIMES 
ONE --

SO LET ME JUST DO ONE MORE LITTLE CHANGE ON THIS QUESTION.  
STUDENT: WHERE DID YOU GET NEGATIVE 16.

PROFESSOR: SO I GOT IT FROM KNOWING THAT WHEN I TOOK THE 
ACCELERATION, THIS IS A, SO I GOT IT BY MAGIC AS FAR AS THIS 
CLASS IS CONCERNED. BUT I GOT DID FROM PHYSICS SEVEN A. SO 
THAT'S WHERE I JUST WROTE DOWN THE ANSWER. BUT THE 
JUSTIFICATION IS THAT IF THE ACCELERATION OF GRAVITY IS CONSTANT. 
32 FEET PER SECOND PER SECOND PULLING STRAIGHT DOWN. SO THAT'S 
WHY MINUS SIGN. SO WHATEVER FUNCTION I PICKED UP THERE I HAD TO 
MAKE SURE THAT THE SECOND DRIVE E DRIVEN IS EXACTLY MINUS 32.
AND THAT'S WHY YOU GOT 16.

STUDENT: SO T-EQUALS FOUR THEN YOU PLUG IT BACK INTO.

PROFESSOR: THEN I PLUG T-EQUAL FOUR BACK IF THE EQUATION FOR S.

ANY OTHER QUESTIONS? NOW I'M GOING TO DO ALMOST THE SAME

PROBLEM. SO SUPPOSE THAT INSTEAD OF THROWING THE BALL STRAIGHT

UP, THE PERSON IS ACTUALLY GOING TO BAT IT SLIGHTLY SIDEWAYS. SO

HERE THE PERSON AND THE BALL IS GOING TO BE HIT AT TIME T-EQUAL

ZERO, UP, STILL GOING TO GO 128 FEET PER SECOND UP. BUT ALSO

GOING TO HAVE A SIDEWAYS VELOCITY. AND TO THE RIGHT AT 64 FEET

PER SECOND. SO THE MOMENT THE AT TIME T-EQUAL ZERO THE BAT HITS

THE BALL AND IT'S GOING OFF LIKE THAT. AND IT'S GOING UP AT 128

AND SIDEWAYS AT 64 AND GOING TO FOLLOW SOME PATH. INTUITIVELY

THAT'S WHAT THE PATH LOOK LIKE. I WANT IT FIGURE OUT WHAT THE

PATH IS. SO LET ME DRAW SOME AXES. AND THERE'S THE STARTING

POINT RIGHT THERE. THIS IS, SIDEWAYS AXIS I'LL CALL X. AND UP,

I CALLED THIS S-BEFORE BUT I'LL EQUAL IT Y. USUALLY Y-IS UP. SO

I WANT TO WRITE DOWN THE EQUATION OF AT THAT CURVE THAT THE BALL

FALLS. SO WHAT PATH Y-EQUALS F-OF X, SO THAT'S GOING TO BE THE

EQUATION FOR THE PATH OF THE BALL, DOES THE BALL FOLLOW AND HOW

FAR DOES IT GO WHEN IT HITS THE GROUNDS. OKAY. SO THAT'S THE

QUESTION. WE ALREADY KNOW PART THE ANSWER. WE ALREADY KNOW

HOW HIGH IT IS, BECAUSE WE ALREADY SOLVE THAT PROBLEM. IT'S THE

SAME ANSWER AS BEFORE. I CALLED IT S-THE LET MET JUST CHANGE IT

to Y. SO IT WAS MINUS 16 T-SQUARED, PLUS 128 T-PLUS SIX. THAT

HASN'T CHANGED BUT NOW I NEEDS TO SAY HOW FAR HAS IT GO. HOW

FAR HAS IT GONE TO THE RIGHT AS A FUNCTION OF TIME. ALL WE KNOW

WHEN IT STRARTS IT'S GOING 64 FEET PER SECONDS PSI WAYS. SO
WHAT IS, HOW FAR DOES IT GO AFTER T-SECONDS? IF THIS IS ITS VELOCITY SIDEWAYS? SO IF IT'S GOING 64 FEET PER SECOND CONTINUOUSLY AS IT MOVES FROM LEFT IT RIGHT AFTER T-SECONDS HOW FAR IT GONE. 64 T. SO THERE WITHOUT WORKING VERY HARD IS LA THIS CURVE LOOK LIKE AS A FUNCTION OF TIME. IF YOU TELL ME T, I'LL TELL YOU AT WHAT POINT YOU ARE ALONG THE CURVE. BUT WHAT I HAVEN’T DON'T YET IS WRITTEN THIS AS Y-AS SOME FUNCTION OF X. I WANT IT WRITE THIS AS A REGULAR OLD FUNCTION. SO HERE ONE WAY IT REPRESENT THE CURVE. BUT I WANT TO KNOW FIDDLE WITH IT. NOT VERY HARD AND CHANGE IT TO Y-EQUALS F-OF X. ELIMINATE T.

OKAY. SO WHAT I WANT TO DO IS I WANT TO WRITE Y-EQUALS F-OF X, I NEED TO ELIMINATE THE VARIABLE T. SO T-GOES AWAY. SO WHAT SHOULD I DO? SO I KNOW THAT X-IS 64 T, RIGHT? SO I WROTE IT UP THERE, X-IS 64 TIMES T. SO T-IS X-DIVIDED BY 64. SO Y-WHICH IS THIS, (ON BOARD) JUST PLUG THAT IN. MINUS 16 TIMES X-DIVIDED BY 10 64 (ON BOARD). PLUS SIX. THAT WASN'T TOO BAD. AND IF I WRITE THAT ALL OUT I THINK IT COMES OUT TO THIS. (ON BOARD). SO THERE'S THE EQUATION OF THAT PATH. AND WHAT IS THE NAME FOR THIS KIND OF CURVE? Y-IS A QUADRATIC. THAT'S CALLED A PARABOLA. SO PEOPLE NOTICE THIS IN THE MIDDLE AGE WHEN THEY INVENTED PHYSICS AND CALCULUS. PARABOLAS. SO THERE IT IS. IF THE PERSON THAT DID IT AT DIFFERENT SPEEDS UP AND DOWN YOU'D STILL GET A PARABOLA. THOSE NUMBERS WOULD BE DIFFERENT BUT YOU'D STILL GET'S PARABOLA. WE STILL HAVEN’T ANSWERED THE QUESTION HOWEVER DOES IT GO BEFORE IT HITS THE GROUND.

SO HOW DO WE DO IT WITHOUT ANY EFFORTS AT ALL. ANSWER RIGHT
Up there. At what time $t$ does it hit the ground? Four seconds. So how do I figure out how far it's gone? Evaluate $x$-of $t$ at $t$-equal four. When it hits the grounds. And you get whatever that is. 256 feet. Sorry, not four. Sorry. What time does it hit the ground? I copied the wrong number. It hits the ground at 8.4 owe second. Four is when it got to the top. And so it's 64 time eight-point owe 47 and that's about 515 feet. That's the easiest way to do it. But if you know this, what's the only way to do it? It's harder but you could do it. If you start from this equation how would you figure out how far it went had it hit the ground? Just starting with this. So it hits the grounds when $y$-equals zero. So what I get if I plug in $y$-equals zero, I get a quadratic and I can solve with the quadratic formula. So this is the one way to do it. And the other way to do it is solve zero equals that thing (on board). Solve the quadratic. And you get the same answer. Either way is fine. So that's a bit of calculus for nice, it's a home run. So the person hit a home run am 515 feet is pretty far. Are there any questions about that example? So this one, all the unit work, time was measured in second everywhere. Distance was feet. No problem. I'm going to do one more example about driving a car where you have to pay a little closer attention to the units do get the right answer.

Student: Given that the horizontal (inaudible).

Professor: Because it moving at constant velocity, 64 feet per seconds sideways. That was just given. And in the real world the air would slow it down a bit but I wasn't worried about that.
SO HERE’S THE SECOND EXAMPLE. YOU’RE DRIVING YOUR CAR. AT 55 MILES PER HOUR. OKAY. AND YOU SLAM ON THE BREAKS. AND I WANT TO KNOW HOW FAR DO YOU GO? OKAY TO ANSWER THAT QUESTION WE HAVE IT KNOW HOW MUCH THE BREAK ARE SLOWING YOU DOWN. SO HERE’S, THE BREAK ARE RATED TO SLOW YOU DOWN, 11 MILES PER HOUR FOR EVERY SECOND YOU HOLD ONTO THE BREAK, YOU’RE GOING SLOWER 11 MILES PER HOUR PER SECOND SO LONG YOU HAVE THE BREAK ON. THAT’S A, THAT MAKE SENSE PHYSICALLY. YOU’RE HOLDING YOUR FOOT ON THE BREAK AND AFTER YOU STARTED 55. AT ARE AFTER ONE SECOND HOW FAST ARE YOU GOING? FIFTY-FIVE MINUS 11 OR 44. TWENTY-SECOND YOU’RE GOING 33. SO THAT’S EASY. SO LET’S, SO WHAT WE HAVE HERE, LET’S USE OUR NOTATION, SO A-OF T-YOU’RE SLOWING DOWN AND SLOWING DOWN AT CONSTANT LEVEL, 11 MILES PER HOUR PER SECOND. SO IT'S CONSTANT DECELERATION. YOU'RE SLOWING DOWN AND THAT'S WHY IT'S A MINUS SIGN. SO NOW WHAT IS THE VELOCITY GOING TO BE? REMEMBER, WHATEVER THIS FUNCTION IS, THIS DERIVATIVE BETTER BE MINUS SEVEN. IT BETTER START AT 55. SO IT STARTS AT 55, AND THEN I HAVE TO ADD SOMETHING TO IT. SO WHEN I DIFFERENTIATE I GET MINUS 11. SO SHOULD IT BE? MINUS 11 TIMES TIME. SO LET'S JUST CHECK. IF YOU PLUG IN T-EQUALS ZERO WHEN THE WHOLE THING STARTS YOU'RE GOING 55 AND EVERY SECOND YOU'RE GOING A LITTLE BIT MORE SLOWLY. SO FAR SO GOOD. AND WHAT ARE THE UNITS ON THIS ONE? MILES PER HOUR, THAT'S MAKES SENSE. AND SO NOW WE HAVE TO GET THE DISTANCE, BECAUSE WE EVENTUALLY WANT IT KNOW HOW FAR WE GO. SO UNDO ONE MORE DERIVATIVE. SO WE HAVE INITIALLY, SO WE NEED A FUNCTION WHO'S DERIVATIVE IS THIS. HOW DO WE DO THAT? SO IT'S,
JUST ANOTHER POLYNOMIAL. I HAVE TO PUT A FEW MORE T-AND STUFF IN THERE. SO 55 TIMES T, IF I DIFFERENTIATE THAT I GET 55. HOW DO I GET IT ONE. MINUS 11 TIMES T-SQUARED IS THAT ENOUGH? DIVIDED BY TWO. IF I, AT S-EQUALS ZERO, TO START THIS IS ZERO, SO THAT'S WHEN I BEGIN. AFTER AWHILE I GO SOME DISTANCE. AND SO WHAT ARE THE, LET ME NOT ASK WHAT THE UNITS ARE. THEY'RE TRICKY. LET ME JUST WRITE. WE AGREE IF I TAKE THAT AND DIFFERENTIATE I GET THAT. I DIFFERENTIATE THAT I GET THAT. SO V OF T-S-(ON BOARD). OKAY. SO FAR SO GOOD. LET'S TRY TO SOLVE THE PROBLEM. SO WE SHOULD HAVE EVERYTHING WE NEED TO SAY HOW FAR WE GO. SO WHAT IS THE STOP? YOU STOP IF VELOCITY IS ZERO.

SOLVE V OF T-EQUALS ZERO. (ON BOARD). WHO CAN TELL ME WHERE T-IS? SOLVE THAT FOR ZERO AND T-IS FIVE. FIVE WHAT? SECOND. OKAY. THAT'S RIGHT. AFTER FIVE, SECONDS YOU STOP. AND SO THE DISTANCE TRAVELED DOES GOING TO BE THE DISTANCE WHEN YOU STOP, TIME T-EQUAL FIVE, SO LET'S JUST PLUG THAT IN. AND THE ANSWER IS 137 AND A HALF. 137 AND A HALF WHAT? MILES? IF YOU'VE GONE THAT FAR IN FIVE SECONDS, NO. NO. THERE'S NO FEET IN SITE. THAT'S A GOOD GUESS THOUGH. THERE'S SOME WEIRD UNIT THAT WE DIDN'T PAY ATTENTION TO THE UNITS. THIS IS 137 AND A HALF. THERE'S SOME WEIRD UNIT WE HAVEN'T FIGURED OUT. SO WE HAVE TO DO THAT. SO THE CLUE THAT WHEN I WROTE DOWN THESE EQUATIONS I HAD SOME TIMES IS IN HOURS AND SOMETIMES IT'S IN SECONDS. AND THAT MAKE THE UNIT WEIRD. SO LET ME TRY AGAIN AND CHANGE EVERYTHING TO SECONDS AND WE'LL SEE WHAT WE GET AND WE'LL UNDERSTAND WHY. SO LET ME WRITE, WHAT ARE THE UNITS?

SO LET'S DO THIS AGAIN, CHANGE ALL THE UNITS TO SECOND.
JUST SO THERE'S NO WEIRDNESS FROM THAT. SO LET'S START WITH THE, HOW MUCH YOU SLOW DOWN. SO THAT'S GOING TO BE SOMETHING, BUT I KNOW IT MEASURED IN MILES PER SECOND PER SECOND?

A THAT'S WHAT I MEAN BY ALL UNITS CHANGE. HOW MANY MILE PER SECOND DO YOU SLOW DOWN IN ONE SECOND. HOW MANY SECONDS IN AN HOUR. SIXTY SQUARED. SO THAT'S HOW MUCH YOU SLOW DOWN. IN ONE SECOND MEASURED MANY MILES PER SECOND. SO CHANGING. SO THIS, SO PUT A MINUS SIGN HERE. SO THIS IS A-OF T. MEASURED IN THOSE UNITS. SO NOW I HAVE TO FIND A FUNCTION THAT, WHOSE DERIVATIVE, THE VELOCITY, WHOSE DERIVATIVE IS THAT. SO IT'S GOING TO BE THE SAME THING AS BEFORE AM MULTIPLY THAT BY T-. SO THE DERIVATIVE WORKS. BUT I HAVE TO GET THE INITIAL VELOCITY RIGHT. YOU'RE GOING 55 MILES PER HOUR WHEN YOU START BUT NOW I WANT IT IN MILE PER SECOND. HOW MANY MILE PER SEXD IS 55 MILES PER HOUR. HAVE TO DIVIDE IT BY SOMETHING. IF YOU GO 55 MILES AN HOUR IN ONE SECOND YOU GO 13,600TH. BECAUSE THERE'S -- SO THIS IS ALL MEASURED IN MILES PER SECOND. THAT'S THE UNIT. AND IF YOU GIVE OF DIFFERENTIATE THIS FUNCTION YOU GET THAT. SO THE CALCULUS IS STILL RIGHT. YOU TAKE THIS FUNCTION AND DIFFERENTIATE IT. THAT'S A CONSTANT. GOES AWAY. SO THE CALCULUS IS THE SAME. AND SO NOW I HAVE TO GO ONE MORE STEP AND GET THE DISTANCE FUNCTION OF HOW FAR I'VE GONE. AND IT'S GOING TO BE THE SAME THING AS BEFORE. WHAT DOES T-TURN INTO WHEN I UNDO ONE MORE DERIVATIVE? SO THERE'S THE T-SQUARED OVER TWO. SO WHEN I DIFFERENTIATE THIS I GET THAT TERM. AND THEN, TIME T. WHEN I DIFFERENTIATE THIS I GET THAT. SO THE CALCULUS WORK. V IS STILL
S-PRIME OF T, DIFFERENTIATE THAT AND GET THAT. AND S-IS STILL THE DERIVATIVE OF THE VELOCITY. SO THE CALCULUS WORK EXACTLY THE SAME. AND NOW WHAT ARE THE UNITS OF DISTANCE? MILES. NO AMBIGUITY THERE. MILES PER SECOND TIMES SECOND. EVERYTHING WORK. AND THIS IS, MILES PER SECOND TIME SECOND. AND THAT'S MILES PER SECOND SQUARED TIMES SECOND SQUARED, SECONDS'S CANCEL OUT AND IT'S JUST MILES. SO MAYBE I SHOULD BE MORE CAREFUL. SO WHAT ARE THE UNIT HERE. SO THIS IS MILES PER SECOND.

MULTIPLIED BY SECOND. SECONDS CANCEL. SORRY. THIS IS MILES. THIS IS SECONDS TIMES SECONDS, SECONDS SQUARED. AND THIS T-SCARED IS ALSO SECOND SQUARED AM SECONDS SQUARED CANCELLING AND I GET MILES. SO ALL THE UNIT WORK RIGHT. AND THIS ONE I'LL WRITE, THIS WAS MILES, THE 11TH. THIS WAS SECOND SQUARED. MULTIPLY BY SECONDS, SECONDS DIVIDED BY SECONDS SQUARED I GET ONE SECOND LEFT. MILE PER SECOND. SO YOU CAN CANCEL YOU WANT JUST LIKE YOU DO ANYTHING ELSE. SO WE CHECKED, IT WORK. LET'S SEE IF WE GET THE RIGHT ANSWER. SO WE HAVE TO SOLVE, SO WHEN DOES IT STOP? I STILL HAVE TO SOLVE V OF T-EQUALS ZERO. WHEN DOES THE VELOCITY HIT ZERO, THAT'S WHEN WE STOP. SO SOLVE IT Z-EQUALS ZERO. SO, V OF T-EQUALS ZERO, 3600 CHANGE THAT. NOES IT'S THE SAME ANSWER. T-EQUALS FIVE. WE HAD THAT BEFORE. SO YOU STOP. THAT WAS RIGHT. SO NOW WE'RE GOING TO PLUG IN S-OF FIVE, INTO THAT THING. AND THE UNIT ARE GOING TO BE MILES.

AND I'LL JUST WRITE IT DOWN .0382 MILES. IS THAT WHAT HAPPENS WHEN YOU PLUG IT IN. AND IF YOU WANT IT CHANGE THAT TO FEET IT'S ABOUT 202 FEET ROUGHLY. AND THAT'S THE ANSWER AND HOW FAR YOU GO. THAT MAKE PHYSICAL SENSE TOO. SO THIS IS THE RIGHT ANSWER.
O ver here, so where is this 137.5 come from? That's the right answer but in weird units. So let's go back and figure out what the units are.

I want to leave this. Okay, so I think I can just look at this, these are the equations before. So 11 was how much you slowed down, unit as I said before were miles per hour. Per second. That's right from before. Let's just look at this.

Let's make sure the units work. 55, that 55 miles per hour minus 11, the units of 11 are miles per hour per second. And the units of time were seconds, so there's the 11, there's the seconds. Seconds cancel. I get miles per hour, miles per hour, and fine. That's miles per hour. So that make perfect sense. Those are the units of velocity. It's the last one that's weird. Let me write it down again. So let me again write down what the unit are carefully. So 55 is miles per hour times seconds. And then what is 11? Eleven was miles per hour per second. And what are the units of t-squared am we're measuring t-in second. So that's second squared am so those are the units of this thing. And I have a second squared in the second. That cancels. And I get miles, let me write it this way, miles times seconds per hour. That's a weird unit being isn't it. I can't cancelling those. They're both measuring time. But there's a second an hour. How do I make that cancel? How do you change. So I want to cancel this because I want it to be miles. I'm supposed to get miles and you want. This is disanls but it's miles times something weirder am how do I change this unit to
SECONDS? 3600 SECONDS. CHANGE SECOND TO 3600 SECONDS. NOW I CAN CANCEL SECONDS AND I GET MILES DIVIDED BY 3600. THAT'S THE UNITS AND 130,600TH OF A MILE. SO THIS 137.5 IS IN UNITS OF 136TH HUNDREDTH OF A MILE. SO LET'S SEE, IT BETTER BE THE SAME ANSWER AS WE GOT BEFORE. LET'S CHECK. SO THAT WAS THE ANSWER WE GOT BEFORE. .03 82 MILES. SO THE QUESTION IS, 137.5 IN UNIT OF A MILE, THAT'S WHAT THAT ANSWER IS. AND OVER HERE THE ANSWER WAS .0382 MILES. YES. THAT'S THE SAME THING. I GOT THE SAME ANSWER BOTH WAYS BUT I HAVE TO DIVIDE 137 AND A HALF BETTER -- SO YOU CAN GET THE RIGHT ANSWER BOTH WAYS BUT YOU HAVE TO BE CAREFUL OF THE UNITS. EASIER IF YOU KEEP IT ALL IN SECONDS. EVERY TIME IF DISTANCE IT'S MILES. EVERY TIME IT'S TIME USE SECOND. BUT CALCULUS, EVERY TIME IT SOLVE A SERIOUS PROBLEM YOU HAVE TO GET THE UNIT RIGHT.

SO ARE THERE ANY QUESTIONS ABOUT THESE TWO EXAMPLES? SO ONE OF THE THING WE'VE BEEN COUNTING ON HERE, WE'VE BEEN USING, IS THAT IF YOU KNOW THE VELOCITY, THEN WE TRY IT WRITE DOWN FORMULAS FOR THE DISTANCE. SO OUR STARTING POINT IS WE HAVE A FORMULA FOR THE VELOCITY. AND THEN WE'VE GONE AND WRITTEN DOWN BY UNDOING THE DERIVATIVE A FORMULA FOR THAT. SO FOR EXAMPLE, WEDS V OF T-EQUALS MINUS 11 T-PLUS 55 AND WE TRIED TO DRAW THE CONCLUSION THAT THIS WAS TRUE. WE USED AT THAT ONE JUST NOW. IS THAT THE ONLY FUNCTION S-OF T? WRITE THIS DOWN HERE. IS THIS THE ONLY FUNCTION S-OF T-WHOSE DERIVATIVE GIVES YOU THE RIGHT VELOCITY? IT WAS A RIGHT ONE FOR OUR PROBLEM. BUT WE HAVE SOME EXTRA INFORMATION FOR OUR PROBLEM. AT T-TIME EQUALS ZERO WE WERE STARTING AT ZERO. WE USED THAT. BUT IF I DIDN'T TELL YOU THAT,
THEN YOU JUST HAD TO WRITE DOWN ANY OLD FUNCTION S-OF T. IS THERE ANOTHER ONE YOU CAN THINK OF? WITH THE PROPERTY S-PRIME -- I COULD ADD PI OR SUBTRACT 37 OR ADD ANY CONSTANT. RIGHT. THEY ALL WORK. BECAUSE WHEN I DIFFERENTIATE THE CONSTANTS GOES AWAY. THAT'S, ALL I'M POINTING OUT IS YOU CAN ADD ANY CONSTANT TO S-OF T. AND IT STILL WORKS. BECAUSE WHEN YOU DIFFERENTIATE THE CONSTANT GOES AWAY. SO WHEN YOU DO YOUR PROBLEM YOU NEED EXTRA INFORMATION TO PICK CONSTANT. SO IF THE FIRST EXAMPLE THE EXTRA INFORMATION WAS YOU STARTED 6 FEET OFF THE GROUNDS. SO I ADDED SIX. SO THERE'S SOME EXTRA INFORMATION. BUT, SO I JUST WANT IT POINT THAT OUT SO YOU DO IT RIGHT. WHETHER YOU HAVE TODAY EYE REAL PROBLEM YOU USE CALCULUS TO MODEL THE REAL PROBLEM.

ARE THERE ANY QUESTIONS ABOUT APPLYING DERIVATIVES. I WANT TO GO BACK TO EARLIER IN THE CHAPTER AND USE STUFF FROM BEFORE. I HAVE A BUNCH OF POSSIBILITY BUT ARE THERE ANY QUESTIONS BEFORE I GO BAG.

STUDENT: WRAFS THE CONSTANT BE AT THE STARTING POINT.

PROFESSOR: IF YOU KNEW YOU WERE STARTING AT POSITION, STARTING AT ONE AT THAT WOULD MEAN S-OF ZERO WAS ONE. SO YOU ADD ONE. JUST THE STARTING POINT. WE WRITE THAT DOWN. YOU NEED THE STARTING POINT.

STUDENT: STILL NOT SURE WHEN YOU'RE UNDOING THE DERIVATIVE HOW YOU GET NEGATIVE 11 T-SQUARED OVER TWO.

PROFESSOR: SO WHAT I HAVE, AS I START WITH THIS, AND I KNOW IT PAY THE FUNCTION WHOSE DERIVATIVE IS THAT. SO I JUST HAVE V OF T-EQUALS MINUS 11 T. AND I WANT ANOTHER FUNCTION THAT WHEN I
DIFFERENTIATE IT I GET THAT. SO IF I LOST OFF THIS CONSTANT WHAT FUNCTION WOULD I DIFFERENTIATE JUST TO GET T. THE DERIVATIVE OF 19

WHAT IS T? IN THE ANSWER WAS, IT SQUARED OVER TWO. BECAUSE IF I DIFFERENTIATE T-SQUARED OVER TWO, WHAT I SION POWER RULE SAY, THIS IS A GOOD QUESTION AM I'M ABOUT TO SHOW YOU WHERE THE POWER RULE COMES FROM. TO DIFFERENTIATE THIS YOU PULL DOWN TWO. SUBTRACT ONE FROM THE COMPONENTS YOU GET T-TO THE FIRST POWER. AND SO THIS TURNS INTO THAT AND THEN I MULTIPLY BY MINUS 11 BECAUSE THAT'S PART OF THE GAME.

STUDENT: IS THERE ANY ADVANTAGE TO DOING THAT OVER JUST DIVIDED THE (INAUDIBLE) BY TWO.

PROFESSOR: SO THE WORD FOR UNDOAG AGO DELIVERY POINT OUT IS CALL INTEGRATION AND THAT'S PART OF COURSE TOO ALTHOUGH I HAVEN'T USED THE WORD YET AND I AM GOING TO GETS THAT IS CALL GAS IF YOU LIKE. IF THAT'S A FAMILIAR NOTION AS OPPOSED TO UNDOING A DERIVATIVE, YOU'RE FREE TO THINK OF IT THAT WAY ATTITUDE WHAT I MEANT SAY IEWRG GOING FROM MINUS 11 TO SQUARE. WHETHER YOU'RE TALKING IT UP SEASON IT EASY (INAUDIBLE).

PROFESSOR: YOU, ONCE YOU'VE DONE IT YOU CAN WRITE IT ANYWAY YOU LIKE. IT'S ALL THE SAME THING. WHATEVER'S EASIER. SO SINCE, I'M GRAD YOU ASKED ABOUT IT BECAUSE I WANT TO EXPLAIN WHERE POWER RULE COMING FROM. SOMETHING YOU MAY OR MAY NOT BE FAMILIAR WITH FROM HIGH SCHOOL, I'M NOT SURE.

TAKE X-TO SOME POWER, PULL THE POWER DOWN, SUBTRACT FROM COMPONENT AND THAT'S HOW YOU DIFFERENTIATE. THAT'S ONE OF OUR BASIC RULES. SO THIS IS FOUR, THIS IS TRUE FOR ANY KNOWN ZERO, IF R-IS ZERO THIS IS THE FUNCTION ONE. THAT WAS FOR ANY OLD R.
AND I'M GOING TO TELL YOU WHERE IT COME FROM, WHEN R-IS A
POSITIVE INTEGER. SO IT'S ONE, TWO, THREE, ... WHEN R-IS
POSITIVE INTEGER I'LL SHOW YOU WHERE THIS COME FROM. AND WHO
REMEMBERS PASCAL'S TRIANGLE? OKAY. I'M GOING TO REMINDS YOU
MUCH PASCAL'S TRIANGLE AND THAT'S WHERE IT COMES FROM.

OKAY. SO WHAT I'M GOING TO TRY TO DO, HERE'S MY FUNCTION.
(ON BOARD). AND I WANT TO COMPUTE THE DERIVATIVE. AND THAT'S A
LIMIT. AS H-GOES TO ZERO OF, SO THERE'S THE GENERAL DEFINITION
AGAIN. AND LET ME PLUG IN R-F-OF X. SO IT'S X-PLUS H-TO THE
POWER R-MINUS X-TO THE R-DIVIDED BY H. SO I HAVE TO WORRY ABOUT
H-PLUS H-TO SOME INTEGER, THAT'S WHAT I NEED TO FIGURE OUT. AND
PASCAL'S TRIANGLE TELLS YOU THE ANSWER. (ON BOARD). SO LET ME
MAKE A TABLE JUST TO REMIND YOU OF WHAT IT IS. AND SO A LOT OF
IMAGINE AND SCIENCE COMES FROM WRITING DOWN A BUNCH OF DISAJ AND
SAYING THERE'S A PATTERN AM AND THEN YOU FIGURE OUT WHY THE
PATTERN IS TRUE. COMES DOWN TO RECOGNIZING THERE'S A PATTERN.
SO START WITH JUST NUMBER ONE. JUST TO HAVE, TOP OF TRIANGLE.
THEN I'M GOING TO TAKE X-PLUS H-TO THE FIRST POWER. THERE'S NOT
MUCH TO IT. BUT WRITE THAT AS ONE TIMES X-PLUS ONE TIMES H-.
AND WRITE DOWN THOSE COEFFICIENTS OVER HERE IN THE SECONDS ROW
THE TRIANGLE. ONE AND ONE. ?ESM TAKE X-PLUS H-SQUARED AND IF I
MULTIPLY THAT OUT, X-QIERTD PLUS TWO (ON BOARD) AM THERE'S A
COEFFICIENT OF ONE IN THERE. AND GOING TO WRITE DOWN THOSE
NUMBERS AS THE NEXT ROW OF TRIANGLE. NOW YOU MAY OR MAY NOT
REMEMBER THIS, BUT YOU WILL BY THE TIME I'M DONE. MULTIPLY IT
OUT. YOU'RE GOING TO GET ONE TIMES X CUBED PLUS THREE X-(ON BOARD). AND IF I KEEP FILLING IN THE TRIANGLE I GETS ONE, THREE, THREE, ONE. AND MAYBE I'LL JUST DO ONE MORE. FOURTH POWER. ONE TIMES X-TO THE FOURTH PLUS FOUR X-CUBED H-PLUS SIX X-SQUARED H-SQUARED PLUS FOUR X-H-CUBED PLUS ONE H-TO THE FOURTH AND I'LL WRITE DOWN ALL THOSE COEFFICIENTS ONE MORE TIME. AND TRUST YOUR INTUITION, WHAT'S THE NEXT ROW OF THIS TRIANGLE? (ON BOARD). THE INTUITION IS YOU TAKE THE TWO NUMBERS ABOVE AND ADD THEM. OR IF ON THE ENDS YOU PUTS ONE AND THAT'S PASCAL'S TRIANGLE. A SIMPLE PATTERN. AND THOSE ARE EXACTLY THE NUMBERS. THAT YOU GET WHEN YOU TAKE THESE POWERS OF X-PLUS H. SO LET'S JUST ACCEPT THAT PATTERN FOR NOW. I WILL EXPLAIN IT IN A MOMENT. BUT LET'S JUST ACCEPT IT AND GO FINISH DOING THE DERIVATIVE.

X-to the r. (on board). These cancel. This term, copy it over here am so I get r-x-to the r-minus one h-divided by h, that goes away. Plus stuff divided by h. (on board). So there's what I want. Good stuff. What is, so what do I know about the stuff? It has in it either h-squared or h-cubed or h-to the fourth am that's what is in it. So what is stuff over h-have in it? H-to what power? So remember stuff has something plus h-squared -- divided by h. So what powers of h-are left? H-h-squared, h-cubed. H-squared term turns into h-. If I divide this by h, if I divide this by h-I get h. If I divide that by h-I get h-squared. But there is at least one h-in every one of these terms. (on board). Okay. So now, what do I need to do? I need to take the limit, as h-goes to zero of that, so what happens to this now when I take the limit as h-goes to zero in this doesn't depend on h at all. (on board). What about this stuff? It all has an h-in it. I'm done. So it's just that first little piece of Pascal's triangle, and that's what, that's where the power rule comes from. If you can remember the picture then you can remember the power rule. So I want to take, what I want to do now is say, it's really easy to see why this is true. So why does Pascal's triangle work? Just show you how to get from one line to the next. So here, let's suppose I've already figured out, next line is one plus h-plus h-to the fourth so many let me write it this way. (on board). There's the previous line. So

Let me write that out. (on board). So there's the previous line. So I'm going it take this, multiply it bhi x. That always
ONE TO ALL THESE COMPONENTS. X-CUBED TURNS INTO X-TO THE FOURTH
AM X-SQUARED TURNS INTO X-CUBED. X-TURNS INTO X-SQUARED. AND
THEN I GET, THAT'S X-TIMES THIS WHOLE THING. AND NOW MULTIPLY
IT BY H-. ADD IT. LINE THEM UP SO I CAN ADD THEM RIGHT. (ON
BOARD). THERE'S H-TIMED X-CUBED AM AND I GET H-TIMES THIS. AND
I GET H-TIMES THIS. PLUS H-TIMES THAT. AND I LINE THEM ALL UP
SO THAT I CAN ADD ALL THE TERMS LINE UP, I CAN ADD THEM. SO I
GET ONE TIMES X TO THE FOURTH. AND THEN I GET THREE PLUS ONE,
FOUR TIME X-CUBED H-. AND THEN I GET THREE PLUS HE THREE EQUALS
SIX (ON BOARD) AM AND THEN I GET THREE PLUS ONE X-H-CUBED AND
THEN I GET ONE H-TO THE FOURTH, (ON BOARD). THEY JUST LINE UP.
THOSE ARE THE SAME NUMBERS MUCH 1331, THERE'S ONE, THREE, THREE,
ONE, SHIFT THEM OVER AND ADDS THEM. THAT'S WHAT IT MEANS TO ADD
YOUR NEIGHBORS UP THERE. TAKING TWO COPIES OF 1331. SHIFT THEM
OAF. SO IT'S JUST A LITTLE BIT OF ALGEBRA.

ANY QUESTIONS? ONE LAST EXAMPLE AND CALL IT QUITS.

STUDENT: REVIEW NEXT TUESDAY. ARE WE GOING TO DO REVIEW NEXT
TUESDAY.

PROFESSOR: NEXT TUESDAY IS ALL REVIEW. PROBABLY SPEND TIME
DOING OLD SAMPLE MIDTERMS. THAT SORT OF THING.

STUDENT: WHEN ARE YOU POSTING THE ANSWERS.

PROFESSOR: EVERYTHING'S BEEN POSTED. SO SAMPLE MIDTERMS HAVE
BEEN POSTED ON THE WOULD BE PAGE. FROM PREVIOUS SEMESTER AND

OAFNLG ONE COULD I FIND ANSWERS. SO THERE ARE THREE MIDTERMS
FROM OLD SEMESTERS AND ONE OF THEM HAS ANSWERS AND TWO DO NOT.
THAT'S JUST THE WAY IT'S.

STUDENT: MIDTERM GOING TO BE THE SAME THING.
PROFESSOR: I put them because they will be very similar in style.

STUDENT: Are you only deducting point for true false seconds.

PROFESSOR: Yeah. Right. You can opening statement get a A-in gnawed and the in true or false because it's too -- let me the rules for all milled term closed book, closed network, closed computer am open minds. And there will be three versions of midterm so you'll always be sitting next to someone with different versions.

Last example. So we've seen functions f-of x-where we may not be able it differentiate. We have f-prime of x-may not exist. And basically if you have a corner, something can go wrong. Here was our example before. At that corner there's no unique tangent line. So we've done that. So here's the question. Can you think of a function, let's call it g-of x, where you differentiate it so that it's t exist everywhere, so no corners, but g-double prime of x-does not exist everywhere. If you can differentiate it once, no corners but you can't differentiate it twice. So the hint is something's wrong with g-prime so maybe it has a corner. Okay. So maybe g prime has a corner even though g-does not. So let's just try this example.

Try g-prime equals absolute value. Can you find x, can you find g-so at that g-prime is the absolute value?

So let me draw it here am so here's our function g-prime of x-equals absolute value. So the question is with a would g-look like? So let's figure out what g-has it look like over here,
OVER HERE, G-OF X-IS NEGATIVE X. SO ON THIS SIDE THAT'S WHAT THE
ABSOLUTE VALUE IS. G-PRIME OF X. G-PRIME OF X-IS NEGATIVE
X-OVER HERE. SO G-PRIME OF X-IS X, WHAT IS G-OF X? WE DID THIS
ONE EARLIER TODAY. SO IF I, YOU KNOW THE DERIVATIVE IS X, WHAT
DO I PUT HERE? X-SQUARED DIVIDED BY TWO. ON THAT SIDE WHEN X-IS
POSITIVE. WHAT ABOUT OVER HERE? SO G-OF X-IS GOING TO EQUAL
WHAT? F-OF NEGATIVE X-SQUARED. WE KNOW -- OVER ON THIS SIDE I
GET A PARABOLA. ON THIS SIDE I GET THE PARABOLA GOING DOWN. (ON
BOARD). AND FORTUNATELY THEY MEET IN THE MIDDLE SO G-OF ZERO
EQUALS ZERO. SO LET'S, THAT'S MY PROPOSED FUNCTION WHERE YOU CAN
DIFFERENTIATE IT ONCE, YOU GET ABSOLUTE VALUE BUT THEN YOU'RE
STUCK, YOU CAN'T DIFFERENTIATE ANYMORE BECAUSE YOU GET A CORNER
AM SO THERE'S TO QUESTION HERE WHEN X-IS BIGGER THAN ZERO. SO
G-PRIME OF X-IS X-OVER HERE. AND OVER HERE, G-PRIME OF X-IS
NEGATIVE X-IF X-LESS THAN ZERO. SO THE OWN POINT WE HAVE TO
CHECK IS RIGHT THERE. WE HAVE TO CHECK TO THE LIMIT OF (ON
BOARD). WE HAVE TO DO THAT. THERE'S A DIN RULE FOR WHAT G TO
THE LEFT AND RIGHT, DIFFERENT RULE FOR WHAT G-IS TO THE LEFT AND
RIGHT. SO G-OF ZERO IS ZERO. AND G-OF ZERO PLUS H-DEPENDS ON
THE LEFT OR RIGHT. IT'S EITHER H-SQUARED OVER TWO IF H-IS

POSITIVE. AND IT'S MINUS H-SQUARED OVER TWO IF H-IS NEGATIVE.
THAT'S WHAT THE FUNCTION IS. SO NOW I HAVE TO DIVIDE BY H. SO
THAT'S H-OVER TWO IF H-IS POSITIVE. AND NEGATIVE H-OVER TWO IF
H-IS NEGATIVE. AND I THINK THE LIMIT OF THAT AS H-GOES TO ZERO.
ANY PROBLEM WITH THAT LIMIT? H-OVER TWO ON THE PLUS SIDE -- AND
THE LIMIT IS ZERO. SO THE LIMIT EXIST, THE DERIVATIVE EXIST AND
THERE IS ABSOLUTE VALUE. OKAY. GOOD PLACE TO STOP.