
GSI: AN ANNOUNCEMENT.

GSI: MY NAME IS CASPER AND THIS IS MELISSA. N-TAKE THIS OPPORTUNITIES TO INVITE ALL OF YOU TO THE FIRST EVENT THIS SEMESTER. THIS WILL BE A GOOD OPPORTUNITIES FOR YOU TO EXPLORE DIFFERENT INDUSTRIES OF BUSINESS. MANY OF OUR ALUMNI -- SO ONCE AGAIN EVENTS 730 IN WHEELER AND THANK YOU FOR YOUR TIME AND HOPE TO SEE YOU THERE.

PROFESSOR: SO WELCOME BACK. I WAS IN THE MIDDLE OF TALKING ABOUT CONTINUITY IN DERIVATIVES. LET ME BACK UP A TINY LITTLE BIT AND REMINDS YOU OF A FEW OF THING I WAS TRYING TO SAY. THE QUESTION WE WERE ASKING OURSELVES WAS, HOW DO WE RECOGNIZE WHEN A FUNCTION F-OF X-HAS A DERIVATIVE. SO WE WROTE DOWN THE DEFINITION. WE HAD IT ASK OURSELVES, A DERIVATIVE AT A PARTICULAR POINTS A. SO WHAT WE DID WAS TOOK (INAUDIBLE).

HERE'S THE DEFINITION. THE QUESTION WAS WHAT DOES IT TAKE FOR THIS TO EXIST. IF IT'S EXIST, THAT'S THE DEFINITION OF F-PRIME OF A. AND EASIEST THICK YOU CAN DO WITHOUT DOING ANY WORK AT ALL IS TO SAY AS H-GETS TINY AND DIVIDE BY ZERO SO THE NUMERATOR BETTER GET DINE I. OTHERWSE I'M GOING TO BE DIVIDING BY ZERO AND IT WON'T WORK. SO WE CLEARLY NEED A NUMERATOR BETTER GO TO ZERO. AND WE GAVE THAT NICE PROPERTY A NAME. WE CALLED IT CONTINUITY. SO F-OF X-IS CONTINUOUS AT A, THAT DENOMINATOR GOES TO ZERO. SO LET ME WRITE DOWN, LIMIT WHEN H-GOES TO ZERO (ON BOARD). AND THE INTUITION IS THAT LET'S SAY THAT'S A. WHAT'S,
YOU CAN DRAW THE FUNCTION THROUGH THE POINT WITHOUT THE LIFTING YOUR CHALK OFF THE BOARD. THAT WAS VERY SIMPLE IDEA. YOU CAN PLOT F-OF X-NEAR X-EQUALS A-WITHOUT LIFTING YOUR PENCIL OR EXAMPLE, RUNNING OUT OF LEAD. BECAUSE THE FUNCTION WIGGLES SO MUCH. ON THE OTHER HAND, AND THIS IS JUST WHERE I ENDED LAST TIME, SO WHAT WE JUST SAID HERE IS THAT DIFFERENT SHABILITY IMPLIES CONTINUITY. SO WRITING DOWN IN ENGLISH THE SAME THING WE JUST SAID. SO IF IT'S DIFFERENTIABLE THE NUMERATOR THAT'S TO GO TO ZERO, IT HAS TO BE CONTINUOUS. ANOTHER WAY TO SAY THAT IS CONTINUITY IS NECESSARY FOR DIFFERENT SHAIBILITY. SO THE VERY LAST THING I SAID WAS IT'S NOT ENOUGH ALL BY ITSELF. NOT ENOUGH BY ITSELF. SO LET ME DO THE EXAMPLE THAT REMIND YOU OF THE EXAMPLE WE HAD LAST TIME. LET'S TAKE THE FUNCTION F-OF X-EQUALS ABSOLUTE VALUE OF X. IT ASKS, THE PLOT OF THIS IS THERE, THERE, THERE IT IS, OVER HERE, F-OF X-EQUALS X-AND OVER HERE F-OF X-EQUALS NEGATIVE X. SO THE QUESTION IS IS THIS FUNCTION DIFFERENTIABLE AT X EQUALS ZERO. AND SO LET ME APPEAL TO YOUR INTUITION. SO WHAT DOES THAT MEAN, I-E, DOES IT HAVE A TANGENT LINE. AND WHAT'S THE TANGENT LINE? IS THERE A LINE THROUGH THIS POINT, THROUGH X-COMMA F-OF X-WHICH IS JUST THE ORIGIN, IS THAT, SO WHAT THE TANGENT LINE HAVE TO BE THAT ONLY INTERSECTS THE CURVE, ONLY INTERSECTS THE GRAPH ONCE NEAR THAT POINT. CAN'T GO THROUGH IT TWICE. AND IT'S UNIQUE. THERE'S ONLY ONE WAY TO DRAW IT. ONLY ONE WAY TO DRAW A TANGENT LINE. SO LOOK AT THIS POINTS AND ASK CAN I DRAW A LINE THROUGH THERE THAT ONLY THE INSECT AT ABSOLUTE VALUE PLOT ONCE. YEAH, SHIEWMPLET AND THERE'S
ONLY ONE WAY TO DO IT. NO, I CAN ALSO DRAW THAT ONE. THAT ONE WORKS, AND THAT ONE WORK. THERE'S NO SINGLE TANGENT LINE, THERE'S NO LIMIT. UNIQUE, THE DEFINITION OF A LIMIT SAYS THE SLOPE, THERE'S EXACTLY ONE SLOPE. WHEN YOU WRITE DOWN THAT LIMIT UP THERE, THAT THING, DIFFERENT VALUES. JUST ONE VALUE THAT IT APPROACHES. JUST ONE TANGENT LINE AM AND HERE YOU CAN SEE THERE'S A BUNCH OF THEM. SO THAT'S THE INTUITION THAT SAYS CORNERS ARE PROBLEMS. YOU DON'T GET TANGENTS IN CORNERS. SO THAT'S THE GEOMETRIC INTUITION. BUT LET JUST PLUG IT IN THE ALGEBRA. AND ASK DOES THIS LIMIT EXIST? SO HERE IT IS. THERE'S, THAT'S THE LIMIT YOU WANT TO TALK, EVALUATING IT AT X-EQUALS ZERO. SO LET ME JUST KEEP GOING. THIS EXIST. SO IF IT DOES, LET ME PLUG IN H-GOES TO ZERO, ABSOLUTE H-MINUS ABSOLUTE ZERO, AND, DIVIDED BY H. OKAY. (ON BOARD). SO THAT'S THE FUNCTION. WHAT DOES THIS FUNCTION LOOK LIKE? SO THERE'S THE FUNCTION. I WANT TO KNOW DOES IT APPROACH LIMIT AS P-GET SMALL. SO LET ME PLOT IT. GOING TO HAVE H-ON THAT AXIS. AND PLOT OF GRAPH OF THIS FUNCTION. WHAT DOES THIS FUNCTION EQUAL WHEN H-IS POSITIVE? AND WHAT HAPPENS WHEN H-IS NEGATIVE? THERE'S PLUS ONE. WHAT HAPPENS WHEN H-IS NEGATIVE? IT'S NEGATIVE ONE. AND SO THAT'S WHAT THE GRAPH OF THIS THING LOOKS LIKE. DOES IT HAVE A LIMIT? AS H-GOES TO ZERO? NO BECAUSE ON THIS SIDE IS GETS CLOSER TO ONE, IT'S ONE. ON THIS SIDE IT'S NEGATIVE ONE AM REMEMBER LIMIT HAVE TO APPROACH A SINGLE NUMBER AS YOU GET CLOSE TO THE POINTS. SO YOU WANT TO GO BY YOUR INTUITION WHICH SAYS AT A CEARN THERE IS NO UNIQUE WAY TO DRAW A TANGENT. OR YOU DO THE
ALGEBRA. THIS FUNCTION DOESN'T HAVE A LIMIT. EITHER WAY YOU CONVINCED YOURSELF THAT IT WAS NOT ENOUGH THAT THIS FUNCTION WAS CONTINUOUS. THAT I CAN DRAW THE ABSOLUTE VALUE FUNCTION JUST BY LEAVING CHALK ON THE BOARD. SO ARE THERE ANY QUESTIONS ABOUT THAT?

I SEE A FEW PUZZLED FACES SO IT'S OKAY TO ASK.

FOR EXAMPLE, NO DERIVATIVES AT CORNERS. OKAY. SO YOU HAVE TO BE CAREFUL. LET ME DO ANOTHER EXAMPLE. LEAVE THAT DEFINITION UP THERE. ERASE THIS ONE. LET ME DEFINE A FUNCTION. AND WE'RE GOING TO ASK IS IT CONTINUOUS. AND IS IT DIFFERENTIABLE. SO HERE IT IS. IT'S GOING TO BE X-SQUARED IF X-IS LESS OR EQUAL TO ONE. AND TWO X-MINUS ONE IF X-IS GREATER THAN ONE. SO THERE'S ANY FUNCTION. IT HAS TWO DIFFERENT DEFINITIONS, TWO DIFFERENT DEFINITIONS DEPENDING ON WHERE YOU ARE. SO HERE'S POINTS ONE. ON THIS SIDE THERE IS JUST A PARABOLA. SO THERE'S THE X-SQUARED FUNCTION. OKAY. AND ON THIS SIDE X-IS GREATER THAN ONE, IT'S THE FUNCTION TWO X-MINUS ONE THAT'S A STRAIGHT LINE. SLOPE IS, STRAIGHT LINE IS TWO. AND WHAT IS IT'S EQUAL WHEN X-EQUALS ONE, TWO TIMES ONE MINUS ONE SO. IT STARTS HERE AND THEN IT'S JUST A STRAIGHT LINE. TWO X-MINUS ONE. SO THAT'S WHAT IT LOOKS LIKE. SO THE FIRST QUESTION IS F-OF X-CONTINUOUS FOR ALL X? SO LET ME ASK THAT QUESTION. NOT JUST AT ONE POINT. IF WE PICK ANY OLD X, IS IT CONTINUOUS? SO I CAN PICK THAT X-OR THAT X-OR THAT, OR THE INTERESTING X OF COURSE IS RIGHT THERE. SO IF X-IS LESS THAN ONE, SO THAT F-OF X-EQUALS X-SQUARED THAT'S JUST A POLYNOMIAL. NICE AND SMOOTH. EASY, IT IS CONTINUOUS.

IF I MAKE X-COLORSER AND CLOSER. -- SO THIS IS CONTINUOUS.
AND IF X-IS GREATER THAN ONE, SO THAT F-OF X-EQUALS TWO X-MINUS ONE, THAT'S A STRAIGHT LINE, THAT IS CONTINUOUS. THE ONLY INTERESTING POINT IS WHAT TO ASK IF YOUR EXACTLY AT THE POINTS ONE ARE YOU CONTINUOUS? THAT'S THE ONLY INTERESTING QUESTION. BUT SINCE WE HAVE THESE TWO DIFFERENCE FORMULAS. ONE ON THE LEFT AND ONE ON THE RIGHT. SO ASK THE QUESTION ARE WE CONTINUOUS DOES THAT LIMIT EXISTS. WE HAVE TO ASK OURSELVES WHAT HAPPENS AS WE GET CLOSER FROM THE RIGHT AND WHAT HAPPENS AS WE GET CLOSER FROM THE LEFT. AND WE HAVE TO MAKE SURE THEY APPROACH THIS NUMBER. I'VE DRAWN IT SO THERE'S NO MYSTERY HERE. AND THE ANSWER IS YES. IT IS CONTINUOUS. SO LET ME JUST WRITE THAT DOWN. SO THERE'S TWO FORMULAS. ON THE LEFT AND THE RIGHT. OKAY. AND SO LET JUST USE EACH ONE, AS X-INCREASES TO ONE, SO IT STARTS OVER HERE AND MOVES GRADUALLY THAT WAY, WHAT HAPPENS TO X-SQUARED? IT ALSO APPROACHES (INAUDIBLE). SO ON THIS SIDE THE FRUNGS IS X-SQUARED AM AS X APPROACHES ONE X-SQUARED APPROACHES ONE. AND AS X-DECREASES, SO ON THE RIGHT-HAND SIDE NOW. AS X-DECREASES TO ONE, WHAT HAPPENS F-OF X-IS EQUAL TO TWO X-MINUS ONE, IT ALSO DECREASES TOWARD ONE. AND SO YOU GET THE SAME VALUE YOU GET CLOSE TO ON EITHER SIDE SO IT IS CONTINUOUS. IT'S OBVIOUS FROM THE PICTURE. I JUST WROTE IT DOWN. THEL THESE FORMULAS WASN'T THERE AOE I, YOU'D HAVE TO DO THE WORK. YOU'D HAVE TO SAY WHAT HAPPENS AS I GET CLOSER TO THE LEFT AND WHAT HAPPENS AS I GET CLOSER TO THE RI. SO THE ANSWER IS YES. NOW, THE NEXT QUESTION IS WHAT ABOUT (INAUDIBLE). THAT'S GOING TO REQUIRE A LITTLE BIT MORE THINKING BECAUSE IT'S ISN'T OBVIOUS THAT THERE'S NOT'S CORNER
THERE. WE HAVE TO BE A LITTLE BIT MORE WORK TO MAKE SURE THERE'S NO CORNER THERE. YOU SHOULDN'T TRUST YOUR DRAWING FOR THAT. YOU HAVE TO DO THE ALGEBRA. IS THERE A CORNER THERE OR NOT? OKAY. SO LET ME DO THAT.

SO I'LL ASK THE SAME QUESTION. ABOUT DIFFERENT SHABILITY, IS THIS FUNCTION OVER THERE IS IT DIFFERENTIABLE EVERYWHERE? ALL DIFFERENT VALUES OF X. SO AS BEFORE THERE ARE GOING TO BE THREE CASES WHERE YOU'RE OVER TO THE LEFT OF ONE, TO THE RIGHT OF ONE AND EXACTLY EQUAL TO ONE. SO IF YOUR TO THE LEFT OF ONE, F-OF X-EQUALS X Squared, PERIOD, THERE'S NO DOUBT ABOUT IT. SO WE CAN DIFFERENTIATE THAT BECAUSE WE KNOW HOW TO DIFFERENTIATE THIS RULE. DIFFERENTIATING POWER. SO WHAT'S THE DERivative. TWO X. SO YOU'RE FINE THERE. IF X-IS GREATER THAN ONE WE HAVE ANOTHER FORMULA THAT WORKS. SO F-PRIME OF X-EQUALS TWO. OKAY. NOW FINALLY IF X-EQUAL ONE YOU DON'T HAVE A NICE FORMULA. WE HAVE TWO FORMULAS ON THE LEFT AND RIGHT. SO WHAT WE NEED TO DO IS TAKE THE LIMIT AS H-GOES TO ZERO OF F-OF ONE PLUS H-MINUS F-OF ONE DIVIDED BY H- AND ASK DOES THAT APPROACH A LIMIT? WHAT DO WE HOPE THE LIMIT IS? WHAT'S YOUR BEST GUESS? BASED ON WHAT I'VE WRITTEN UP? ON, IF WE'RE ON THE RIGHT, THE SLOPE IS TWO ALL THE TIME, IT'S A STRAIGHT LINE. AND IF WE'RE ON THE LEFT X-IS REALLY CLOSE TO ONE, WHAT HAPPENS TO THIS? TWO X-GETS CLOSER AND COLORS TO WHAT AS X-GUESS COLORS TO ONE? IT GETS CLOSER AND COLORS TO TWO. AND WE'RE GOING -- THERE IS ACTUALLY A TANGENT LINE, NOT A CORNER. YOU HAVE TO WORK IT OUT. SO THE POINTS IS, LET ME WRITE THIS DOWN, SO THIS IS WHAT WE HAVE TO ASK. DOES THIS LIMIT EXIST. F-OF ONE IS ONE. SO LET ME PLUG THAT IN. DOES THIS
EXIST? AND THE TROUBLE IS OF COURSE WE HAVE A DIFFERENT FORMULA ON THE LEFT AND RIGHT. SO WE WILL JUST DO THOSE. ON THE LEFT, THAT MEANS H-IS GOING TO BE JUST NEGATIVE, WE'RE GOING TO HAVE ONE JUST A LITTLE BIT LESS THAN ONE. SO LET'S LIMIT. SO H-IS GOING TO INCREASE TO ZERO. H-IS GOING TO BE NEGATIVE IN AND GROW UPWARDS TOWARD ZERO AND WE'RE GOING TO ASK DID THIS WORK. DOES IT HAVE A VALUE? SO THIS CASE IS APPROACH ONE FROM THE LEFT. AND NOW SINCE I, I'M ON THE LEFT I KNOW THERE'S A FORMULA FOR THIS AND IT'S ONE PLUS H-SQUARED BECAUSE IT'S THE SQUARE FUNCTION RP MINUS ONE DIVIDE BY H-. NOW I CAN ACTUALLY WORK WITH THAT. AND MULTIPLY IT OUT. ONE PLUS H-QUANTITIES SQUARED? WHAT IS IT, ONE PLUS TWO H-PLUS H-SQUARED SUBTRACT ONE OVER H. NOW LET'S ME DO THE ALGEBRA, PLUS ONE MINUS ONE THEY CANCEL. I GET AN H-FACTOR. I CAN CANCEL H-S. I CAN CANCEL THAT H-WITH THAT H-AND AND WHEN I CANCEL H SQUARED DIVIDES DOLLARS BY H-THE -- SO I HOPE I CAN JUST DO ALL THAT ALGEBRA. SO WITH A DO I HAVE LEFT ONCE I CANCEL EVERYTHING, I HAVE TWO PLUS H-. THAT'S EASY, WHAT'S THE LIMIT OF TWO PLUS H? TWO. SO FROM THIS SIDE IT ALL WORKS. (ON BOARD). FROM THE OTHER SIDE WE'RE GOING TO GO ON THE RIGHT NOW. SO H-IS GOING TO BE POSITIVE AS APPROACH ONE FROM THE RIGHT. SO I'M GOING TO HAVE, ASK IS THE LIMIT AS H-GETS CLOSER AND CLOSER TO ZERO FROM FROM ABOVE, TINY POSITIVE NUMBER, IT'S GOING TO BE THE SAME DIVIDED DIFFERENCE. ARROW POINTING DOWN MEANING H-IS GETTING SMALLER, SO I GET F-OF ONE PLUS H-MINUS ONE DIVIDED BY H, BUT NOW THAT H-IS POSITIVE, I KNOW THAT I CAN PLUG IN FORMULA. FOR F-OF ONE PLUS H-AND IT IS TWO TIME ONE PLUS
H-Minus one minus one divided by H. So I just, that's the formula for here am what do I have here. I have two plus two H-minus two, put it all together, and two's cancel. H's cancelling and what I have left here is two. So the limit coming from the right is two. The limit coming from the left is two. That mean the limit exist because you get two no matter when way you come from. (On board). So the overall limit as H goes to zero (on board). Exist because the fraction here gets close to two from the left and the right. Okay.

Student: Did you notify f-of one equals one because both x squared.

Professor: So the definition here is when x is less than or equal to one is x squared am so I can pluck in one.

Student: It just so happened on the right half of one --

Professor: So that well I built the function so that you come at it this way and turn to one. Came down at that way it toward one. But this is the definition. The equal sign is there. I couldn't but the it there because I can't define something twice. I made it so it was continuous.

Student: Can you have just plugged the one in at the ends after leaving it x-and solve it and plug one in.

Professor: So this case was special. I couldn't have done it this way because I have different formulas for the left and right. When x is strictly less than one, this formula is true on both sides of x. If I'm over here the function looks like x-squared either to that side or that sides. And so the only time where I have to worry about having different formulas on the
LEFT AND RIGHT IS WHEN X-EQUALS EXACTLY ONE. THAT'S THE ONLY COMPLICATED ONE.

SO THIS WAS HAMMERING IN THE DEFINITIONS TO MAKE SURE YOU UNDERSTAND WHEN ALL THESE THINGS EXISTS AND DON'T. BUT FORTUNATELY MOST OF THE TIME WE HAVE RULES YOU CAN APPLY TO MAKE IT EASY. THAT IS THE SUBJECTS OF SECTION 1.6, WHICH WE'RE ABOUT IT START, RULES FOR DIFFERENTIATING.

SO ALMOST ALL THE TIME WHEN YOU GET A FUNCTION.

STUDENT: SO FOR THE EXAMPLES YOU GAVE UNLESS IT ASKS FOR X-EQUALS ONE YOU DON'T HAVE TO FIND LIMITS.

PROFESSOR: SO IF THE FUNCTION IS DEFINED BY SOMETHING SIM BLUL PULL LIKE THIS, YES IT WAS DERIVATIVE. YOU -- THE ONLY TIME IT GET MESSY IS WHEN YOU HAVE DIFFERENT FORMULAS FOR THE LEFT AND RIGHT.

STUDENT: YOU PLUG THE (INAUDIBLE) INTO THE ORIGINAL FUNCTION WHEN FINDING THE LIMIT.

PROFESSOR: SO THIS FUNCTION HAS N-IN IT. SO WHEN I DO IS I GO BACK TO THE ORIGINAL FORMULA UP THERE. SOMETHING TO THE LET'S AND SOMETHING TO THE RIGHT AND ASK AM I ON THE LEFT AND RIGHT AND USE THE RIGHT FORMULA. AND THAT ONLY HAPPENS WHEN YOU HAVE DIFFERENT FORMULAS FOR THE LEFT AND RIGHT. MOST OF OUR FUNCTIONS ARE GOING TO BE ONE FORMULA THAT'S TRUE EVERY WRSM AND THEN WE'VE (INAUDIBLE). BUT YOU HAVE TO KNOW WHEN YOU'RE ALLOWED TO DO THAT. THAT'S WHY YOU HAVE TO KNOW, NOT JUST THE RULES BUT WHETHER IT'S LEGAL TO USE THEM. THE RULES OF ROAD, WHEN THEY APPLY AND WHEN THEY DON'T.
SO SO MOST THE TIME WE WILL USE RULES TO MAKE THE PEOPLE FIGURED OUT THAT THEY'RE ALREADY NICE AND SAFE TO USE, TO MAKE DIFFERENTIATION EASY. FIGURE IT OUT ALREADY AND YOU HAVE TO REMEMBER THE RULES AND WHEN IT'S OKAY TO APPLY THEM. IT'S A LITTLE LIKE SOMEBODY HAS WRITTEN A NICE PIECE OF SOFTWARE THAT'S COMPLICATED ON THE INSIDE. AND YOU CAN GO AHEAD AND USE IT. LIKE USING MICROSOFT WORD AM I AM JUST GO AHEAD AND USE IT AND NOT WORRY ABOUT IT. BUT THAT'S NOT EXACTLY TRUE. IS USING MATH LIKE USING WORD? THERE ARE SOME DIFFERENCES. ONE IS ONCE YOU KNOW MAT-H, IT'S FREE. THAT'S ONE. AND THERE'S FEWER BUGS IN MATH. WE KNOW THAT SORT OF A SILL I COMPARISON BETWEEN THE TWO. IMAGINE IS A TOOL, COMPUTER ARE A TOOL. BUT TOOLSZ WE'RE LEARNING HERE HAVE SLIGHTLY DIFFERENT PROPERTIES AM THEY'RE FREE AND ALWAYS TRUE. BUT YOU HAVE TO KNOW HOW TO APPLY THEM SO YOU DON'T APPLY THEM TO THE WRONG SITUATION.

SO LET'S START BY TELLING YOU WHAT SOME OF THESE RULES ARE NOW. IF I CAN GET THIS TO MOVE. SO RULES ALL LOOK A LIKE IN SOME SENSE. THEY ANSWER THE FOLLOWING QUESTION. AND THAT IS IF I KNOW HOW TO DIFFERENTIATE ONE FUNCTION, AND I CHANGE THE FUNCTION A LITTLE BIT, I CHANGE THE FUNCTION A LITTLE, HOW DOES THE DERIVATIVE CHANGE? SO A LITTLE, THAT'S WHAT THE RULE WILL TELL YOU. SO IF YOU THINK HOW TO DIFFERENTIATE JUST A FEW BASIC FUNCTIONS PLUS THESE RULES YOU CAN DIFFERENTIATE A GA DISTILL I DON'T KNOW FUNCTIONS. SO KNOWING HOW TO DIFFERENTIATE A FEW BASIC FUNCTIONS PLUS THESE RULES, WILL LET YOU DIFFERENTIATE LOTS OF FUNCTIONS. OKAY SO THAT'S THE IDEA. SO LET'S GO ON WITH THE RULES IF THERE'S NO QUESTIONS.
So we'll have a lot of them eventually. I'll just do a few of them today. This is the easiest one. I'll write did down and then explain where it comes from later. And that says that if I have a function that I know how to differentiate, x. X-. Remember this sim all I do is multiply it by a constant then I can factor the constant out. f-of x-(above). So that's a really simple rule. And a little example would be if I want it differentiate 17 x-squared I know how to differentiate x-squared, that's two. So all I have to do is factor out the 17, I get 17 times two x-and that's 34 x. So that's, that makes it easy. So the next one is, says, suppose I have two functions whose derivatives and know. f-of x-and g-of x. I add them up. And gets a new function. g. I want it differentiate that, I just add the derivatives. I differentiate f-because I know that and differentiate g-because I know that and and just add them up. So you can move the plus sign it you like outside the derivative. And the way people usually remember this or say it is the derivative of a sum equal the sum of the derivatives. So there's the derivative of the sum. Here's the sum of the derivatives. And so that's the english language way of saying the rule. So let's try that one out.

I will eventually explain why these rules are true but not just yet. Let's take two functions that I know. f-plus and one I just figured out. 17 x-squared. And the rule says that I can move the sum outside like that. So I can just have to differentiate x-and that's one. This one I already did. It was
34 X. FROM THE LAST EXAMPLE. AND HERE'S A SLIGHTLY MORE
COMPLICATED ONE. THE GENERAL POWER RULE. SO I HAVE A FUNCTION
NOW WHOSE DERIVATIVE I KNOW AND TAKE IT TO SOME POWER, R. SOME
POSITIVE POWER R. AND I WANT TO DIFFERENTIATE THAT. SO HERE'S
HOW IT WORKS. IF IT'S GOING TO BE R-TIMES MY FUNCTION TO THE
POWER OF R-MINUS ONE, TIMES THE DERIVATIVE THAT I KNOW. THE
DERIVATIVE OF F-ITSELF. OKAY SO THAT'S CALLED THE GENERAL POWER
RULE. LET ME DO AN EXAMPLE. HOW ABOUT F-OF X-EQUALS X. I KNOW
HOW TO DIFFERENTIATE THAT FUNCTION. SO WHAT DOES MY RULE TELL
ME? IT SAYS THAT D-F-OF X-TO THE R-IS R-TIMES F-OF X-WHICH IS
JUST X-TO THE POWER R-MINUS ONE TIMES, WHAT IS THE DERIVATIVE OF
F-OF X? ONE. DOES THAT LOOK FAMILIAR? WE ALREADY DID THAT, WE
TALKED ABOUT THAT ONE LAST WEEK. I THINK WE CALLED IT THE POWER
RULE. AND SO IT'S JUSTIFY GIVES YOU BACK THAT OLD RULE BUT GIVES
YOU --

STUDENT: ISN'T THAT ALSO (INAUDIBLE).

PROFESSOR: AND IF YOU'VE HAD CALCULUS BEFORE YOU'LL RECOGNIZE
THIS IS A SPECIAL CASE OF A CHAIN RULE WHICH WE WILL INDEED
COVER. BUT I WASN'T GOING TO DO THE MOST GENERAL CASE RIGHT
AWAY. I WAS GOING TO DO SOME SPECIAL CASES FIRST. THAT'S A
GOOD QUESTION THOUGH.

SO LET ME TAKE D-D-X-PLUS QUANTITY PLUS TWO SQUARED. SO
NOW I'M GOING DO LET F-OF X BE X-PLUS TWO. SO I'M GOING TO BE
SQUARING AT THAT FUNCTION?

A SO MY RULE SAYS HERE R-IS TWO. SO I WRITE TWO DOWN AM TAKE
ANY FUNCTION F-OF X--- (ON BOARD). AND THEN I HAVE TO
DIFFERENTIATE THE FUNCTION ITSELF. SO THAT'S WHEN IT IS. WHEN

SO NOW WE CAN DO AN EXAMPLE THAT WE SHOWED YOU BEFORE WE WE FIGURED IT OUT JUST FROM GEOMETRY WHICH IS THE SLOPE OF TANGENT LINE TO A CIRCLE. SO HERE'S ANOTHER EXAMPLE. THERE'S NICE UNIT CIRCLE AM HERE'S THE ORIGIN. HERE'S A POINTS. AND I WANT, SO WHAT'S THE FUNCTION HERE? THIS IS (ON BOARD). X-SQUARED PLUS Y-SQUARED EQUALS ONE. SO THERE I'M GOING TO SOLVE FOR Y. SO
THAT IS ANY FUNCTION. F-OF X, SO I'M JUST GOING TO TAKE THE TOP
OF THE CIRCLE. AND I WANT TO KNOW WHAT IS THE DERIVATIVES OF
THIS FUNCTION. AND THAT'S THE SLOPE OF TANGENT LINE. SO THE
DERIVATIVE OF THE SLOPE OF THE TANGENT LINE AM DO YOU RECALL HOW
WE DID THIS ONE BEFORE? WE SAID THE OTHER TRICK THAT WE CAN USE
IS TO SAY THAT TANGENT LINE IS GOING TO BE ORTHOGONAL TO THE
RADIUS BECAUSE THAT'S HOW CIRCLES WORK. REMEMBER WE DID THIS.
AND SO IF I CAN FIGURE OUT THE SLOPE TO THAT RADIUS, HOW DO I GET
THE SLOPE TO THE TANGENT LINE WHICH IS PERPENDICULAR? HOW DO I
GET FROM THIS SLOPE TO THAT SLOPE? TAKE THE NEGATIVE RECIPROCAL.
SO LET'S JUST REMEMBER HOW WE DID THAT. HOW DO I FIGURE OUT THE
SLOPE OF THIS LINE? I TAKE THIS DISTANCE AND DIVIDE BY THAT
DISTANCE. SLOPE OF THE STRAIGHT LINE. I NEED IT KNOW WHAT
THAT POINT IS. SO F-BECOMES (ON BOARD). SO WHAT'S THE SLOPE OF
THIS LINE? I JUST GAVE YOU THE COORDINATES OF THE POINT. SO THE
SLOPE OF THIS RADIUS SEGMENT IS WHAT? SO IT'S THAT DISANGS
DIVIDED BY THAT DISTANCE WHICH IS SQUARE ROOT OF ONE MINUS
X-SQUARED OVER, I DID THIS WRONG, SORRY. YOU SHOULD HAVE CAUGHT
ME HERE. I DID MY ALGEBRA WRONG. THANK YOU. OKAY. GOT DO
YELL AT ME WHEN I DO THAT. SO THERE'S THE SLOPE OF THAT LINE.
AND SO WHAT'S THE SLOPE OF THIS LINE? NEGATIVE RECIPROCAL. SO
I JUST TAKE THE RECIPROCAL AND PUT A MINUS SIGN. THAT'S HOW WE
DI Did IT LAST TIME. LET'S NOW USE THE GENERAL POWER RULE TO
DIFFERENTIATE THIS FUNCTION. OKAY. HOPE WE GET THE SAME ANSWER.
SO I WANT D-D-X-OF F-OF X, DERIVATIVE, OF THE SQUARE ROOT OF ONE
MINUS X-SQUARED. I THINK I GOT IT RIGHT NOW. SO HOW DO WE MAKE
THIS LOOK LIKE THE GENERAL POWER RULE? THE SQUARE ROOT IS JUST
THE ONE-HALF POWER AM SO GENERAL POWER RULE WITH R-EQUALS A HALF. SO I GET A HALF. PULL THAT OUT AM TAKE THIS THING TO THE POWER ONE-HALF MINUS ONE. AND THEN I HAVE TO DIFFERENTIATE THE THING HERE, ONE MINUS X-SQUARED. SO LET ME JUST (INAUDIBLE) WHAT I GET. I GET A HALF TIMES ONE MINUS X-SQUARED TO THE MINUS ONE-HALF. AND THE DERIVATIVE OF THIS IS THE DERIVATIVE OF ONE WHICH IS ZERO. MINUS A DERIVATIVE OF X SQUARED. SO WHAT IS THIS? IT'S MINUS TWO X. GOING FROM THERE TO THERE. AND SO THE TWOS CANCEL AND I GETS A MINUS X. AND WHAT DOES IT MEAN IT TAKE MINUS ONE-HALF? IT MEANS THERE'S SOMETHING IN THE DENOMINATOR. AND IT'S THE HALF. SO IT'S THE SQUARE ROOT AGAIN. AND FORTUNATELY I GET THE SAME ANSWER TWO DIFFERENT WAYS. THEY'RE THE SAME. ALWAYS GOOD TO GET THE SAME ANSWER WHEN YOU DO IT TWICE.

STUDENT: DERIVATIVE OF JUST THE ONE MINUS X-SQUARED.

PROFESSOR: SO THE RULE THAT I USED WAS, AND MAYBE MY NOTATION HERE COULD BE BETTER. SO HERE'S THE FUNCTION I WANT IT DIFFERENTIATE. AND I RECOGNIZE IT'S SOMETHING TO THE HATCH POWER AM SO I GO OVER HERE TO THIS RULE AND I SAY TAKING THE DERIVATIVE OF SOMETHING TO A POWER.

STUDENT: WHY ARE WE STARTING WITH THAT FUNCTION TO DERIVE.

PROFESSOR: IT'S THE CIRCLE. I TOOK THE FUNCTION, Y-EQUALS THE SQUARE ROOT OF ONE MINUS X SQUARED BECAUSE THAT IS THE FUNCTION THAT YOU GET WITH THE CIRCLE. AND THE CIRCLE WAS A NICE EXAMPLE SINCE WE HAD DONE IT BEFORE.

OKAY. ALL RIGHT I'LL ERASE IT. HOPE I REMEMBER IT WHEN I
NEED IT.

SO LET ME TRY TO EXPLAIN WHY THESE TWO RULES ARE TRUE.

WOULDN'T TRY IT EXPLAIN THE ONE I ERASED WAS TRUE. SOMEONE

OBSERVED -- LET ME TRY IT EXPLAIN WHY RULES ONE AND TWO WORK. SO

IT'S A PROOF OF RULE ONE. AND I HAD TO GO BACK TO SOMETHING THAT

WE LEARN BEFORE, WELL I TOLD YOU BEFORE, USING THOSE OLD LIMIT
THEOREMS AM SO THAT RULE UP THERE IS JUST A CONSEQUENCE?

SOMETHING I TALKED ABOUT WITHIN OR TWO LECTURE AGO. AND SO

SO THIS IS WHAT I WANT TO COMPUTE. AND I'M JUST GOING DO WRITE

DOWN THE DEFINITION, THE LIMIT AS GOES TO DISTORTION PLUSES
CONSTANTS (ON BOARD). THAT IS THE FUNCTION I WANT IT
DIFFERENTIATE DIVIDED BY H-. THAT'S JUST DEFINITION OF THIS
DERIVATIVE. SO LET ME WRITE DOWN. SIMPLY THE DEFINITION. SO

NOW LET ME JUST DO A TINY LITTLE BIT OF ALGEBRA. FACTOR OUT
CONSTANT C-Since THIS IS A COMMON FACTOR. AND NOW I'M GOING TO

APPEAL TO THE OLD LIMIT RULE WHICH SAID THAT IF YOU'RE TAKING THE
LIMIT SOMETHING AND THERE'S A CONSTANT FACTOR, YOU CAN FACTOR IT
OUT. SO WHAT I'M USING IS ONE OF THOSE LIMIT RULES BEING I THINK
IT MIGHT BE LIMIT RULE NUMBER ONE. SO NOW I RECOGNIZE THIS
THING HERE IS JUST THE DERIVATIVE OF F. AND I'M DONE. SO I'VE
COME FROM SAYING THE DERIVATIVE OF A CONSTANT AND FUNCTION IS A
CONSTANT TIMES DERIVATIVE AND THAT WAS THE FIRST RULE OUT THERE.

AND IT JUST CAME DOWN, THE MAIN THING WAS USING THE OLD
LIMIT RULE. AND NOW FOREVER MORE YOU CAN JUST GO AHEAD AND USE
THAT CONSTANT MULTIPLIER RULE WITHOUT OF WORRY ABOUT WHETHER IT'S
TRUE OR FALSE. YOU'VE SEEN IT.

AND I'LL JUST DO THE OTHER ONE. AND AGAIN IT'S GOING DO
Come back to one of the limit rules. So what I want to compute is \( D-DF, F-OF X-PLUS G-OF X \). So the definition says if that's the limit as \( H-GOES TO ZERO \), of \( F-OF X-PLUS H-PLUS G-OF X-PLUS H \), there's a function I want to differentiate, (on board). That's again just definition of what a derivative means. That's just the definition. And I will rearrange this of so slightly. (on board). All I'm doing is putting these terms in a slightly different order am all of the same denominator \( H- \). Just put these two terms together and these two terms together. Nothing changed. So now I recognize I can apply one of the limit rules that says the limit of a sum is the sum of the limits. So going to break this into two parts. So this is the limit theorem two. That I just applied here to the sum of the, the limit of the sum is the sum of the two limits and I recognize that I'm done. So I have the derivative of the sum is the sum of the derivatives and that's what I want to do. And so now forever more you can use rule two without of worrying about whether it's true or false.

Student: Do we have to know all these proofs.

Professor: I am not be asking you it prove thing very much on midterms. But I will ask you to do some limits where thing are different on the right and left. So you do have to know those techniques. I will ask you to do them. So if I put these two rules together, I can differentiate something that's very general. And it covers most of the functions you'll see in this semester or any other. Because if all I'm allowed to do is multiply and add, all I can ever do is get polynomials. So let
PRETTY CLEAR. SUPPOSE YOU HAVE A POLYNOMIAL OF DEGREE N. THE
SUM COEFFICIENT. SO THERE'S AN X-TO THE N-TERM. X-TO THE
N-MINUS ONE TERM AND SO ON AND SO ON. I'M NOT GOING TO WRITE
THEM ALL DOWN AM I'M GOING TO WRITE ... BECAUSE I'M TOO LAZY TO
WRITE THEM ALL DOWN. THERE'S A COMPLETES WILL I GENERAL
POLYNOMIAL OF DEGREE N-. AND I'M GOING IT DIFFERENZIATE IT AND
IT'S GOING TO LOOK PRETTY MUCH LIKE THE THING UP THERE. TAKE THE
EXPONENT. PUT IT UP FRNT. KEEP AT THAT FACTOR A-N. AND
SUBTRACT ONE FROM THE EXPONENT. THAT TERMS TURNINGS INTO THAT
ONE. PULL DOWN THE EXPONENTS. PUT IT OUT FRONT. SUBTRACT ONE
FROM THE EXPONENT. AND HERE IS THE MOST GENERAL CASE IF YOU OF
NEED IT. THAT'S A REALLY GENERAL CASE AND ALL WE NEED TO KNOW,
WERE THE TWO RULES UP THERE. THE SUM RULE AND THE PRODUCT RULE,
SO THEY' RE REALLY POWERFUL, WHICH IS WHY WE TEACH THEM.

ANY QUESTIONS? OKAY. THAT IS THE END OF SECTION 1.6.

OKAY. WE'RE GOING TO BE TALKING ABOUT --

STUDENT: WAS THE FINAL AND MIDTERM COVER.

PROFESSOR: IT'S GOING TO COVER EVERYTHING THROUGH THE HOMEWORK
THAT IS DUE NEXT WEEK AND I JUST POINTED THAT. AND EVERYTHING TO
THE ENDS OF SECTION 1.7. SO THIS WILL BE ON THE SECOND MIDTERM.
BUT THE FIRST MIDTERM WILL ONLY COVER THROUGH THE END OF SECTION
1.6 WHICH IS WHAT THE HOMEWORK WILL BE DUE ON NEXT TUESDAY. IS
THAT TOO RELAXING NOW? I STILL WANT YOU TO PAY ATTENTION. ANY
OTHER QUESTIONS? OKAY. SO I WANT TO TALK A LITTLE BIT ABOUT
NOTATION. JUST DO MAKE SURE WE DON'T GET CONFUSED WHEN I WRITE
Down all these. So let me ask a question that does not have an answer. And it's just a notation problem. So what is the derivative of \( x + 2y \)? Is that a well posed question? I haven't told you enough. What have I not told you? What's the variable and what's the constant? I can't tell without knowing if \( x \)-variable that you're differentiating with respect to, and \( y \)-is just some constant or if \( y \)-is the variable and \( x \)-is the constant. So let just do those two cases. So if \( x \)-is the variable, then I have a function \( f \)-of \( x \)-that's equal to \( x + 2y \). And I want to differentiate this. \( x \)-is variable. As you can see it's sitting there. Notation tells you \( x \)-is the variable. \( y \)-is just some constant. And so what's \( f \)-prime of \( x \)? It's just, it's the derivative of that plus of derivative of that. Derivative of that is one. Plus what is the derivative of that? \( y \)-is a constant. So it's zero, so the derivative is one. Okay. Now if \( y \)-is the variable, then this thing, let me give it a different name. I'm thinking of it as a function of \( y \)-. So \( g \)-of \( y \)-is \( x + 2y \). So now I can ask what is the derivative, what's \( g \)-prime of \( y \)? Well, so what's the derivative of this constant with respect to \( y \)? It's zero. And what is the derivative of two \( y \)-with respect to \( y \)? Two. So you get a different answer, right? You have so you have to know which is variable and which is constants. So if I just ask you this question, it's a nonsense question because I haven't total you enough. But once you write down an \( x \)-there you say \( x \)-sat

variable, oh, \( y \)-is a variable. So when we do nor complicated
PROBLEMS, MAKE SURE YOU PAY ATTENTION TO THE SYMBOLS. LET ME JUST DO ONE MORE EXAMPLE. SO HERE IS A FUNCTION. (ON BOARD) OKAY. AND I WANT ITS DERIVATIVE. I HAVEN'T TOLD YOU WHAT IT IS VARIABLE IS YET. BUT IF I WRITE D-D-T, WHAT IS THE VARIABLE. IT'S SIGHT RIGHT THERE. THE NOTATION TELLS YOU T-IS THE VARIABLE THE EVERYTHING ELSE IS CONSTANTS. SO WHO CAN TELL ME WHAT'S THE DERIVATIVE. SO WHAT IS THE DERIVATIVE WITH RESPECT TO T-OF THIS EXPRESSION? SO A-IS A CONSTANT AND S-IS CONSTANT. SO, TWO A-T-PLUS MINUS S-OVER T-SQUARED PLUS ZERO. OKAY. SO BUT IF I WERE TO WRITE D-D-S WHAT WOULD I GET? FIRST THERE'S NO S-S THERE SO IT WOULD BE ZERO PLUS, SO WAS THE DERIVATIVE OF THIS WITH RESPECT TO S. ONE OVER T. PLUS TWO S, OKAY DIFFERENT ANSWERS. AND HERE AGAIN THE NOTATION TELLS YOU THE VARIABLE. OKAY. SO ENOUGH ON THAT.

NOW WE RPT CONTENT WITH DIFFERENTIATING FUNCTIONS JUST ONCE. WE'RE GOING TO DIFFERENTIATE THEM MORE THAN ONCE. AND A GOOD REASON, I'LL EXPLAIN. IF F-OF X-IS A FUNCTION OF X, THEN I CAN DIFFERENTIATE IT, I CAN WRITE DOWN F-OF X, F-PRIME OF X. THEN THIS IS ALSO A FUNCTION, RIGHT, ANOTHER FORMULA. SO I CAN DIFFERENTIATE IT IF I WANT. AND I CAN DIFFERENTIATE THIS THING HERE D-D-X IS A FUNCTION F-PRIME OF X. AND THAT'S CALLED THE SECOND DERIVATIVE. (ON BOARD). AND THE NOTATION IS F, THEN WE PUT TWO PRIMES UP THERE. TWO PRIMES MEAN I DIFFERENTIATE TWICE. AND IT'S READ F-DOUBLE PRIME OF X. SO THAT'S THE WAY YOU RAID THAT NOTATION. AND OF COURSE I CAN KEEP ON DIFFERENTIATING FOR AWILE BUT LET ME JUST STOP THERE. SO LET ME DO SOME EXAMPLES.

OKAY SO I WANT TO MOTIVATE WHY I'M BOTHERING TO TELL YOU ABOUT THIS. AND MAYBE THE BIGGEST REASON IS THAT MANY EQUATIONS THAT EXPLAIN HOW THE WORLD WORKS AND IN ALL DIFFERENT WAYS HAVE SECOND DERIVATIVES IN THEM. AND PERHAPS THE MOST FAMOUS ONE THAT YOU'VE ALL SEEN BEFORE IS F-EQUALS MA. EVEN IF YOU DON'T REMEMBER WHAT THAT MEANS FROM PHYSICS. THAT EQUATION WHICH DESCRIBES MANY THING IN THE UNIVERSE HAS A SECOND DERIVATIVE IN IT. SO F-IS THE FORCE ON OBJECT. M IS THE MASS. AND WHAT IS A? A-IS THE ACCELERATION. AND THE ACCELERATION MEASURES HOW FAST THE VELOCITY CHANGES. THE DERIVATIVE OF THE VELOCITY. WHAT'S THE VELOCITY? IT MEASURES HOW FAST SOMETHING ELSE CHANGES. YOUR POSITION. THE DISTANCE. SO THE VELOCITY IS THE DERIVATIVE OF LET'S CALL IT POSITION DISTANCE, WHATEVER YOU WANT TO CALL IT, SO IT IS THE DERIVATIVE OF THE DERIVATIVE OF POSITION. AND SO THAT'S THE SECOND DERIVATIVE. AND SO THAT'S ENOUGH OF A REASON TO DO IT. I HAD A BUNCH OF EXAMPLES ACTUALLY ON THE FIRST DAY OF CLASS. AND MAYBE I'LL JUST REMIND YOU QUICKLY THERE.

I'LL JUST REMIND YOU OF ONE OF THOSE EXAMPLES THAT YOUR LIFE
Depended on it which was will this building collapse in an earth equation. That is when people solve F-equals ma which F-equals from the earthquake and it's the position of the girders and all the that of the building. So just to give you one example of that sort. So what's the notation? Let me do the notation. So there are two different ways. One is with primes. And the other one is with d-x-s. And I want to explain, take y-equals f-of x. We use y-prime and we use f-prime of x. And we've also used d-d-x-of f-of x-much that's how we did the first derivative. For the second derivative I've already talked about f-double prime of x. Also write y-double prime. You can also do it this way. And I didn't leave myself enough room. But here's the symbols. They're actually, let me try to make this fit a little better. So it's either d-d-x-of f-of x-or d-y-d-x-much these with both symbols of derivatives. And for the second derivative it looks like this. (On board). So this, again this is just symbols it mean take the derivative twice. So you're not squaring a number call d-. All of these are synonyms for one another it take the derivative twice and all these appear in the book and your homework. So they're all just synonyms for the same thing? A

This kind of notation is good because you have to remember which variable to use in the derivative. Differentiate with respect to something else like a-or s-or y-you get the wrong answer. And occasionally there is a need for third derivatives and it's pretty natural. You just have three primes, triple
PRIME OF X. AND JUST MAKE IT D-CUBED D-X-CUBE OF F-OF X. (ON BOARD). AND SOMETIMES THERE ARE EVEN MORE. IN THIS CLASS WE WON'T GO ANY HIGHER THAN THIS. SO LET ME JUST GIVE SOME EXAMPLES.

(ON BOARD) X-CUBED MINUS FIVE X-SQUARED PLUS ONE OVER X. AND THEN Y-PRIME OR F-PRIME OF X-THAT WOULD BE THREE X-SQUARED, MINUS PULL DOWN THE TWO, MULTIPLY BY FIVE, GET A TEN. SO THERE IS THE FIRST DERIVATIVE. AND HERE'S THE SECOND DERIVATIVE. (ON BOARD) AND DO ONE MORE FOR FUN. I FORGOT TO TURN MINE OFF TWO. FORTUNATELY NO ONE HAS CALLED. AND YOU CAN KEEP ONGOING. NOW LET ME ASK THE FOLLOWING QUESTION. JUST TO MAKE SURE WE UNDERSTAND THIS. WHAT HAPPENS WHEN YOU TAKE A POLYNOMIAL AND KEEP DIFFERENTIATING A POLYNOMIAL. SO HERE THE GENERAL CASE, I PUUC THE POLYNOMIAL. AND I DIFFERENTIATE IT AND I GOT ANOTHER POLYNOMIAL. YOU ALWAYS GET POLYNOMIALS. THAT'S FOR SURE? A IF YOU LOOK -- THE HIGHEST POWER OF X--- WHAT HAPPENS IF I DIFFERENTIATE IT AGAIN? IT GOES TO MINUS TWO. N-MINUS THREE. WHAT HAPPENS IF YOU KEEP ON DIFFERENTIATING LONG ENOUGH? EECHT WILL I YOU GET, HIGHEST POWER IS ONE., HIGHEST POWER IS ZERO. THAT'S A CONSTANTS AND AND WHAT HAPPENS WHEN YOU DIFFERENTIATE A CONSTANT? YOU GET ZERO. SO ALL POLYNOMIALS GET VERY SIMPLE EVENTUALLY. IF I THAT I CAN POLYNOMIAL DEGREE N-WHEN AM I GARN FEED TO GET TO ZERO. I DO IT ONE AND GET TO -- HOW MANY TIME DO YOU HAVE TO DIFFERENTIATE TO GET -- SO LET'S DO THIS. X-PLUS ONE. HOW MANY TIMES DO I HAVE TO DIFFERENTIATE THAT TO GET TO ZERO? TWICE. ALL RIGHT. IF I STARTED WITH N-AS A EXPONENT HOW MANY TIMES DO I HAVE TO DO IT?
N-TIMES. IF I DIFFERENTIATE IT, THE DEGREE PLUS ONE TIMES.

OKAY. LET ME SEE WHAT I WANT TO TRY TO ACCOMPLISH ... OKAY. ONE LAST LITTLE TINY TOPIC. BEAR WITH ME.

SO I WANT TO RELATE ALL THIS NOTATION TO IDEAS IN ECONOMICS. JUST GOING TO WRITE DOWN SOME DEFINITIONS THAT I WON'T HAVE TIME TO DO. SO LET'S SUPPOSE THIS IS THE COST OF PRODUCING A CERTAIN NUMBER OF OBJECTS, X-OF THEM. THEN WHAT THE ECONOMIST OFTEN CARE ABOUT IS THAT FUNCTION, WHICH IS THE COST OF PRODUCING ONE MORE AFTER DOING X. OKAY. COSTED PRODUCING ONE MORE, THAT'S AN INTERESTING ECONOMIC FUNCTION. AND WE KNOW FROM CALCULUS THAT YOU CAN APPROXIMATE THIS FUNCTION BY SOMETHING. IF THIS IS A COST FUNCTION?

A HERE'S C-OF X-OF HOORS X-. THERE'S X-PLUS ONE. THERE'S C-OF X-. THERE'S C-OF X-PLUS ONE. I WANT TO APPROXIMATE THIS DISTANCE AND DRAW A SEDATE LINE AND USE THAT STRAIGHT LINE IT ESTIMATE WHAT THAT IS. SO WHAT IS THAT DISTANCE? USING IN TERM OF IF I KNOW THE DERIVATIVE RIGHT THERE? IT'S C-PRIME OF X. THAT'S THE SLOPE OF THE TANGENT LINE. SO THIS IS WIG WILL I SIGN MEANS IT'S ABOUT EQUAL TO C-PRIME OF X. AND ECONOMIST LIKE TO USE MATH. AND SO THEY USE THIS FUNCTION ALL THE TIME. IT'S CALLED MARGINAL COST FUNCTION. AND THEY USE THIS AS APPROXIMATION TO WHAT THE NEXT THING WILL COST. NOW IN ADDITION TO COST, YOU ALSO WANT IT MAKE MONEY. SO YOU HAVE TO THINK ABOUT THE REVENUES. SO IT'S THE REVENUE FROM SELLING X-OBJECTS AND FOR THE SAME REASON AS BEFORE, WE'RE REALLY INTERESTED IN HOW MUCH MONEY ABOUT I MAKE BY SELLING ONE MORE. SO THAT'S R OF
X-PLUS ONE MINUS R-OF X-. THAT'S HOW MUCH I MAKE FOR SELLING THE NEXT ONE. FOR THE SAME REASON YOU APPROXIMATE THAT BY THE DERIVATIVE. AND MANCH NATIONAL, THAT'S CALL MARGINAL REVENUE FUNCTION. AND FINALLY, WHEN YOU REALLY WANT TO KNOW IS THE PROFITS. AND THE PROFIT IS HOW MUCH YOU BRING IN MINUS HOW MANY IT COST YOU TO MAKE IT. AND YOU KNOW TO KNOW WHAT YOUR PROFIT BY SELLING ONE MORE. AND THAT'S GOING TO BE, P-PRIME OF X, THAT'S CALLED PROFIT MARGIN OR THE MARGINAL PROFIT FUNCTION. AND IF I KNOW THAT PROFIT IS REFER NEW MINUS COST HOW DO I FIGURE OUT THE MARGINAL PROFIT FUNCTION. IF I KNOW THE MARGINAL REVENUE FUNCTION AND MARGINAL COST FUNCTION I JUST SUBTRACT THE DERIVATIVE. SO THE MARGINAL PROFIT MINUS THE MARGINAL -- THIS IS WHAT ECONOMISTS USE TO TALK ABOUT BUSINESS AND IT'S JUST DIFFERENT WORDS FOR ALL THE THE CALCULUS STUFF AS YOU CAN SEE. AND THAT'S A GOOD PLACE TO STOP.