GSI: GOOD AFTERNOON. I WANTED TO MAKE AN ANNOUNCEMENT. I AM A CAL CAMPUS AMBASSADOR. I GIVE TOURS. AND BESIDES THAT WE WORK AT THE CAMPANILI. AND THE CAL PARENT GAME. AND I WANT TO TELL YOU THAT APPLICATIONS ARE UP. IF YOU'RE INTERESTED IN BEING A TOUR GUIDE, SPREAD THE WORD. APPLICATION YOU CAN GO ON-LINE. AT WWW.VISITORS.BERKELEY.EDU. I HAVE FLIERS UP IN FRONT. SPREAD THE WORDS AND IF YOU HAVE ANY IMMEDIATE QUESTIONS YOU CAN ASK NOW BUT OTHERWISE, APPLY. ANY QUESTIONS? STUDENT: IS EVERYTHING DONE IN ENGLISH OR DO YOU NEED TRANSLATORS.

PROFESSOR: IF YOU KNOW ANOTHER LANGUAGE, IT'S HELPFUL. I HAVEN'T GIVEN ANY SPANISH TOURS YET. THE TREE SITTERS ARE GONE SO WE DON'T HAVE AS MUCH TO SAY OR TALK ABOUT. EXCEPT BEATING STANFORD. APPLY AND HAVE A GREAT THANKSGIVING AND GO BEARS. THANK YOU.

PROFESSOR: I DETECT A SLIGHTLY SMALLER TURN OUT JUST BEFORE THANKSGIVING. ALL RIGHT, SO I'M ALMOST DONE WITH CHAPTER FIVE. THERE'S ONE LAST LITTLE THING I WANTED TO TELL YOU BEFORE I GO ON TO INTEGRATION. AND SO LET ME JUST DO IT. SO WE STUDIED THIS VERY GENERIC MODEL OF MANY THING IN THE WORLD WHICH STARTS BY GROWING EXPONENTIALLY. APPROACHES THE LIMIT ASYMPTOTICALLY. AND THE LIMIT IS CALLED M. IF WE WROTE DOWN A PLOT OF Y-T-VERSUS T-. THIS SATISFIES A NICE SIMPLE DIFFERENTIAL EQUATION. THERE'S A CONSTANT WHICH TELLS YOU HOW FAST IT'S
RISING. AND THERE'S THE CONSTANTS THAT TELLS YOU IT CAN'T GET
PAST M BECAUSE IT STOPS GROWING WHEN IT GETS TO M. WE ALSO
WRITE DOWN THE EXPLICIT ANSWER, WHICH I'LL WRITE HERE. WHERE
THERE'S A CONSTANTS YOU GET TO PICK AND IT DEPENDS ON WHERE YOU
START. IF YOU TELL ME WHERE YOU START RIGHT HERE, THEN I CAN
WRITE DOWN THE WHOLE EQUATION. SO THERE'S ONE QUESTION WE,
THAT'S PERFECTLY NATURAL TO ASK ABOUT THIS ONE. BUT WE DIDN'T,
WHICH IS WHERE IS THE INFLECTION POINT. WHERE DOES IT STOP
GROWING EXPONENTIALLY AND START APPROACHING THE ASYMPTOTE. SO
WHAT I'D LIKE TO DO IS WRITE DOWN AN INCREDIBLY SIMPLE FORMULA
FOR THE INFLECTION POINTS. SO TWO WAYS I CAN DO IT. THE HARD
WAY AND THE EASY WAY. I'M GOING TO DO IT THE EASY WAY. THE
HARD WAY IS I COULD COMPUTE Y-DOUBLE PRIME OF T-FROM THE FORMULA
AND SOLVE Y-DOUBLE PRIME OF T-EQUALS ZERO THAT'S THE DEFINITION
OF THE INFLECTION POINTS. BUT I'M GOING TO DO IS AN EASIER WAY
AND SAVE MYSELF SOME WORK. I'M GOING TO COMPUTE Y-DOUBLE PRIME
OF T-FROM DIFFERENTIAL EQUATION. AND SOLVE FOR THAT EQUALS
ZERO. SO LET ME START HERE. I'M GOING TO DIFFERENTIATE THAT
USING THE PRODUCT RULE. SO JUST TAKE THIS THING OVER HERE, AND
WRITE DOWN THE PRODUCT RULE.

STUDENT: IS THAT ONE TERM K-Y-OF T-

PROFESSOR: SO THIS IS K-TIMES Y, I WROTE Y-UP THERE BUT IT'S A
FUNCTION OF T-. TIME QUANTITIES M MINUS Y-OF T-. WE DON'T
FORGET IT'S A FUNCTION OF T-. SO I'M GOING TO DIFFERENTIATE
THIS. AND NOW WE USE THE PRODUCT RULE. SO THE PRODUCT RULE
Says it's going to be the derivative of the first part times second part. Plus the first part time the derivative of the second part. That's the product rule. So let's write that down. That's going to be, what's the derivative of k-time y, it's k-times y-prime times m minus y plus k-times y-times and what's the derivative of m minus y-of t? m is constants, remember. So what's the derivative of this guy, it's? minus y-prime, okay. So that's what it's there. That's just the product rule. (On board). Let's collect all the terms. Let me write it all out here. It's k-times m times y-prime minus k-times y-prime times y, so k-times y-prime times y, plus k-times y-times minus y-prime. It's another k-times y-prime times y, so multiply it all out. I left off of t-. So let me keep going. These two terms are the same. So I get minus two times y-prime (on board). So this is the second derivative. I want to find, solve for this equal to zero. That's, that will tell me how to find the inflection point. So let me keep going. This turns out to be really easy. There's a common factor in these two terms. They both have a y-prime in common and both have a k-. So let me factor that out. K-times y-prime and what's left? m minus two y-. Okay. (On board). I have to figure out when is this zero. So if I have a product of things the only way this can be zero is if that factor is zero and that factor is zero. Can y-prime of be zero? No because it's increasing, always going up, the curve. Y-prime is always positive. So the only way that this can be zero, only if m equals two y-. 
ONLY IF THIS TERM EQUALS ZERO. OR M OVER TWO EQUALS Y-. OR AND THIS IS WHAT MAKES IT VERY SIMPLE TO UNDERSTAND, IF THIS IS M, IF THE INFLECTION POINTS EXACTLY WHEN IT’S HalFWA YAWHEN IT HITS M OVER TWO, IT GOES FROM CONCAVE UP IS TO CONCAVE DOWN. PERFECTLY NATURAL. HALFWAY, IT SWITCHES. I DIDN’T HAVE TO, THAT WAS A LOT EASIER CALCULUS THAN DIFFERENTIATING THIS GUY TWO TIMES AND SETTING IT EQUAL TO ZERO. HALFWAY. THAT’S THE END OF CHAPTER FIVE. ANY QUESTIONS? WHAT’S THAT? OKAY.

THE FUNCTION THERE. THAT'S GOING TO BE THE SECOND GOAL. IT DOESN'T SOUND LIKE IT HAS MUCH TO DO WITH THE FIRST ONE PERHAPS BUT, IN FACT, IT'S THE OPPOSITE OF THE FIRST ONE. EVERYTHING YOU LEARN YOU HAVE TO DO BACKWARDS FOR THE REST OF THE SEMESTER.

SO TURNS OUT TO GET H-OF X, THE AREA FUNCTION, FROM THE EQUATION OF THE CURVE, FROM G-OF X, WE'RE GOING TO DO THE OPPOSITE OF DIFFERENTIATION. SO WE KNOW ONE WAY, WE'RE GOING TO GO BACKWARDS THE OTHER WAY. SO LET ME DO AN EXAMPLE. IF THE CURVE UNDER WHICH WE WANT TO FIND THE AREA IS THREE X-SQUARE, THEN H-OF X-IS GOING TO BE THE FUNCTION WHOSE DERIVATIVE IS THREE X, THAT'S X-CUBED. TO GET FROM THERE TO THERE YOU HAVE TO DIFFERENTIATE. BUT WE WANT TO GO FROM THERE TO THERE, MORE GENERAL IF G-OF X-IS THE DERIVATIVE OF SOME FUNCTION AND H-OF X-IS THE FUNCTION THAT YOU DIFFERENTIATE. THAT GOES FROM -- ALL SEMESTER WE LEARN HOW TO TAKE F-AND GET F-PRIME. NOW WE'RE GOING TO DO THE OPPOSITE. GIVEN F-PRIME. WE HAVE TO FIGURE OUT WHAT F-WAS. SO THIS PROCESS OF GOING FROM F-PRIME TO F-HAS TWO NAMES, AND BECAUSE IT'S THE OPPOSITE, IT'S CALL ANTI DIFFERENTIATION. THAT'S ONE NAME FOR IT. WHICH MAKE IT OBVIOUS BECAUSE IT'S THE OPPOSITE OR CALL INTEGRATION. I HAVE, THAT'S SORT OF EASY. YOU GO ONE WAY OR THE OTHER. BUT YOU HAVEN'T EXPLAINED WHAT THAT HAS TO DO WITH AREA. WHY THAT HAS ANYTHING TO DO WITH AREA. THAT'S A BIG FACT. THE FACT THAT DOING THIS ANTI DIFFERENTIATION STUFF GIVES AN AREA IS A BIG RESULT. FAMOUS RESULT, IN FACT, IT'S SO FAMOUS, IT'S CALLED THE FUNDAMENTAL THEOREM OF CALCULUS. WHAT COULD BE A FANCIER NAME.
THE MAIN RESULT THIS WHOLE SEMESTER THAT THESE TWO THING ARE
OPPOSITE ONE ANOTHER. IN SECTION 6.2.

SO LET ME DO A REALLY EASY EXAMPLE JUST TO MAKE THE POINT.
WHERE WE DON'T HAVE TO DO ANY CALCULUS. JUST TO FIGURE OUT THE
AREA BECAUSE WE KNOW HOW TO FIND AREAS OF TRIANGLES. SO LET'S
TRY TO FIND THE AREA UNDER THE CURVE Y-EQUALS X-. JUST A
STRAIGHT LINE THROUGH THE ORIGINS. SO THERE IT IS. THAT'S
SUPPOSED TO HAVE SLOPE ONE BETWEEN LET'S SAY ZERO AND B-.
BETWEEN THERE AND THERE. I WANT TO FIND THE AREA. SO LET ME
CALL IT ZERO AND X-. BETWEEN ZERO AND X-. I WANT THE AREA OF
THAT TRIANGLE. SO WHAT'S THE AREA OF THAT NICE SIMPLE TRIANGLE?
FROM HIGH SCHOOL GEOMETRY? X-SQUARED OVER TWO, IT'S HALF THE
AREA OF A SQUARE. LIKE X-ON THE SIDE. AND SO WHAT HAPPENED IF
I DO TAKE THE AREA, AND DIFFERENTIATE IT? I CAN DO THAT. I GET
X-WHICH IS EXACTLY WHAT THE EQUATION OF CURVE WAS. HERE THE
AREA IS X-SQUARED OVER TWO. AND THE DERIVATIVE IS -- THAT'S
TRUE IN GENERAL. WHERE A TRIANGLE, IT'S EASY. (ON BOARD).

SO MOST OF THIS SECTION IS GOING TO BE ABOUT SOME RULES,
BASICALLY FOR EVERY RULE FOR HOW TO DIFFERENTIATE YOU GET A RULE
ABOUT HOW TO INTEGRATE. YOU JUST DO IT THAT WAY. BUT I'LL DO
A BUNCH OF EXAMPLES. BUT LET ME JUST MAKE A LITTLE LIST HERE OF
WHAT THIS IS GOOD FOR. WHAT IS INTEGRATION GOOD FOR? OR ANTI
DIFFERENTIATION. SO WE JUST DID ONE EXAMPLE AREAS UNDER CURVES
OKAY. THAT'S THE FIRST EXAMPLE. SO OH, ACTUALLY LET ME DO ONE
MORE THING. I'M GOING TO WRITE DOWN A DEFINITION TO MAKE IT A
LITTLE EASIER TO SEE MY LITTLE EXAMPLES. SO HERE THE
DEFINITION. Suppose \( f(\mathbf{x}) \) is given and \( \mathbf{F}(\mathbf{x}) \) satisfies \( \mathbf{F}'(\mathbf{x}) = f(\mathbf{x}) \). I want to give a name to this thing. Then \( \mathbf{F}(\mathbf{x}) \) is called the anti derivative of \( f(\mathbf{x}) \). So that's what I mean by that.

Let me use that, those words now, finding the area under the curve, is given by, you need to know the function the anti derivative. That was the starting point. That's the fundamental theorem of calculus. We'll get back to it. What else can you do? So let's suppose you know the acceleration of something. You drop a ball, it accelerates at some rate, then the anti derivative of \( a(t) \) is another quantity called the velocity. Why? Because the way you define the acceleration is it's the derivative the velocity. If somebody hands you acceleration you can figure out how fast something's going by doing this process. That's useful. Let's suppose you now know the velocity. What is its anti derivative? It's anti derivative is the distance moved. How far you've moved. And that's because how do you figure out the velocity? It's define to be the derivative of where you are. How fast where you are is changing. So anti derivatives are good for figuring out where you are and how fast you're going given your acceleration.

This is what we often know. Easy to figure that out, like gravity, it's constant. Constant acceleration. So let me give a couple of other examples. I will come back and do this concrete example later. Drop balls and figure out where they are. So here's another example. If \( f(\text{now}) \) is time and I did an
EXAMPLE LIKE THIS LAST TIME, IS THE RATE OF INFECTION, THEN IT'S ANTI DERIVATIVE F- OF T-IS THE NUMBER OF INFECTED PEOPLE. SO YOU CAN TELL ME HOW MANY PEOPLE ARE GETTING INFECTED PER DAY AS A FUNCTION AND I INTEGRATE, TAKE THE ANTI DERIVATIVE, THAT'S HOW MANY INFECTED PEOPLE I HAVE, WHICH IS A USEFUL THING TO KNOW IF YOU'RE BUYING HOSPITAL BED OR SOMETHING. AND JUST ONE MORE LITTLE EXAMPLE. IF F-OF T-IS THE RATE OF INFLATION OF LET'S SAY THE PRICE OF OIL, THEN ITS ANTI DERIVATIVE IS THE PRICE OF OIL. WHICH IS AN INTERESTING THING TO KNOW TOO. SO GOING BACKWARDS FROM THE DERIVATIVE TO THE ORIGINAL FUNCTION.

STUDENT: A LOT OF TIMES YOU NOTE THAT IT'S A DERIVATIVE BY PRIME IS THERE A WAY TO DO THAT WITH THE ANTI DERIVATIVE.

PROFESSOR: YES. I WILL WRITE THAT NOTATION DOWN. AM I GOING DID WRITE IT DOWN RIGHT NOW? SURE. WHY NOT. THE OTHER WAY YOU WRITE THIS DOWN, I WANT TO, I WILL COME BACK TO THIS LATER, BUT, SO, THIS FUNNY BIG TALL S-CALLED THE INTEGRAL SIGN. YOU CAN ALWAYS ADD ANY OLD CONSTANT TO IT THAT YOU WANT. BECAUSE WHEN YOU DIFFERENTIATE THE CONSTANTS GOES AWAY. AND YOU GET BACK -- THAT'S FAMILIAR TO SOME PEOPLE AND NOT OTHERS. THAT'S THE USUAL NOTATION, THAT FUNNY THING. LET ME DO SOME REALLY CONCRETE EXAMPLES OF SIMPLE FUNCTIONS AND THEN I'LL GIVE YOU THE GENERAL RULES.

STUDENT: DON'T YOU MEAN F-PRIME OF T-. FOR NUMBER FIVE.

PROFESSOR: SO THIS, THAT IS THE DERIVATIVE. AND THAT'S THE ORIGINAL FUNCTION. THE INTEGRAL. SO IT HAS THIS RELATIONSHIP RIGHT THERE. LET'S GIVE SOME EXAMPLES, FINDING ANTI
DERIVATIVES. SO IF YOU'VE BEEN READING THE NEWS THEY KEEP TALKING ABOUT THESE POISONOUS DERIVATIVES ON WALL STREET. THE WORD COMES FROM THE SAME MEANING BUT I WON'T GO THERE. SO LET'S TAKE A REALLY SIMPLE ONE. F-OF X-EQUALS X SQUARED LITTLE F-OF X-. IF YOU HAVE A TWO UP THERE THEN YOU CAN PROBABLY GUESS THAT YOU'RE GOING TO START WITH SOMETHING AN X-CUBED BECAUSE WHETHER YOU DIFFERENTIATE THAT, THAT WILL TURN INTO X-SQUARED BUT WE DON'T KNOW WHAT TO MULTIPLY BY. SO IT'S GOING TO BE SOME CONSTANT WE'RE GOING TO FIGURE OUT IN A MOMENT. SO TRY FIND K-NOW TO MAKE THIS TRUE. WHAT DO I NEED TO HAVE BE TRUE? I NEEDS F-PRIME OF X-HAS TO EQUAL F-OF X-. SO LET ME JUST DIFFERENTIATE THAT THING. HOW DO I DIFFERENTIATE THIS? IT'S GOING TO BE THREE TIME K-TIMES X-SQUARED. I WANT THAT TO EQUAL X-SQUARED. SO I NEED THREE TIMES K-TO BE ONE FOR THAT TO BE TRUE. THAT'S F-OF X, DIFFERENTIATE IT, THE THREE COMES DOWN AND GETS THREE TIMES K-TIMES X-SQUARED. I WANT THAT TO BE EQUAL TO X-SQUARED. SO K-EQUALS ONE THIRD. AND I GET THAT THE ANTI DERIVATIVE, AN ANTI DERIVATIVE IS ONE THIRD X-CUBED. SO THAT'S THE EXAMPLE FOR VERY GENERAL RULE. BUT THERE'S ANOTHER POSSIBILITY. I COULD ADD SEVEN TO IT. WHY IS THAT A POSSIBLE ANSWER? BECAUSE IF I DIFFERENTIATE ONE THIRD X-CUBED PLUS SEVEN, CERTAIN GOES AWAY, AND I STILL GET X-SQUARED. SO I COULD HAVE ADDED ANY CONSTANT THERE THAT I LIKE. THAT'S JUST A GENERAL PRINCIPLE. AND I'LL MAKE THAT CLEAR LATER. AND WE CAN PROBABLY SEE WHAT THE GENERAL RULE IS. WE'LL DO IT LATER. FOR INTEGRATING X-TO ANY OLD POWER LATER. LET ME DO ONE MORE
EXAMPLE LIKE THIS.

\[ F(x) = e^{-2x} \] so you can probably imagine because we know, if you differentiate this thing you get it back. so probably going to be an \( e \) to the minus two \( x \) in the answer. again for some constant we don't know. let's try to find, to make it work. find \( k \)-so that when you differentiate this function I want it to be \( F(x) \), so it better be \( e^{-2x} \). but what happens when I differentiate this? it's, you know how to differentiate this, you get minus two comes down out fronts. we get minus two times \( k \)-times \( e^{-2x} \). (on board). I want a big \( k \)-to make that equally true. I need minus two \( k \)-equals one. or \( k \)-equals minus one-half. or \( F(x) \)-equals \( e^{-2x} \). and so if you differentiate that, you get the original \( e^{-2x} \)-and just as before, this also works, you can add any constant. because when you differentiate the minus three goes away. you still ends up with \( e^{-2x} \). and we'll do the general rule later. (on board). first let me say in

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General why it's always possible you can always subtract a constant, add a constant. it always works. that's the only possible variation on getting the anti derivative.

Let me just make a theorem now that just writes down formally what we're observed. you can all go ahead and add a constant. but what's maybe new is that's the only way you can have a different kind of answer. so suppose the function we want to anti differentiate is continuous on some interval,
CAPITAL U-. SO IT’S A INTERVAL FROM SOME NUMBER UP TO SOME OTHER NUMBER. SUPPOSE F-ONE OF X-AND F-TWO OF X-ARE BOTH ANTI DERIVATIVES. THERE’S TWO DIFFERENT ANSWERS. WHAT DO THEY HAVE IN COMMON? AND THE ANSWER IS BASICALLY ONE IS A CONSTANT PLUS THE OTHER. IT’S THE ONLY DIFFERENT WAY TO DO IT. THEN THERE’S CONSTANT C-SUCH THAT YOU GET ONE OF THE FUNCTIONS FROM THE OTHER FUNCTION AND ADDING CONSTANT. THAT’S THE ONLY AMBIGUITY. SOME CONSTANT. THIS IS THEOREM ONE. DOESN’T TAKE MUCH EFFORT TO PROVE THAT. I’LL JUST TAKE A MOMENT. WHY IS THIS TRUE? IT’S GOING TO FOLLOW FROM AN EVEN MORE BASIC FACT. WHICH SAYS IF YOU HAVE SOME FUNCTION WHOSE DERIVATIVE IS ZERO, FROM A-TO B, SO IT’S FLAT EVERYWHERE, HOW CAN A FUNCTION POSSIBLY BE FLAT, THEN F-OF X-HAS TO BE A CONSTANT FOR SOME CONSTANT C-. THAT’S INTUITIVE. IF YOU HAVE TO BE FLAT YOU GET A HORIZONTAL LINE AT SOME HEIGHT. THE HEIGHT C-. THAT’S ALL THERE IS. I’M GOING TO ASSERT THIS BY INTUITION. AND THEN I’M GOING TO PROVE THAT IN ONE LINE. THIS IS JUST INTUITIVELY TRUE. THE ONLY THING THAT ALWAYS HAS A ZERO DIFFERENT IS A HORIZONTAL LINE WHICH IS AT SOME HEIGHT C-. NOW LET ME EXPLAIN WHY THEOREM ONE IS TRUE, GIVEN THE INTUITION BEHIND THEOREM TWO.

SO HERE’S THE IDEA. I’M GOING TO WRITE, DEFINE MY FUNCTION TO BE, TAKE ANY. THOSE TWO ANTI DERIVATIVES AND SUBTRACT THEM. I CAN DEFINE THIS FUNCTION NOW. THESE ARE TWO ANTI DERIVATIVES WHERE F-ONE AND F-TWO ARE ANTI DERIVATIVES. SO LET ME DIFFERENTIATE THAT. I KNOW HOW TO DIFFERENTIATE. JUST USE THE FORMULA. (ON BOARD). SO WHAT DOES IT MEAN TO BE ANTI
DERIVATIVE. WHAT IS D-D-X-OF F-ONE OF X-. IT'S GOING TO BE F-OF X-. AND THAT'S GOT TO BE F-OF X-. I SUBTRACT THEM AND I GET ZERO. (ON BOARD). AND SO BY THEOREM TWO THAT FUNCTION HAS TO BE CONSTANT. THE DERIVATIVE IS ZERO. AND I JUST SOLVE FOR F-ONE AND IT'S EXACTLY WHAT I SAID, IT'S F-TWO PLUS A CONSTANT. SO THE ONLY WAY I CAN GET FROM ONE ANTI DERIVATIVE TO ANOTHER IS BY ADDING OR SUBTRACTING A CONSTANT WHEREVER THEY'RE CONTINUOUS. AND NOW THAT I'VE EXPLAINED WHY THERE'S ALWAYS THIS CONSTANT I WILL REMIND YOU OF THE NOTATION THAT I WROTE DOWN BEFORE. SO WE'RE TO WRITE IT DOWN THIS WAY. IT MEANS, WE WRITE DOWN THIS FUNNY THING HERE CALL THE INTEGRAL OF F-OF X-WITH RESPECT TO X-. IF YOU WANT TO SAY IT IN LONG WINDED ENGLISH, IS EQUAL TO ANY ONE PARTICULAR ANTI DERIVATIVE PLUS CONSTANT. IT'S THIS FUNNY SYMBOL WE'RE GOING TO USE FROM NOW ON. LET ME JUST USE THIS NOTATION. MAKE SURE WE UNDERSTAND IT. SUPPOSE I TAKE MY FUNCTION, AND I INTEGRATE IT, AND THEN I DIFFERENTIATE IT. WHAT DO I GET BACK? IT'S THE ANTI DERIVATIVE PLUS ANY CONSTANT YOU LIKE, CONSTANT GOES AWAY, AND WHAT ANTI DERIVATIVE MEANS IS I GET THE ORIGINAL FUNCTION BACK. SO IF YOU INTEGRATE AND THEN DIFFERENTIATE, THEY JUST CANCEL ONE ANOTHER OUT. BACKWARDS. I SAID THAT AT THE BEGINNING. THESE TWO OPERATIONS, DERIVATIVES AND INTEGRATIONS ARE OPPOSITE. THAT'S JUST NOTATION.

STUDENT: AFTER PROVE WHERE DID YOU GET THE C-FROM.

PROFESSOR: THEOREM ONE TELLS ME IF THIS FUNCTION F-HAS A DERIVATIVE OF ZERO IT HAS TO EQUAL SOME CONSTANT C-.

STUDENT: THAT'S JUST FROM THE DEFINITION.
PROFESSOR: THAT'S FROM THEOREM TWO. SO I SHOULD HAVE SAID BY
THEOREM TWO, THERE EXISTS A C, A CONSTANT C-SUCH THAT F-OF
X-EQUALS C-. AND THEN I JUST PLUG IN F-OF X, EQUALS F-ONE MINUS
F-TWO. THAT'S HOW I DEFINED IT. OKAY. SO LET ME NOW JUST
REMIND YOU OF ALL OF THE RULES FOR DIFFERENTIATION AND TURN
EVERYONE OF THEM INTO A RULE FOR INTEGRATION.

SO EVERY DIFFERENTIATION RULE BECOMES AN INTEGRATION RULE.

OKAY. LET'S JUST FIGURE THEM OUT. SO LET'S SUPPOSE I TAKE
INTEGRAL OF THE SUM OF TWO FUNCTIONS. HOW AM I GOING TO DO
THAT? JUST GOING TO DO ONE INTEGRAL AT A TIME AND ADD IT UP.
(ON BOARD). THAT'S JUST EXACTLY THE WAY DIFFERENTIATION WORKS.
JUST SUMS, TURN BY TURN. SO YOU SEE WHY THAT FOLLOWS RIGHT AWAY
FROM DIFFERENTIATION? OR SHOULD I DO A ONE LINE PROOF? I'LL DO
A ONE LINE PROOF.

STUDENT: WHAT'S THE SYMBOL.

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PROFESSOR: THAT'S THE INTEGRATION SIGN AND THAT'S JUST A SQUARE
BRACKET. INSTEAD OF PARENTHESES BECAUSE THERE'S TOO MANY
PARENTHESES. SHOULD I DO A ONE LINE PROOF TO JUSTIFY THIS?
THEY'RE ALL PRETTY MUCH THE SAME. SO GOING TO TAKE BOTH SIDES
HERE AND DIFFERENTIATE. I HAVE TO GET THE SAME ANSWER. LET ME
JUST CONFIRM THAT. THAT'S THE DERIVATIVE OF THIS SIDE. AND I
WANT TO MAKE SURE IT'S THE SAME AS THE DERIVATIVE OF THE OTHER
SIDE. AND NOW I KNOW HOW TO DIFFERENTIATE THE SUM. I JUST DO
IT IT TURN BY TURN. (ON BOARD). SO I WANT TO MAKE SURE THAT'S
TRUE. NOW I'M GOING TO USE THE FACT THAT IF I HAVE A DERIVATIVE
AND INTEGRAL, THEY JUST CANCEL. SO THIS SIDE TURNS INTO F-OF
X-PLUS G-OF X-. AND THIS TURNS INTO F-OF X-AND THIS TURNS INTO
G-OF X-. AND THAT'S CERTAINLY IS TRUE. F-OF X-PLUS G-OF X-IS
F-OF X-PLUS G-OF X-.

STUDENT: WHY DO YOU HAVE A D-X-THERE AT THE END OF EACH TERM.

PROFESSOR: BECAUSE THERE MIGHT BE OTHER VARIABLES IN HERE, LIKE
Y-S AND THING. JUST TO REMINDS YOU WHICH VARIABLE IS THE ONE
YOU'RE TALKING ABOUT. IF I HAD F-OF X-EQUALS Y-TIMES X-I WANT
IT TELL YOU Y-IS A CONSTANT. AND X-IS THE VARIABLE. JUST
NOTATION. HERE'S ANOTHER RULE THAT SUPPOSEDLY MATCHES WHAT WE
KNOW FROM DIFFERENTIATION. IF I WANT TO INTEGRATE A CONSTANT
TIMES A FUNCTION, I CAN JUST FACTOR OUT THE CONSTANT AND THEN
DIFFERENTIATE IT. THAT JUST MATCHES THE RULE FOR
DIFFERENTIATION. AND THE WAY I'LL DO IT IS CONFIRM THAT I GET
THE SAME ANSWER IN BOTH SIDES. I WANT TO MAKE SURE THAT WHEN I

DIFFERENTIATE, I GET THE SAME ANSWER ON BOTH SIDE. SO LET ME
WRITE A QUESTION MARK THERE AND JUST SAY LET'S MAKE SURE AM THESE
GUYS CANCELING AND I GET K-TIMES F-OF X-ON THIS SIDE. I KNOW IF
I'M DIFFERENTIATING I CAN FACTOR OUT CONSTANTS. LET ME JUST DO
THAT. AND NOW THAT CANCELS AND I GET K-TIMES F-OF X-ONLY BOTH
SIDES AND THAT'S CERTAINLY TRUE. THERE ISN'T MUCH TO IT.

THESE RULES JUST WORK BACKWARDS.

X-TO THE POWER R-D-X. WE'RE INTERESTED IN FUNCTIONS. AND
THE ANSWER IS X-TO THE R-PLUS ONE OVER R-PLUS ONE PLUS ANY
CONSTANT YOU CARE TO NAME IF, NOW I HAVE A RESTRICTION ON R-.
WHAT DO YOU THINK THE RESTRICTION IS IF R-DOES NOT EQUAL WHAT?
NEGATIVE ONE. BECAUSE THEN I'D BE IN TROUBLE. LET'S CONFIRM.
I need to confirm if I differentiate both sides I get the same thing. C-goes away. And when I differentiate x-to the power r-plus one I get r-plus one times x-to the power one lower, and that works. Right? I get x-to the r-the way I should. I get x-to the r-equals x-to the r-. This is worth remembering.

Just like you remember how to differentiate, you should remember that rule for integrating any old power. So next comes exponentials. e-to the k-times x-d-x-is x-is constants. and the answer's going to be plus any old constants you care to name. and we'll just check it. differentiate both sides and make sure I get the same thing. I better get k-x-on both sides. (on board). I factor out the one over conclusion. differentiate that guy. I get k-times e-to the k-x-and fortunately the k-s cancel and it all works. (on board). Let me just say, there's a one difference between integration and differentiation. If I can write down a formula with exponentials and polynomials I know how to differentiate it. We've got all the rules. That's not true with integration. there's no formula for that one. You can prove that there's no formula. I've never going to ask you to integrate it but it actually come if are up in statistics all the time. In statistics you use that function all time. But there's no formula for it. so integration is a little different that way. But we won't go there.

Student: there's not an integral for that.

Professor: you can find the area under this curve. That's perfectly well define. It's a function, there's just you can't
WRITE IT DOWN WITH EXPONENTIALS AND POLYNOMIALS, YOU CAN'T. IT'S VERY USEFUL. IT COMES UP ALL THE TIME BUT THERE'S NO FORMULA FOR IT. IN STATISTICS IT'S CALLED THE NORMAL DISTRIBUTION OR BELL CURVE.

LET'S JUST DO A LITTLE EXAMPLE HERE. HOW ABOUT THE INTEGRAL OF X-CUBED PLUS ONE OVER SQUARE ROOT OF X-. LET'S JUST APPLY RULES HERE (ON BOARD). FIRST I'LL SAY I KNOW HOW TO DO SUMS. DO IT TERM BY TERM. AND I KNOW THAT HOW TO DO X-TO A POWER AM LET ME WRITE ONE OVER THE SQUARE ROOT IS ONE OVER X-TO THE MINUS ONE-HALF SO I CAN APPLY THE RULE. NOW THE RULE SAYS WHEN I HAVE X-TO SOME POWER WHICH IS NOT NEGATIVE ONE, SO I CAN JUST THAT AS X-TO THE FOURTH OVER FOUR PLUS SOME CONSTANTS. (ON BOARD).

AND WHAT DOES THIS ONE TURN INTO APPLYING THE SAME RULE? X-TO WHAT POWER? SO TAKE THIS THING, THAT'S THE R-OVER THERE, ADD ONE TO IT. SO I GET X-TO THE PLUS ONE-HALF DIVIDED BY ONE-HALF WHICH, OF COURSE, I CAN SIMPLIFY. PLUS SOME OTHER CONSTANT. AND SO I GET THAT TERM PLUS THAT TERM, WHEN I SIMPLIFY IT, PLUS ADD TWO CONSTANTS I GET ANOTHER CONSTANTS. SOME CONSTANT. OKAY.

SO LET ME JUST SAY JUST TO GIVE YOU A LITTLE ILLUSTRATION OF ONE OF THE LITTLE TRICKY POINTS OF SOMETHING I ERASED. SO LET ME.

STUDENT: JUST COMBINE THE C-S FOR THAT ONE.

PROFESSOR: IF I DO IT TERM BY TERM I GET TWO DIFFERENT --

STUDENT: WHAT WOULD YOU GET IF YOU DID ONE OVER X-

PROFESSOR: I'M GOING TO DO THAT. YOU GET THE LOG BECAUSE WE
KNOW IF YOU DIFFERENTIATE THE LOG YOU GET ONE OVER X-. LET ME DO THAT MORE CAREFULLY IN A MOMENT. THIS ONE IS A LITTLE TRICKIER. LET ME JUST SHOW YOU WHY. BECAUSE WHAT HAPPENS AT ZERO? IT'S NOT CONTINUOUS THERE, IT BLOWS UP. IF I GRAPH THIS THING WHAT'S IT LOOK LIKE? IT'S GOING TO GO, BLOW UP ON THAT SIDE. AND IT'S GOING TO GO DOWN ON THAT SIDE. IT LOOKS LIKE THAT. SOMETHING FUNNY HAPPENS THERE. REMEMBER I SAID THE RULE WAS YOU GET TO ADD A CONSTANT, THERE'S ONLY ONE ANSWER IF THE FUNCTION IS CONTINUOUS. THIS GUY IS NOT CONTINUOUS, IT'S GOING TO CHANGE A LITTLE BIT. X-TO THE MINUS ONE THIRD IS NOT CONTINUOUS AT ZERO. SO HERE, WHAT DOES THE RULE TELL US? THE RULE SAYS, YOU CAN GET, YOU GET ONE OVER HERE IT'S CONTINUOUS. SO I GET A CONSTANT. OVER HERE IT'S CONTINUOUS. I GET MAYBE A DIFFERENT CONSTANTS. SO LET ME WRITE THAT DOWN. SO YOU GET PLUS C-ONE, BECAUSE IT'S CONTINUOUS HERE, AND YOU GET PLUS C-TWO WHICH COULD BE DIFFERENT BECAUSE IT'S CONTINUOUS OVER THERE BUT THEY DON'T HAVE TO BE THE SAME. HOW DO I DO THIS? I TAKE THE EXPONENT, ADD ONE. AND SO I GET MINUS ONE THIRD PLUS ONE, THAT'S TWO THIRDS. NOW I DIVIDE BY TWO THIRDS. AND THAT'S THE SAME ADS MULTIPLYING BY THREE HALVES. JUST SIMPLIFY. SO THAT'S THE RULE. THERE'S GOING TO BE TWO CASES. ON THE LEFT OR ON THE RIGHT. THAT PART'S ALWAYS THE SAME. BUT ON THIS PART, I CAN HAVE C-ONE. ON THAT PART I CAN GET A DIFFERENT C-TWO. SO LET'S PLOT THIS SO WE KNOW WHAT WE'RE LOOKING AT. LET ME JUST, TO WARM UP, PLOT X-TO THE TWO THIRDS. WHAT DOES THAT LOOK LIKE? ROUGHLY? SO IT COMES DOWN LIKE THIS, AND GETS
VERY STEEP. BECAUSE THE SLOPE IS GOING TO INFINITY THERE. AND THEN IT GOES UP LIKE THAT. IT HAS A LITTLE POINTY PART THERE. A LITTLE CUSP. AND SO THAT'S JUST X-TO THE TWO THIRDS. AND SO WHAT DOES THIS LOOK LIKE? WELL, I'M GOING TO THAT I CAN PART AND ADD C-ONE TO IT. SO START AT C-ONE. AND THEN THIS ONE I CAN START AT, ANYWHERE ELSE, CALL THAT C-TWO. IT KIND OF CAN BE BREAK IN THE MIDDLE. THAT'S WHAT THE ANTI DERIVATIVE LOOKS LIKE. ONLY ONE NICE SMOOTH POSSIBILITY THERE. ANOTHER NICE SMOOTH POSSIBILITY THERE. THIS IS ANY CONSTANT I CARE TO NAME IT.

STUDENT: JUST BREAK UP THE GRAPH.

PROFESSOR: YEAH. THIS PART SLID UP AND DOWN AND THAT PART CAN SLID UP AND DOWN INDEPENDENTLY. BECAUSE THEY GET DECOUPLED HERE IN THE MIDDLE BECAUSE THE THING IS NOT CONTINUOUS. MOST OF THE FUNCTIONS WE'RE GOING TOO DEAL WITH DON'T LOOK LIKE THAT. JUST SO YOU KNOW. SOMEBODY POINTED OUT THIS DOESN'T WORK IF R-IS NEGATIVE ONE. WE STILL NEED A RULE FOR THAT. THAT RULE THREE DOESN'T WORK IF R-EQUALS MINUS ONE. SO HERE'S THE RULE, INTEGRAL OF ONE OVER X-D-X, ANOTHER WAY, SO WRITE IT THIS WAY. SAME THING. (ON BOARD). THAT'S GOING TO BE THE LOG OF THE ABSOLUTE VALUE OF X-PLUS ANY OLD CONSTANT. AND WHY? THE USUAL THING. DIFFERENTIATE BOTH SIDE AND CHECK THAT WE GET THE SAME THING. OF COURSE, THIS IS GOING TO GIVE ME ONE OVER X-. NOW HERE I HAVE TWO CASES. X-POSITIVE, X-NEGATIVE. SO I'M GOING TO HAVE TO CASES, I BETTER CHECK THEM BOTH. SO WHEN X-IS POSITIVE, X-IS JUST X-. AND WHEN X-IS NEGATIVE, ABS X-IS
NEGATIVE X-. THAT'S WHAT THAT FORMULA MEANS. SO WHAT IS THE
DERIVATIVE OF LOG OF X? IT'S ONE OVER X-. THAT ONE WORKS OUT.
NOW WHAT'S THE DERIVATIVE OF LOG OF NEGATIVE X? YOU HAVE TO USE
CHAIN RULE. SO FIRST I HAVE TO TAKE ONE OVER THIS. I GET ONE
OVER NEGATIVE X-. THEN I HAVE TO DIFFERENTIATE NEGATIVE X-WHICH
IS TIMES NEGATIVE ONE AND THAT'S ONE OVER X-. IT WORK ON BOTH
SIDES. THERE ARE TWO CASES, FORTUNATELY YOU GET THE SAME
ANSWERED.

STUDENT: BACK UP ON THE GRAPH WHERE SPLIT UP INTO C-ONE AND
C-TWO. DO YOU HAVE TO WORRY ABOUT ONE BEING OPEN CIRCLE AND ONE
BEING CLOSED.

PROFESSOR: IN A PARTICULAR PROBLEM YOU MIGHT ONLY, SO DEPENDING
ON YOUR PROBLEM, IF YOU ONLY WANTS THE AREA ON ONE SIDE OR OTHER,
THEN YOU HAVE NO PROBLEM. IF YOU ONLY WANT THE AREA FROM THERE
to THERE, OR THE AREA FROM THERE TO THERE, THEN THERE'S NO
AMBIGUITY. THE ONLY WAY YOU CAN GET INTO TROUBLE. SUPPOSE YOU
WANT THE AREA FROM THERE TO THERE, THAT'S A WEIRD QUESTION
BECAUSE THAT AREA WILL BE PLUS INFINITY AND THAT AREA WILL GO
MINUS INFINITY. SOME AREAS YOU CAN'T COMPUTE BECAUSE THEY'RE NOT
FINITE. THAT'S WHAT HAPPENS. I WILL NOT ASK YOU TO COMPUTE
INFINITE AREAS. YOU COULD WRITE DOWN THE QUESTIONS BUT THAT'S
THE ANSWER. LET'S JUST PRACTICE THE RULES AGAIN.

SO X-TO THE MINUS THREE PLUS SEVEN E-TO THE FIVE X-PLUS FOUR
OVER X-(ON BOARD) LET'S JUST TRY THAT. THE FIRST THING IS I'M
GOING TO BREAK IT UP INTO PIECES BECAUSE I KNOW HOW TO DO EACH
PIECE. (ON BOARD). SO THE NEXT RULE SAYS, YOU CAN FACTOR OUT
THE CONSTANTS.  LET'S GET THOSE OUT.  AND FINALLY I'M IN A PLACE WHERE YOU CAN APPLY MY RULES.  SO X-TO THE MINUS THREE, WANT TO INTEGRATE THAT, THAT'S RULE THREE UP THERE, SO WHAT DO I GET? X-TO WHAT POWER?  ADD ONE TO MINUS THREE AND I GETS MINUS TWO. OVER MINUS TWO.  AND HERE WHAT ABOUT E-TO THE FIVE X?  I GET E-TO THE FIVE X-AGAIN BUT MULTIPLIED BY OR DIVIDED BY WHAT? DIVIDED BY FIVE.  AND THEN INTEGRAL OF ONE OVER X-IS WHATEVER I HAD OVER THERE. LOG OF ABSOLUTE VALUE OF X-PLUS A BIG CONSTANTS OF YOUR CHOICE.  OKAY.  SO MANY EXAMPLES, IT'S SORT OF THE SAME MACHINERY AS DIFFERENTIATION.  DO IT TERM BY TERM.  FACTOR OUT CONSTANTS.  APPLY RULES TO EACH TERM.  SO IN PRACTICE WE SOMETIMES WANTS TO KNOW WHAT THAT CONSTANT IS.  SO LET ME DO ANOTHER SIMPLE EXAMPLE.  AND WHERE WE CAN FIGURE OUT THE CONSTANT.

FIND X-OF X-IF I TELL YOU THE DERIVATIVE AND I TELL YOU THAT IT GOES, IT'S VALUED AT ONE POINT.  SO I'VE GIVEN YOU ONE MORE PIECE OF INFORMATION THAN BEFORE.  WHAT I'M GOING TO DO IS I'M GOING TO INTEGRATE THE DERIVATIVE TO GET MY FUNCTION.  THAT'S GOING TO BE INTEGRAL OF X-SQUARED MINUS THREE D-X-.  SO INTEGRAL OF X-SQUARED D-X-MINUS THREE TIMES INTEGRAL OF ONE D-X-.  WE'LL FIGURE THAT OUT.  WHAT'S THE INTEGRAL OF X-SQUARED?  X-CUBED OVER THREE. WHAT'S THE INTEGRAL OF X-SQUARED? X-CUBED OVER THREE.  WHAT'S THE INTEGRAL OF ONE? X-TO THE, X-TO THE ZERO SO IT'S -- PLUS THE CONSTANT I DON'T HAVE.  SO HERES X OF X.  I HAVE ONE MORE FACT NOW.  F-OF ONE IS BEGIN TO BE ONE THIRD, SO IT'S GOING TO BE ONE CUBED MINUS THREE TIMES ONE PLUS C-.  (ON BOARD).  AND I THINK I CAN SOLVE THAT FOR C-.
C-EQUALS THREE. I CAN SOLVE THAT ONE. FINALLY I GET F-OF X-EQUALS X-CUBED OVER THREE MINUS THREE X-PLUS THREE. SO A LITTLE BIT MORE INFORMATION TO FIGURE OUT THE CONSTANT. (ON BOARD). SOMETIMES I DO EXAMPLES FROM ECONOMICS AND SOMETIMES I DO EXAMPLES FROM HEALTH. I'LL DO ONE EXAMPLE FROM BOTH JUST TO DO THIS.


TIMES E TO THE .12 T-ROUGHLY. PLUS A CONSTANT. (ON BOARD).
AND SO I DON'T KNOW THE CONSTANT BUT I'M NOT SURE I CARE.
BECAUSE WHAT I WANT IS ACTUALLY, SORRY, I WROTE THIS DOWN
SLIGHTLY WRONG. BECAUSE YEARS, THIS IS TRUE I WANT, BUT THIS
IS, LET ME WRITE DOWN WHAT I MEANT TO SAY. THIS IS HOW MUCH
WE'VE SPEND UP TO THE YEAR 2000, BUT I'M STARTING COUNTING AT
T-IS THE YEAR 2000. WHAT I WANT IS T-OF TEN TIMES T-OF ZERO.
THAT'S GOING TO BE 3167, SORRY, 3167 TIMES E-TO THE .12 TIMES
TEN. PLUS C-. MINUS 3167 E-TO THE MINUS .12 TIMES ZERO PLUS
C-. (ON BOARD). THE C-S CANCEL, DON'T THEY. I DON'T NEEDS
TO KNOW C-BECAUSE THERE'S A MINUS C-AND A PLUS C-. ONE, AND I
CAN GO AHEAD AND DECIDE THAT IT'S THE ENORMOUS SUM OF
$735,000,000,000,000. ACCORDING TO THIS MODEL. THEY MAY RAISE
THE BAIL OUT TO THAT LEVEL BY TOMORROW BY WHO KNOWS, SOUNDS LIKE
A LOT.
STUDENT: WHY IS IT IN THE SECOND ONE E-TO THE NEGATIVE .1 --
PROFESSOR: OOPS BECAUSE I WAS WRONG. ONLY ONE MINUS SIGN BUT
IT'S TIME ZERO. SO PLUS .12.
STUDENT: WHAT ABOUT THE LINE BEFORE THAT.
PROFESSOR: THIS? THAT'S A POINT. OKAY. SO THAT'S ENOUGH
ABOUT MONEY. I'M GOING TO DO.
STUDENT: IS THAT ONE RIGHT THERE, WHY WHEN INTEGRATED ONE D-X-DO
YOU GET X-
PROFESSOR: SO THAT, LET ME GIVE TWO ANSWERS. ONE IS THAT THE
INTEGRAL OF ONE D-X-IS THE SAME THING AS THE X-TO THE ZERO D-X-. 
AND THE RULE SAYS THAT'S X-TO THE ZERO PLUS ONE DIVIDED BY ZERO PLUS ONE PLUS CONSTANTS OR X-PLUS CONSTANTS. THAT'S JUST APPLYING THE RULE. THE OTHER THING IS I WANT TO FIND THE AREA UNDER THE CURVE WHICH IS JUST THE STRAIGHT HORIZONTAL LINE AT ONE, WANT TO FIND THE AREA BETWEEN ZERO AND X-. THIS IS THE AREA OF THE RECTANGLE. THE BASE IS X, THE HEIGHT IS ONE. ANSWERS IS X-. OKAY. COMING TO ARREST ME ... (ALARM SOUNDED).

STUDENT: WHY ISN'T IT V OF ZERO EQUALS EIGHT.

PROFESSOR: BECAUSE I JUST HAPPEN TO KNOW THAT THE ROCKET, I COULD HAVE GIVEN YOU SEVERAL PIECES OF INFORMATION BUT THE PIECE OF INFORMATION I HAD WAS THAT THE ROCKET SITTING ON THE GROUNDS IN THE BEGINNING AND I KNOW HOW TALL THE ROCKET IS. BECAUSE THE ROCKET IS 8 METERS UP. I COULD USE THAT INFORMATION TOO.

THERE'S DIFFERENT KINDS OF INFORMATION YOU CAN USE. I USED THAT ONE. SO NOW I KNOW THE FUNCTION. I KNOW THAT S-OF T, IS THREE T-SQUARED PLUS T-OVER TWO PLUS EIGHT. THAT'S MY FORMULA FOR ANY OLD T-. THE QUESTION WAS HOW HIGH IS IT AFTER TEN. SO I PLUG IN TEN SECONDS AND I GET, AFTER I DO ANALYSIS THERE, 313.

THAT'S HOW HIGH IT IS OFF THE GROUNDS AND 313 METERS UP AFTER TEN SECONDS.

NOW LET'S SUPPOSE THE ROCKET RUNS OUT OF FUEL AFTER TEN SECONDS. SO THE ONLY THING LEFT IS GRAVITY. SO WHERE DOES THE ROCKET GO? SO IN OTHER WORDS I WANT A FORMULA FOR S-OF T-. FOR IN THE FUTURE. RIGHT AFTER IT RUNS OUT OF FUEL. IT STOPS ACCELERATING. SO WHAT'S THE ONLY THING I KNOW? I KNOW THE ACCELERATION. IT'S GRAVITY'S PULLING IT DOWN. AND THERE'S THE FORMULA, IT'S CONSTANT. ACCELERATION OF GRAVITY. IT'S NEGATIVE BECAUSE IT'S PULLING IT DOWN, I THINK. SO WHAT I WANT TO DO NEXT IS FIGURE OUT THE VELOCITY. AND FROM THE VELOCITY I'LL GET THE HEIGHT. USE THE FACT THAT THE DERIVATIVE OF THE VELOCITY IS ACCELERATION. THAT'S HOW THEY'RE RELATED TO ONE ANOTHER. I WILL GET THE VELOCITY BY INTEGRATING ACCELERATION.

AND SO WHAT'S THE INTEGRAL OF MINUS TEN? MINUS TEN T-PLUS SOME
CONSTANT. OKAY. HOW DO I FIGURE OUT THE CONSTANT? I NEED TO KNOW WHAT THE VELOCITY IS RIGHT AT THE MOMENT IT RUNS OUT OF FUEL. SO COME BACK OVER HERE, HOW FAST IS IT GOING THE MOMENT IT RUNS OUT OF FUEL? PLUG IN T-EQUAL TEN, 60.5. SO IT'S SIX TIMES TEN PLUS ONE-HALF IS 60.5. SO V OF T-EQUALS MINUS TEN T-PLUS, AND WHAT'S THE CONSTANT? WHEN T-EQUALS TEN, THIS SUM HAS TO BE 60.5.

STUDENT: WHY DO YOU HAVE TWO DIFFERENT EQUATIONS FOR VELOCITY.

PROFESSOR: BECAUSE THIS IS WHAT'S TRUE FOR THE FIRST TEN SECONDS. WHEN THERE'S STILL FUEL. AND THEN AT TEN SECONDS I RUN OUT OF FUEL AND THING CHANGE. I HAVE ONE FORMULA FOR UP TO TEN SECONDS AND ANOTHER FORMULA FOR AFTER TEN SECONDS.

STUDENT: THAT A SIX.

PROFESSOR: SIX TIMES TEN PLUS A HALF. THIS IS WHAT HAPPENS RIGHT AT THE ENDS OF FIRST TEN SECONDS. AT THE ENDS OF FIRST TEN SECONDS I'M GOING THAT FAST. SO WHAT'S THE CONSTANT? IT'S 160.5. IF I PLUG IN T-EQUALS TEN I GET 100 PLUS 160.5. SO IT MATCHES. SO THIS IS TRUE FOR T-IS GREATER THAN OR EQUAL TO TEN. AND THIS IS TRUE FOR T-LESS THAN OR EQUAL TO TEN, SO WHILE IT HAS FUEL AND AFTER IT HAS NO FUEL. SO TWO DIFFERENT EQUATIONS.

ALMOST DONE. NOW I WANT THE FINAL THING I WANT IS HOW HIGH IS IT OFF THE GROUND.

STUDENT: WHEN YOU SAY WHERE DOES THE ROCKET GO.

PROFESSOR: HOW HIGH. WHAT IS S-OF T-

STUDENT: AFTER T-SECOND, TEN SECONDS.

PROFESSOR: I HAVE THE FORMULA HERE, THIS IS WHAT, THIS IS TRUE
UP TO TEN SECONDS WHILE I STILL HAVE FUEL. IT'S ACCELERATING UPWARDS. THIS IS WHAT, NOW I WANT TO FORMULA THAT'S THROUGH FOR

BIGGER THAN TEN SECONDS, WHEN IT RUNS OUT OF FUEL AND EVENTUALLY GOING TO FALL DOWN.

STUDENT: WHY IS CONSTANT 160.5.

PROFESSOR: BECAUSE, PLUG IN T-EQUALS TEN. AND GET MINUS TEN TIMES TEN. AND I GET 160.5. AT THE MOVEMENT, EXACTLY TEN SECONDS BOTH FORMULAS FOR V HAVE TO BE EQUAL. THEY HAVE TO AGREE. IT'S GOING AT 6.5 METER PER SECOND AT T-EQUALS TEN. I HAVE ONE FORMULA ON ONE SIDE AND ONE FORMULA ON THE OTHER AND THEY HAVE TO MATCH AT THAT MOMENT. I WANT S-OF T-. GOING TO DO THE SAME THING. GOING TO INTEGRATE OF VELOCITY. WHY IS THAT? BECAUSE S-TIMES T-EQUALS V OF T-. GOING TO INTEGRATE THE VELOCITY TO GET THE HEIGHT OFF THE GROUND. TAKE THAT FORMULA. IT'S MINUS TEN T-PLUS 160.FIVE D-T-. SO WHAT IS THE INTEGRALITY OF MINUS TEN T? MINUS FIVE T-SQUARED, DIFFERENTIATE THAT YOU GET MINUS TEN T-. WHAT'S THE INTEGRAL OF 160.5, THE CONSTANT. PLUS SOME CONSTANT THAT I HAVE TO FIGURE OUT. SO HERE'S S-OF T-. HOW DO I FIGURE OUT THE CONSTANT.

STUDENT: MINUS FIVE T-IS NEGATIVE.

PROFESSOR: IF I DIFFERENTIATE THIS, TWO COMES DOWN, MINUS FIVE CHANGES INTO MINUS TEN.

STUDENT: I MEANT LOOK LIKE A NEGATIVE TWO.

PROFESSOR: SORRY. WHERE AM I. THIS IS A T-SQUARED. SORRY. BAD WRITING. SO WHAT I NEED TO DO, SAME IDEA AS BEFORE, I NEED TO FIND C-AND WHAT'S THE FACTOR I'M GOING TO USE. I KNOW HOW
HIGH I AM. AT THE MOMENT, I RUN OUT OF FUEL, WHERE DID I WRITE

THAT DOWN? WHICH BOARD IS IT ON? AT TEN SECONDS THIS IS HOW
HIGH OFF THE GROUND I AM. WHAT I KNOW IS AT TEN SECONDS I'M
313 METERS OFF THE GROUND. THAT'S THE MOMENT I RAN OUT OF FUEL.
SO I JUST PLUG THIS IN. I GET MINUS FIVE TIMES CONTINUE SQUARED
PLUS 160.5 TIMES TEN PLUS C-BETTER EQUAL 313. I CAN SOLVE FOR
THAT. AND I GET IT EQUALS MINUS 792. JUST DO THE ARITHMETIC.
FINALLY I GET THAT AFTER TEN SECONDS, THE HEIGHT OFF THE GROUND
IS MINUS FIVE T-SQUARED PLUS 160.5 T-MINUS 792. AND FOR T-LESS
THAN OR EQUAL TO TEN, S-OF T-EQUALS THREE T-SQUARED PLUS T-OVER
TWO PLUS EIGHT. THAT'S ALL THERE'S TO KNOW ABOUT THAT ROCKET.
HAVE A GOOD THANKSGIVING.