MATH 16A LECTURE. NOVEMBER 18, 2008.

PROFESSOR: SO WELCOME BACK. YES, QUESTION? SO I HAVE NOT GOTTEN ALL THAT DATA FROM GSI'S YET. SOMEONE ELSE ASKED THAT QUESTION. SO -- HIGHER THAN THE FIRST MIDTERM. SO I CAN SAY THAT BUT I'M NOT SURE EXACTLY WHAT HAD.

STUDENT: IF BASED HIGHER, DOES THAT MEAN YOU'LL RAISE THE CURVE.

PROFESSOR: THE HOPE IS THAT I WILL ADJUST THE FINAL SO THAT THE CURVE WILL BE EXACTLY HAD A I STATED IT TO BE ON THE WEB PAGE THE FIRST DAY OF CLASS. WHICH I THINK WAS 85 PERCENT. I DON'T HAVE ALL THE DATA YET SO WE'LL WORK IT OUT. THE TARGET IS TO HAVE PRETTY MUCH THE SAME DIFFICULTY, DISTRIBUTION AS CLASS AS EVIDENCE BY THE MIDTERMS.

STUDENT: IS THE CURVE ON TEST, AS A GENERAL.

PROFESSOR: SINCE THE TEST VARIED IN DIFFICULTY FROM EXAM TO EXAM I WILL PROBABLY TAKE EACH ONE AND ADJUST SLIGHTLY. SUBTRACT THE MEAN AND DIVIDE, TO COME UP WITH A SCORE THAT TAKE INTO ACCOUNT THAT ONE EXAM IS DIFFERENT FROM ANOTHER EXAM. SO BUT DON'T WORRY. THERE'S STANDARD SCIENCE TO MAKE ALL THIS HAPPEN. ANY OTHER QUESTIONS BEFORE WE GO ON WITH SECTION 5.2? OKAY.

SO LET ME THERE'S ONE MORE TOPIC EXAMPLE I WANTED TO DO. SECTION 5.2. LET ME JUST WAIT UNTIL EVERYBODY'S QUIET AND THEN KEEP GOING. SO LET ME PUT UP THE DEFINITIONS AGAIN THAT WE HAD, IN THAT CHAPTER, THAT SECTION LAST TIME. AND THERE'S ONE MORE EXAMPLE. SO HERE WAS A FORMULA WE STARTED WITH. THIS IS THE AMOUNT WE GET FROM INVESTING PRINCIPLE AMOUNT OF MONEY P-AT
YOURSELF 1/12TH OF THE INTEREST PER MONTH. AND I CAN'T NOT MORE
OFTEN COMPOUNDED M TIMES PER YEAR. AND FINALLY WE SAID
COMPOUNDED EVERY MICRO SECOND IF YOU LIKE. TAKE THE LIMIT, AND
TURNS OUT THAT THERE WAS A NICE SIMPLE FORMULA FOR THAT. IT WAS
P-TO THE E R T-. SO THIS FORMULA HERE AND MAKE M LARGER AND
LARGER, FINALLY CONVERGED TO THAT. REVIEW FROM LAST TIME. AND
WE CALLED THAT COMPOUNDED, I'LL WAIT UNTIL EVERYBODY'S QUIET
AGAIN, WE CALLED THAT COMPOUNDED CONTINUOUSLY. AND WE DID AN
EXAMPLE WHERE WE INVESTED $1 FOR 20 YEARS AT 5 PERCENT AND SO IF
YOU PLUG IN THAT FORMULA, YOU GOT THE AMOUNT OF MONEY WAS ONE
times e-to the .05 times 20 which is e-to the one or e-. WE
ACTUALLY DOES EXAMPLES THERE AND I KEPT MAKING M BIGGER AND
BIGGER AND WE SAW IT CONVERGED TO E-WITH OUR VERY OWN EYES. AND
I DID A PROOF. I'M NOT GOING TO DO THE PROOF AGAIN. BUT HERE
WAS THE IDEA THAT WE USED, IT FALLS FROM ANOTHER LIMIT, AND THIS
IS ONE THAT I HOPE EVERYBODY WILL RECOGNIZE FROM NOW ON. ONE
PLUS H-TO THE POWER OF ONE OVER H, IF LET H-GT TYPE YOUR AND
TYPE YOUR THIS DOES CONVERGE TO E-. THIS IS A LIMIT THAT YOU
NEED TO RECOGNIZE WHEN YOU SEE LIMITS LIKE THAT. OKAY. SO
THAT WAS PRETTY MUCH THE SUMMARY OF EVERYTHING WE DID IN SECTION
5.2 LAST TIME. AND I WANT TO DO NOW AN EXAMPLE. TO USE IT.
IT'S SORT OF A INVESTMENT EXAMPLE. SO IS IT BETTER TO INVEST
YOUR MONEY, LET'S SAY A THOUSAND DOLLARS AT 6 PERCENT COMPOUND
ANNUALLY OR 5.9 PERCENT COMPOUNDED CONTINUOUSLY? YOU HAVE TWO
DIFFERENT INVESTMENT OPTIONS THAT YOU CAN PICK AND THAT'S YOUR
CHOICE. SO WHICH ONE IS BETTER? LET'S JUST WRITE THE TWO
POSSIBILITIES DOWN. SO I'M GOING TO WRITE, LET ME CALL IT
P-ONE, LET ME CALL IT M ONE FOR MONEY. M TWO FOR MONEY, TWO
POSSIBILITIES. AND HERE ARE THE TWO. ONE AT 6 PERCENT AND 5.9
PERCENT AM SO WHAT IS THE FORMULA HERE? IT'S GOING TO BE A
THOUSAND DOLLARS TIMES ONE PLUS ONE SIX TIMES VALUE T-. THAT'S WHAT YOU DO INVESTING AT 6 PERCENT AND COMPOUNDED ANNUALLY. WANT TO COMPARE THAT TO INVESTING CONTINUOUSLY AT 5.9 PERCENT. SO WHAT'S THE FUNCTION, E-TO SOMETHING? IT'S E-TO THE .0592. I'D LIKE TO KNOW WHICH OF THOSE IS BIGGER. WHICH ONE DO I GET MORE MONEY? SO WHAT I'M GOING TO DO TO COMPARE THEM IS, TAKE THEIR DIFFERENCE, OF COURSE, AND ASK IF THE DIFFERENCE IS BIGGER THAN ZERO OR LESS THAN ZERO. THIS TURNS OUT TO BE A LITTLE EASIER. I'M GOING TO TO ASK TO THE RATIO BIGGER THAN ONE WHICH IS IN THIS CASE IS THE THING TO DO. OR LESS THAN ONE IN WHICH CASE THAT'S THE WAY TO INVEST. SO LET'S PLUG IN THOSE FORMULAS. (ON BOARD). SO THOUSANDS CANCEL. AND I'LL LEFT WITH E-TO THE .059 DIVIDED BECAUSE OF LOG OF EXPONENTS ALL TO THE POWER T-. SO JUST COMES DOWN TO ASKING DOES THIS NUMBER HERE, IS IT BIGGER THAN ONE IN WHICH CASE THIS, THE 5.9 PERCENT INTEREST RATE WINS, OR LESS THAN. IF YOU USE YOUR CALCULATOR OR ANY OTHER SUCH DEVICE THAT'S WHAT YOU GET. IT'S JUST BARELY BIGGER THAN ONE.

SO THE NUMERATOR IS LARGER SO YOU SHOULD INVEST 5.9 PERCENT AND COMPOUNDED CONTINUOUSLY. ANY QUESTIONS ABOUT HAVING (INAUDIBLE). STUDENT: COULDN'T YOU HAVE DONE M ONE OVER M TWO. PROFESSOR: ABSOLUTELY, SURE. YOU JUST, THAT WOULD TELL YOU, YEAH, IT'S THE SAME IDEA. OKAY. THAT IS ALL I WANTED TO SAY ABOUT COMPOUND INTEREST. BUT IT'S NOT ALL I WANT TO SAY ABOUT MONEY. SO THE NEXT SECTION, FIVE .3. STUDENT: SO BECAUSE ONE YOU CAN ASSUME 5.9 WORKS BETTER. PROFESSOR: SO I TOOK THE RATIO OF HOW MUCH MONEY YOU HAVE FROM THE 5.9 PERCENT INTEREST CONTINUOUSLY DIVIDED BY THE 6 PERCENT ANNUALLY, THE RATIO IS BIGGER THAN ONE. THE NUMERATOR IS BIGGER AM YOU GET MORE MONEY DOING IT IN THE NUMERATOR. SO I WANT TO
TALK MORE ABOUT USING LAW OF EXPONENTS TO MODEL ECONOMICS. SO LET ME TALK ABOUT THAT A BIT IN SECTION FIVE .3. SO WHAT WE WANT TO DO IS USE THE FACT THAT IT'S COMMON TO TALK ABOUT RATES OF CHANGE, THAT'S WHAT DERIVATIVES ARE, BUT MEASURED IN A PARTICULAR WAY, RATE OF CHANGE RELATIVE TO THE CURRENT VALUE. SO IN PARTICULAR WE ALWAYS CARE ABOUT, FOR EXAMPLE, INFLATION, IT WAS UP 2 PERCENT LATE YEAR. THAT MEANS TWO PER MORE THAN IT WAS THE PREVIOUS YEAR, OR THE STOCK MARKET WAS DOWN MAYBE 5 PERCENT LAST WEEK. HOW MUCH DOES IT CHANGE -- I WANT IT EXPRESS THAT IN CALCULUS IN THE WAY THAT MAKES IT CLEAR WE HAVE ALL THE FUNCTIONS LYING AROUND TO SOLVE IT. LET ME DO ANOTHER EXAMPLE HERE.

SO JUST, MAKE IT CLEAR HERE THAT T-IS GOING TO BE MEASURED IN YEARS. SO MEASURING THIS CHANGE OVER YEARS, TO MAKE IT CONCRETE. AND LET F-OF T-BE PRICE PER POUND OF SIR LOIN, OKAY. AND WE'LL SAY THAT THE STARTING POINT MEASURED AT SOME POINT TIME T-NAUGHT IS 5.25. AFTER ONE YEAR THE PRICE GOES UP. IN ONE YEAR THE PRICE GOES UP 75 CENTS. THAT'S OUR DATA. AFTER ONE YEAR IT'S $6. I WANT TO COMPARE THAT AND ASK HOW FAST IS THAT GROW COMPARED TO THIS OTHER FUNCTION WHICH IS THE NATIONAL DEBT. NOW I HAVEN'T UPDATED THIS FOR THE LAST 24 HOURS SO IT'S PROBABLY TOO SMALL. BUT HERE'S THE NUMBERS. LET'S JUST MAKE IT 1.5 TRILLION. OKAY. AND IT'S GOING UP AND LET'S SAY IT WENT UP 100 BILLION IN ONE YEAR. I KNOW THAT. SO THE QUESTION IS WHICH IS GOING UP FASTER? IS THE NATIONAL DEBT GOING UP FASTER OR THE PRICE OF A HAMBURGER GOING UP FASTER? SO I WANT TO MAKE THIS POINT SO IT'S CLEAR HOW WE'RE MEASURING THING. WHETHER INCREASE OR DECREASE. SO THERE ARE TWO THING WE CAN COMPARE. DO WE COMPARE G-PRIME WHICH IS A HUNDRED BILLION DOLLARS TO F-PRIME, WHICH IS 75 CENTS. AND ASK WHICH ONE OF THOSE IS BIGGER? NO, THAT'S NOT WHAT WE'RE GOING TO DO. WE'RE GOING TO
ASK WHICH ONE WENT UP AT A HIGHER RATE? THE ANSWER TO THIS
QUESTION IS DO WE COMPARE THIS? NO. BUT WE'RE GOING TO DO
SOMETHING ELSE. INSTEAD WHAT WE'RE GOING TO DO IS COMPARE THE
FRACTIONAL INCREASE, GOING TO COMPARE F-PRIME OF T-NAUGHT OVER
F-OF T-NIGHT WHICH IS 75 CENTS DIVIDED BY CURRENT PRICE WHICH IS
.143 OR LET'S SAY SINCE PEOPLE LIKE PERCENTAGE IN THIS BUSINESS,
THE PRICE OF WE'VE WENT UP 14 .3 PERCENT AND COMPARE THAT TO THE
NATIONAL DEBT WHICH IS .1 TRILLION DIVIDED BY 1.5 TRILLION WHICH
IS .067 WHICH IS ONLY 6.7 PERCENT. SO THAT IS SMALLER THAN
THAT. SO SIRLOIN, WE'RE GOING TO SAY SIRLOIN IS GETTING LARGER
FASTER. THAN THE NATIONAL DEBT BECAUSE OF THESE RATES, THIS
RATE IS GOING UP FASTER THAN THAT. OBVIOUSLY A HUNDRED BILLION
IS BIGGER THAN 75 CENTS BUT THAT IS WHAT WE'RE GOING TO BE
COMPARING, THE RELATIVE CHANGE. IS THAT CLEAR WHAT I WANT TO
MEASURE? OKAY.

LET ME PUT A DEFINITION HERE. THERE'S THE EXAMPLE. LET
ME WRITE DOWN THE GENERAL DEFINITION. SO F-OF T-IS GOING TO BE
THE VALUE AT TIME T-AND WE'RE GOING TO SAY THAT D-T-OF THE
NATURAL LOG OF F-OF T-IS CALLED THE LOGARITHMIC DERIVATIVE. OR
ANOTHER WORD FOR IT IS THE RELATIVE RATE OF CHANGE. THAT'S THE
QUANTITY THAT WE'RE GOING TO USE, HOW FAST IS THE LOGARITHM
CHANGE. AND IF YOU USE THE CHAIN RULE, TO EVALUATE WHAT THIS
IS, WHAT'S THE CHAIN RULE TELL ME THIS IS? IT'S GOING TO BE
F-PRIME OF T-OVER F-OF T-. WHICH IS WHAT I WAS USING OVER THERE
to MEASURE THE RATE OF CHANGE. HOW FAST IS THE LOGARITHM
CHANGE. THAT'S GOING TO MEASURE HOW FAST SOMETHING CHANGE
RELATIVE. SOMETIMES WE'LL ALSO USE, WRITE IT THIS WAY, (ON
BOARD). USE PERCENTAGES. AND CALL THIS THE PERCENTAGE RATE OF
CHANGE. SO OVER HERE, IT WAS 14 .3 PERCENT. WAS PERCENTAGE
RATES OF CHANGE VERSE 6.7 PERCENTAGE RATES I HAVE CHANGE.
THAT'S GOING TO BE THE QUANTITIES THAT I USE TO MEASURE HOW FAST THE QUANTITY -- INCREASES. SO LET ME DO A FEW MORE EXAMPLES.

. LET ME USE A REAL MODEL FROM ECONOMICS. THIS IS THE GROSS DOMESTIC PRODUCT OF THE U.S. ONCE UPON A TIME, WAS ONE MODELED AS, F-OF T-AND THIS IS MEASURED IN TRILLIONS OF DOLLARS, I WON'T BOTHER TO WRITE THAT DOWN. (ON BOARD). WHERE T WAS THE YEARS, MEASURED IN YEARS SINCE 19905. SO T-EQUALS ZERO IS NINE 90 WHEN PEOPLE WROTE DOWN THIS MODEL. AND AT THAT TIME THE U.S. WAS IN A RECESSION. SO HERE IT WAS 1990. IT WAS DOWN. THE RECESSION WOULD BOTTOM OUT HERE AND GO UP AGAIN. AND THEY WEREN'T PRETENDING. BUT THIS IS GOOD FOR A FEW YEARS AM SO THE QUESTION IS HOW BAD IS RECESSION THERE IN 1990. AND HOW BAD IS THE RECESSION THERE ONE YEAR LATER WHICH IS 1991. SO WE'RE GOING TO ASK HOW FAST IS THE GDP INCREASING AS A PERCENT AND HOW FAST IS IT DECREASING. SO THERE'S F-OF T-. AND PRIME OF T-THAT'S GOING TO GIVE ME .04 MINUS .13. DOING CALCULUS. SO WHAT I WANT TO DO IS LOOK AT THIS QUANTITY, THE RATIO (ON BOARD). AND HERE IT IS. SO I CAN PLUG THIS IN, THERE ARE TWO CASES. AND AT T-EQUALS ZERO AND AT T-EQUALS ONE, WE CAN JUST NOW PLUG IN ZERO, PLUG IN ONE AND YOU GET, MINUS 2.5 PERCENT. IN THAT YEAR, THAT'S HOW BADDED RECESSION WAS, CLINKING AT THAT RATE. BUT A LITTLE WHILE LATER IT BOTTOMED OUT. AND SHRINKING SLIGHTLY. NOT GOING DOWN VERY MUCH.

STUDENT: YOU DID THAT BUT PLUGGING IN ZERO AND ONE.

PROFESSOR: YES ZERO FOR T-AND ONE FOR T-AND EXACTLY RIGHT.

LEAVING OUT THE CALCULATIONS.

STUDENT: THAT'S JUST OUR ANSWER.

PROFESSOR: THIS IS THE NUMERICAL ANSWER. IT SAYS IF THIS,
THERE'S A RECESSION BOTH POINTS BECAUSE DECREASING BUT NOT DECREASING VERY MUCH THERE. ONLY GOING DOWN .2 PERCENT PER YEAR.

STUDENT: WHAT WAS THE ORIGINAL QUESTION.

PROFESSOR: SO THE QUESTION WAS HOW BAD IS THE RECESSION? AND THE WAY PEOPLE MEASURED THAT IS BY WHAT RATE IS THE ECONOMY SHRINKING AND SO THIS YEAR IT'S SHRINKING AT 2.5 PERCENT PER YEAR WHICH IS PRETTY BAD. SHARP DOWNWARD SLOPE. GDP VERSE TIME. HERE IT ALMOST BOTTOMED OUT. AND THE AMERICA METRIC OF THAT IS THE SLOPE IS SMALL, IT'S -- ISN'T CHANGING VERY MUCH. USING PERCENTAGES BECAUSE THAT'S WHAT YOU WOULD USUALLY READ IN THE NEWSPAPER TO MEASURE THIS. IT'S THE UNIT THAT PEOPLE ARE ACCUSTOM TO. IT ALL CAME FROM THE DEFINITION, TAKE THE DERIVATIVE OF A LOG FUNCTION.

STUDENT: YOU WOULD HAVE TO KNOW AT LEAST (INAUDIBLE).

PROFESSOR: ONCE, IN OTHER WORDS YOU'RE ASKING WHERE DOES THIS FUNCTION COME FROM.

STUDENT: I'M SAYING YOU HAVE TO KNOW T-EQUALS ONE.

PROFESSOR: HOW BAD IS THE RECESSION T-EQUAL ZERO AND YOU WOULD COMPUTE THIS FUNCTION. AND ASK IT FOR ANY PARTICULAR VALUE OF T, I JUST CHOSE THOSE TWO FOR ILLUSTRATION. HOW DID THE ECONOMIST FIGURE OUT THIS IN THE FIRST PLACE. THEY HAD A WHOLE BUNCH OF POINTS AND THEY FIGURED IT OUT.

LET ME DO A FEW MORE EXAMPLES HERE. SOME VALUE OF AN INVESTMENT. STARTS OFF AT LET'S SAY SOMEONE IS INVESTED A MILLION. AND IT'S GROWING EXPONENTS WILL I BUT SLOWING DOWN BECAUSE THERE'S A SQUARE ROOT UP THIS. TO E-TO THE .6 SQUARE ROOT OF T-. IF YOU WERE TO ASK WHAT IS THE INTEREST RATE IT'S NOT A CONSTANT. NOT GOING UP AT A CONSTANT INTEREST RATE. SO THE QUESTION IS HOW FAST DOES F-OF T-INCREASE IN YEAR FIVE. SO
LET ME ANSWER BY DOING THE CALCULUS I'VE JUST BEEN TALKING ABOUT.

THEN I'LL DO IT IN, BY BRUTE FORCE.

STUDENT: IS THAT NEGATIVE.

PROFESSOR: THAT'S .6. BECAUSE YOUR MONEY IS GOING, YOU HOPE
THAT'S POSITIVE BECAUSE THAT'S WHERE YOUR MONEY IS INCREASING, IF
IT'S NEGATIVE IT MEANS YOU'RE LOSING MONEY. .6 TIME THE SQUARE
ROOT OF T-. SO I NEED TO COMPUTE F-PRIME OF T-AND THEN DIVIDE
BY F-. SO LET ME DO F-PRIME. SO THE MILLION IS STILL THERE.

WHENEVER I NOW I'M GOING TO USE CHAIN RULE. SO GET EXPONENTIAL
AND I STILL HAVE TO MULTIPLY BY THE DERIVATIVE THE EXPONENT.

USE THE CHAIN RULE. SO I'M WRITING THIS T THIS WAY BECAUSE THIS
FUNCTION WE RECOGNIZE IS JUST F-OF T-. TIMES DIFFERENTIATE THIS
NOW, AND THAT'S GOING TO BE .6 TIMES THE DERIVATIVE OF THE SQUARE
ROOT OF T-WHICH IS ONE-HALF TIMES ONE OVER SQUARE ROOT OF T-.

JUST DOING THE CHAIN RULE ON THIS PARTICULAR OBJECT. NOW I HAVE
TO DO F-PRIME DIVIDED BY F-OF T-. AND F-OF T-IS JUST SITTING
HERE SO IT'S PRETTY EASY TIDE THE DIVISION. AND I'M LEFT WITH
.6 TIMES ONE-HALF TIMES ONE OVER THE SQUARE ROOT OF T-. CALL
THAT .3 OVER THE SQUARE ROOT OF T-. SO IT'S A NICE SIMPLE

AND SO THE WAY WE WOULD ANSWER OUR QUESTION IS HOW
GOOD IS THIS IN YEAR FIVE? SO WHAT WE WOULD DO IS PLUG IN FIVE
F-PRIME OF FIVE DIVIDED BY F-OF FIVE .3 DIVIDED BY SQUARE ROOT OF
FIVE AND THAT COMES OUT TO .134 OR 13.4 PERCENT WHICH IS AWFULLY
GOOD. NOW IF YOU DENY KNOW THIS STUFF AND SIMPLY LOOKING AT THE
RECORD AND HOW MUCH MONEY THERE WAS, THERE'S ANOTHER WAY TO DO
IT. TO ASK HOW MUCH MONEY DID YOU MAKE IN THE FIFTH YEAR, HOW
WOULD YOU DO IT? YOU'D SAY HOW MUCH MONEY DID I MAKE? THAT'S
HOW MUCH I ENDED WITH MINUS HOW MUCH I STARTED WITH, DIVIDED BY
F-OF FIVE. THAT'S THE OTHER WAY TO KEEP THE SAME THING. LET'S
SEE IF WE GET THE SAME ANSWER ROUGHLY. SO F-OF SIX MINUS F-OF
FIVE, THAT TURNS OUT TO BE ABOUT THIS (ON BOARD). AND THAT'S ABOUT 13.6 PERCENT. SO IT'S ACTUALLY VERY CLOSE. THIS IS YOUR ESTIMATE OF HOW MUCH MONEY YOU MADE. RELATIVE CHANGE IN ONE YEAR, 13.4. IT IS NOT A STRAIGHT LINE. I THINK I'LL LEAVE THAT UP.

OKAY.

STUDENT: HOW DID YOU DO THAT THE SECOND WAY.

PROFESSOR: SO THE SECOND WAY WAS TOOK THE FUNCTION F, WHAT'S F-OF SIX MINUS F-OF FIVE. THIS IS HOW MUCH MONEY HE ENDED WITH IN THE FIFTH YEAR. THAT'S HOW MUCH MONEY I STARTED WITH. SO THIS IS THE AMOUNT EARNED, NEW RATE, AMOUNT EARNED IN YEAR FIVE DIVIDED BY START HAVING VALUE. SO THAT'S ANOTHER WAY, HOW MUCH DID I EARN PERCENTAGE WISE? YEAR FIVE. THIS WAS MY ESTIMATE AT THE VERY BEGINNING OF YEAR, HOW MUCH YOU EARNED. I ESTIMATED I EARNED 13.4 AND THIS IS WHAT I ACTUALLY GOT. I WON'T KNOW THAT UNTIL I'M DONE WITH YEAR SIX. THAT WAS MY BEST GUESS AT THE BEGINNING OF YEAR FIVE. LET ME ASK A NON ECONOMIC QUESTION. SUPPOSE THE RELATIVE RATE OF CHANGE THAT, WHAT I JUST DEFINE IN THAT DEFINITION THERE. THE RELATIVE RATE OF CHANGE OF F-OF T-IS A CONSTANT. I'LL CALL IT K-. CONSTANT. WHAT IS F-OF T? SO ALL I'M TELLING YOU ABOUT THIS IS THAT THIS FUNCTION IS GROWING AT A CONSTANT RATE. CONSTANT RELATIVE RATE EVERY YEAR AM CAN YOU TELL ME WHAT FUNCTION IT IS? F-PRIME OF T-DIVIDED BY F-OF T, THAT'S THE RELATIVE RATE OF CHANGE. CAN WE SOLVE THAT FOR F-OF T? WRITE IT THIS WAY. (ON BOARD). DO YOU RECOGNIZE THAT DIFFERENTIAL EQUATION WE'VE TALKED ABOUT? WE CALL THIS THING, IT'S AN EQUATION WHERE THE UNKNOWN IS A FUNCTION. AND WE USED IT BEFORE. IT HAS ONE SOLUTION. WHO REMEMBERS WHAT THIS SOLUTION IS? ALL ABOUT, HOW WE STARTED THE CHAPTER BY WRITING DOWN THIS DEFERENTIAL EQUATION. SOME CONSTANT TIMES E-TO THE
K-T-: THIS IDEA OF A RELATIVE RATE OF CHANGE BEING CONSTANTS
WHICH IS THE SIMPLEST THING IT COULD BE IS JUST THE EXPONENTIAL.
THAT'S ANOTHER WAY TO REMEMBER IT. LET ME GO BACK TO ECONOMICS
NOW AND DEFINE THE TERM FROM ECONOMIC WHICH IS VERY COMMON THERE.
AND RELATE IT TO THIS RELATIVE RATE OF CHANGE. AND IT'S
SOMETHING WE TALKED ABOUT A LITTLE BIT BEFORE BUT I'LL START FROM
SCRATCH AND REDEFINE IT. ELASTICITY OF DEMANDS. AND WHAT THIS
WAS ALL ABOUT IS IT MEASURES HOW MUCH THE DEMANDS FOR A PRODUCT,
HOW MANY YOU CAN SELL, HOW THAT DEMAND DEPENDS ON THE PRICE. IF

I RAISE THE PRICE HOW DOES THAT CHANGE HOW MANY YOU SELL? THAT'S
SOMETHING ECONOMISTS CALLING THE ELASTICITY OF DEMAND AND THE
MAIN IDEA WAS THAT IF YOU RAISE THE PRICE THEN WHAT HAPPENS TO
THE DEMAND? DOES IT GO UP OR DOWN? IT GOES DOWN. IF YOUR
CUSTOMERS ARE MAKING RATIONAL DECISIONS THAT'S WHAT HAPPENS.
LET ME GIVE NAMES TO ALL THESE THINGS. P-EQUALS PRICE. I'M
GOING DID LET Q-BE DEMAND. AND THAT'S GOING TO BE SOME FUNCTION
OF THE PRICE. SO F-OF P-WILL TRYOUT DIFFERENT FUNCTIONS AS TIME
GOES ON. AND WHAT WE JUST OBSERVED IS THAT IF YOU ASK, DOES
DEMAND INCREASE OR DECREASE, HOW DOES THAT COMPARE TO ZERO? IT'S
LESS THAN ZERO. THE DEMANDS GOES DO YOU THINK AS THE PRICE GOES
UP. SO THAT'S AN INEQUAL WE KNOW. IT'S ALWAYS GOING TO BE
TRUE. SO HERE'S A REALLY SIMPLE FUNCTION JUST TO ILLUSTRATE.
SO THERE'S A FUNCTION F-OF P-. IF I DO F-PRIME OF P-IT'S GOING
to BE, ANYBODY WANT IT TELL ME? SO THERE'S A SIMPLE STRAIGHT
LINE LINEAR FUNCTION. WHAT WAS IT? NEGATIVE 100. SO THERE'S
A SIMPLE EXAMPLE OF A POTENTIAL DEMAND CURVE. WAS THERE A
QUESTION? OKAY. SO IT TURNS OUT THAT IN THIS BUSINESS JUST
LIKE THE RESTS OF THIS SECTION PEOPLE LIKE IT TALK ABOUT RELATIVE
CHANGE OF ALL THESE THINGS. SO LET ME ASK ONE OF THOSE
QUESTIONS. ABOUT RELATIVE CHANGES. SO HERE'S THE WAY WE'RE
Going to talk about this. If the price goes let's say up by 1 percent, goes up by a relative amount of one in 100, by what percent does the demand fall? Okay so that's the way the economist, if price goes up by 1 percent does demand fall by 2 percent, by 3 percent, that's the kinds of question you want to ask about. So, for example, if the answer is that the demand falls by 3 percent, that could be the answer, whatever it is, then we're going to define something, we say that the elasticity of demand is the relative rate of change of the demand, let me give myself more space here, is the relative rate of change of demand which is 3 percent divided by the relative rate of change of the price which in this case is 1 percent. So you take the ratio of 3 percent to 1 percent and the answer is three. So that's the way economist like to talk about this. Increase the price by 1 percent, the demand goes down 3 percent, that ratio is three. And that's what they call the elasticity of the demand. Let me write that down as a derivative.

So here's the formal definition there. I wrote it down in English but now I'm going do write it down in derivatives. But it's the same thing. So the elasticity of the demand equals, and here's the simple for it, capital E-for elasticity. So how do I write down the relative rate of change? It's the derivative of something. So what goes in the numerator? The relative rate of change -- the logarithm of f-of p--; that's the definition the relative rate of change in the demands. And what goes in the denominator. The relative rate of change in the price. Is everybody okay with the numerator. Take the definition, relative rate of change and used it up there. Derivative for the logarithm. So that's, but I have to put something in the
DENOMINATOR. I HAVE TO PUT DOWN THE RELATIVE RATE OF CHANGE THE 
PRICE, SO WHAT DO I PUT HERE? SO D-P, USE THAT, D-D-P-OF 
something. OF THE LOG OF THE PRICE. THAT'S THE RELATIVE RATE 
of CHANGE THE PRICE. THE FACT THAT THAT IS THE SAME AS THAT IS 
OKAY. LET ME KNOW THE RELATIVE RATE OF CHANGE OF THE PRICE. 
SO LET ME IS THIS ENGLISH LANGUAGE DEFINITION OKAY. RAISE THE 
PRICE 1 PERCENT, ASK HOW MUCH, WHAT HAPPENS IT DEMANDS YOU GET 
THE RATIO. HOW DO YOU MEASURE RELATIVE RATE OF CHANGE. AND WE 
DID THIS, WE SAY WE MEASURED BY THAT RATIO. F-PRIME OVER F-. 
BUT WHICH IS THE SAME AS THE DERIVATIVE OF LOGARITHM. SO LET ME 
JUST DO IT. IN THE NUMERATOR I HAVE THE DERivative OF LOG 
DEMANDS. HERE I HAVE THE DERivative OF THE LOG OF 
DEMANDS. HERE I HAVE THE DERivative OF THE LOG OF PRICE. AND 
SO NOW LET ME JUST, USE THAT, SO THAT'S GOING TO BE F-PRIME OF 
P-DIVIDED BY F-OF P-. WHAT GOES IN THE DENOMINATOR? WHAT'S THE 
DERivative OF LOG? ONE OVER P-. SO THIS IS, SO SIMPLIFY IT, 
THIS IS P-F-PRIME OF P-OVER P-(ON BOARD). 
STUDENT: I STILL DON'T UNDERSTAND THE TOP THE -- 
PROFESSOR: IT'S THAT FORMULA RIGHT THERE WHICH SAYS, RELATIVE 
RATE OF CHANGE OF FUNCTION F, YOU TAKE THE DERivative OF THE LOG, 
AND THAT'S WHAT I HAVE HERE HERE. THE RELATIVE RATE OF CHANGE 
WHICH IS THE DERivative OF THE LOG OF THE FUNCTION WHICH IS THE 
DEMAND. AND DEPENDS ON THE PRICE. 
STUDENT: FOR THE DENOMINATOR DOESN'T HAVE TO BE ONE OVER P-TIME 
P-PRIME. 
PROFESSOR: AND P-PRIME IS, D-P-D-P-IS ONE, YEAH. D-P-D-P-I 
LEFT THAT OUT. SO THEN I PUT P-UP IN THE NUMERATOR AND BROUGHT 
THAT DOWN. THAT TELL ME IF I CHANGE THE PRICE BY 1 PERCENT WHAT 
PERCENT IS THE DEMANDS CHANGE BY? SO IT'S THIS, NICE SIMPLE 
EXPRESSION. BUT WE NEED SOME EXAMPLES TO MAKE SURE IT'S ALL
CLEAR.

LET ME PICK A PARTICULAR EXAMPLE WHERE LET’S SAY THE DEMAND AND I’LL GIVE IT SOME UNITS TO MAKE IT REALISTIC FOR SOME RAW MATERIAL METAL, IN MILLION OF POUNDS OF THE STUFF PER YEAR. OKAY. SO IF YOU WANT TO ADD A UNIT TO IT, THAT’S GOING TO BE MY FUNCTION F-OF P-AND THAT’S GOING TO BE 100 MINES TWO TIMES P-WHERE P-EQUALS PRICE IN DOLLARS PER POUND. OKAY. NICE SIMPLE FUNCTION. THE DERIVATIVES ARE ALL STRAIGHTFORWARD. LET'S ASK OURSELVES THE QUESTIONS AND WALK THROUGH THE DEFINITION. SO THE FIRST QUESTION IS, IF THE PRICE IS $30 PER POUND HOW MUCH CAN I SELL? THAT’S JUST WHAT THIS FUNCTION TELLS ME, THE ANSWER IS GOING TO BE F OF 30 WHICH IS 100 MINUS TWO TIMES 30 OR 40. 40 THE UNIT IS MILLIONS OF POUNDS. THE NEXT QUESTION IS WHAT IS THAT FUNCTION OVER THERE THAT I JUST WROTE DOWN SO THAT I CAN USE IT TO SAY HOW THE DEMANDS FOR THE METAL WILL VARY WITH PRICE. SO IT'S GOING TO BE P-TIME F-PRIME OF P-OVER F-P-. SO LET'S DO THAT. IT'S P-TIMES AND WHAT'S F-PRIME OF P? I MADE IT VERY EASY. NEGATIVE TWO. AND THEN THE DENOMINATOR IS 100 MINUS TWO P-. SO IT'S MINUS TWO P-OVER 100 MINUS TWO P-. (ON BOARD). LET ME I MADE A LITTLE MISTAKE. I LEFT OUT A MINUS SIGN. I APOLOGIZE. AND YOU’LL, I WANT TO PAUSE. THIS IS JUST TO CONVINCE YOU. SO-AND-SO WHAT THIS FUNCTION TELLS US IS THAT IF THE PRICE GOES UP 1 PERCENT IT WILL GO DOWN THIS MANY PERCENT. SO YOU GET A POSITIVE NUMBER OUT OF IT. I APOLOGIZE. DIDN'T CHANGE ANY OF THE IDEAS. SO THERE'S THE FUNCTION. SO THIS TELLS ME IF I RAISE THE PRICE BY A LITTLE BIT HOW MUCH DOES THE DEMANDS GO DOWN? IT TELLS ME HOW MUCH DEMANDS GOES DOWN.

STUDENT: DOESN'T NEGATIVE FROM THAT EQUATION, IS THERE A NEGATIVE ON THE TOP BOARD ON THE LEFT.
PROFESSOR: NO, NO BECAUSE IT WENT UP. I RAISED PRICE 1 PERCENT AND THE DEMANDS WENT DOWN 3 PERCENT. AND SO I GOT A POSITIVE NUMBER HERE. BUT IT WAS GOING DOWN BUT I JUST PLEASURED HOW MUCH IT WENT DOWN. THAT'S WHAT I MEANT TO DO HERE AND THAT'S WHY I NEEDED THAT NEGATIVE SIGN. SO THAT WAS, THE IDEAS ARE THE SAME. JUST SORT OF, YOU HAVE TO REMEMBER, THIS IS GOING DOWN.

STUDENT: SO IDEALLY YOU CAN PUT A NEGATIVE SIGN IF FRONT OF THAT AND HAVE THE NEGATIVE 3 PERCENT.

PROFESSOR: I COULD HAVE. I'M JUST TRYING TO BE CONSISTENT WITH WHAT ECONOMISTS DO. THIS IS THE WAY THEY DEFINE IT. I JUST LEFT IT OUT BY MISTAKE. TRYING TO USE THE SAME NOTATION YOU PLIGHT SEE IN E-CON CLASS. BUT THE TIME I FINISH WITH THIS EXAMPLE I HOPE IT'S CLEAR WHAT'S POSITIVE AND WHAT'S NEGATIVE. SO THE DERIVATIVE IS NEGATIVE TWO SO DEMANDS GOES DOWN AS PRICE GOES UP. THIS IS GOING DO MEASURE HOW MUCH GOES DOWN. I'LL KEEP GOING UNDER HERE. LET ME LEAVE THE DEFINITION UP. SO THE NEXT QUESTION IS WHAT IS E-AT THE E-OF-P-ELASTICITY AT A PRICE OF $30 A POUND. LET'S PLUG IT IN AND SEE. SO IT'S GOING TO BE TWO TIMES 30 OVER 100 MINUS TWO TIMES 30 WHICH IS 60 OVER 40 OR THREE HALVES. LET'S SEE WHAT THAT MEANS. WHAT THIS MEANS, FOR EXAMPLE, IS THAT IF WE RAISE THE PRICE OF METAL 2 PERCENT, THEN THE DEMAND GOES DOWN BY THREE HALVES TIMES 2 PERCENT OR 3 PERCENT. SO THIS IS, IF YOU THINKING ABOUT RAISING THE PRICE THAT MUCH, THAT'S HOW MUCH YOU MULTIPLY THE DEMANDS BY AND MAKING IT DOWN. IN PARTICULAR WHAT IS RAISING THE PRICE 2 PERCENT MEAN? THE PRICE WAS $30. SO 2 PERCENT OF $30 IS 60 CENTS. SO RAISE THE PRICE 60 CENTS, THAT'S 2 PERCENT THE DEMAND GOES DOWN 3 PERCENT. AND THAT'S 3 PERCENT OF WHAT? IT'S 3 PERCENT OF 40 EQUALS 1.2. SO IT GOES DOWN 1.2 AND THE UNITS OF MILLIONS OF
POUNDS OF METAL PER YEAR.

DID THAT CLARIFY PERHAPS, I HOPE? GOING UP AND GOING DOWN
DO I HAVE IT ALL STRAIGHT IN RAISE THE PRICE 2 PERCENT, DEMAND
GOES DOWN THREE HALVES AS MANY AS WITH 3 PERCENT.
STUDENT: IT'S MORE BENEFICIAL RAISING THE PRICE.
PROFESSOR: I'LL GET THAT. GOOD ONE.
STUDENT: WE GOT 830 EQUALS THREE HALVES. IS IT ALWAYS THAT THE
BOTTOM, ALWAYS THAT THE BOTTOM, THE DENOMINATOR IS WHAT YOU RAISE
TO FIND THE TOP IS WHAT DEMAND (INAUDIBLE).
PROFESSOR: IN THIS FRACTION HERE? SO THIS CAME FROM THIS
FORMULA RIGHT HERE. SO THIS IS THE PRICE TIMES F-PRIME. AND

SO I DID THE --
STUDENT: I GOT THAT. I'M ASKING, SO THEN YOU WERE THAT MEANS
RAISING THE PRICE TO PERCENT MEANS DEMANDS GOES DO YOU THINK
3 PERCENT. IS THAT ALWAYS (INAUDIBLE).
PROFESSOR: IT'S NOT ALWAYS THREE HALVES BUT, I'LL DO IT ANOTHER
EXAMPLE RIGHT NOW. IT'S ALWAYS THE CASE IF YOU RAISE THE PRICE
THE DEMANDS GOES DOWN. THE QUESTION IS HOW MUCH. IN THIS
CASE, IT GOES DOWN BY THIS, THREE HALVES TIMES AS MUCH. YOU
RAISE THE PRICE BEING 1 PERCENT IT GOES DOWN BY THREE HALVES.
IF YOU RISE THE PRICE 10 PERCENT IT GOES DOWN BY THREE HALVES
TIMES TEN OR 15 PERCENT. SO LET ME DO ONE MORE EXAMPLE AND THEN
ASK YOUR QUESTION AGAIN.
STUDENT: WHERE DID YOU GET THE 2 PERCENT.
PROFESSOR: I JUST PICKED IT. I SAID, FOR EXAMPLE, ET CETERA
SUPPOSE I RAISED THE PRICE 2 PERCENT WHAT HAPPENS TO THE DEMANDS.
AND THE ANSWER IS WHATEVER THAT NUMBER IS, TWO, I MULTIPLY BY
THREE HALVES. IF I PLAYED THIS 1 PERCENT I WOULD HAVE HAD THREE
HALVES TIMES 1 PERCENT.
STUDENT: WHAT IS THE 2 PERCENT TIMES $30.
PROFESSOR: SO WHEN I RAISED THE PRICE 2 PERCENT HOW MUCH IS THAT IN ACTUAL DOLLARS. I TOOK 2 PERCENT.

STUDENT: WHAT ABOUT THE 3 PERCENT.

PROFESSOR: THE DEMANDS WAS ACTUALLY 40, 40 MILLION POUNDS PER YEAR. THAT COMES FROM HERE. THIS IS THE ACTUAL SALES. AND SO THE SALES ARE GOING TO GO DOWN BY 3 PERCENT OF 40 WHICH IS 1.2.

1.2. JUST TO BE, PLUGGING NUMBERS TO MAKE ALL THE EXAMPLE VERY CONCRETE.

STUDENT: SO THE TOP NUMBER IS HOW MUCH, ON THE THREE OVER TWO. THE THREE.

PROFESSOR: THAT'S THIS, THREE HALVES IS JUST THE EVALUATION OF THIS FUNCTION. IT TURNS OUT TO BE THREE HALVES.

STUDENT: HOW DO YOU KNOW IT GOES DOWN BY (INAUDIBLE) PERCENT.

PROFESSOR: BECAUSE I TOOK HOW MUCH THE PRICE WENT UP. THE NUMBER I PICKED THE ILLUSTRATION TO PERCENT I PICKED. BUT I HAVE TO MULTIPLY BY THREE HALVES. LET ME DO ONE MORE EXAMPLE TO MAKE THIS CONCRETE.

STUDENT: WHY DID YOU MULTIPLY BY 2 PERCENT INSTEAD.

PROFESSOR: I JUST PICKED AS AN EXAMPLE.

STUDENT: (INAUDIBLE).

PROFESSOR: I COULD HAVE DONE THAT TOO. I'M USING PERCENTAGE BECAUSE IF ECONOMICS PEOPLE USE PERCENTAGE. JUST FOR ILLUSTRATION. LET ME JUST DO ONE MORE TO MAKE IT CLEAR.

SO LET'S TAKE A DIFFERENT PRICE POINT AND I'LL JUST PLUG IN NUMBERS. SO LET'S SUPPOSE THE MARKET IS RUNNING NOT AT $30 A POUND BUT AT $20 A POINT. THAT COULD BE, LET'S SEE WHAT HAPPENS. LET'S SEE WHAT IT DOES. SO E-OF 20 IS GOING TO BE TWO TIMES 20 OVER 100 MINUS TWO TIMES 20. WHICH IS 40 OVER 60 WHICH IS NOW TWO THIRDS. OKAY, IT'S A DIFFERENT ELASTICITY.

SO LET'S ASK THE QUESTION OF WHAT THAT MEANS. SO LET'S ASK WHAT
IMPACT ON DEMAND DOES A 2 PERCENT, I'M PICKING IT TO BE THE SAME 2 PERCENT, I'M PICKING IT TO BE THE SAME.

AS OVER HERE, A 2 PERCENT PRICE RISE HAVE. NOW THE GOING PRICE IS 20 BUCKS. I'M GOING TO RAISE 2 PERCENT. I'M GOING TO ASK WHAT HAPPENS TO THE DEMANDS. AND THE ANSWER IS THE DEMAND GOES DOWN, WE KNOW IT HAS TO BE DOWN, BY HOW MUCH? TWO THIRDS TIMES 2 PERCENT OR ONE .3 3 PERCENT. IT GOES DOWN LESS BECAUSE WE'RE AT A DIFFERENT POINT ON THE CURVE. AND DEPENDING ON, YOU MIGHT MAKE DIFFERENT ECONOMIC DECISIONS. PRICE AND DEMAND RELATE TO ONE ANOTHER DIFFERENTLY HERE. AT THIS POINT THIS RATIO IS TWO THIRDS. TWO THIRDS. OVER HERE IT'S THREE HALVES. MARKETS BEHAVE DIFFERENTLY. SO THE REAL, THIS IS SORT OF LEADING UP TO THE NEXT POINTS WHICH IS WHAT IS THE BEST PRICE? WE WANT IT SAY, WHAT DO WE WANT TO CHARGE? AND THE ANSWER IS YOU WANT TO MAXIMIZE IT. YOU WANT TO PICK A PRICE TO MAXIMIZE REVENUE. I WANT TO MAXIMIZE THE REVENUE. THE NEXT BIG QUESTION IS, USE THIS IDEA TO DEFINE ELASTICITY, CHOOSE P-TO MAXIMIZE THE REVENUE. AND REVENUE, FUNCTION DEPENDS ON THE PRICE, CALL IT R-JUST LIKE WE HAD BEFORE. SO LET'S REMEMBER WHAT THE REVENUE IS? WELL IT'S THE PRICE PER UNIT TIMES NUMBER OF UNITS SOLD AND SO THAT'S GOING TO BE D-TIMES F-OF P-. THOSE ARE THE TWO THINGS WE'VE BEEN USING, THE PRICE TIMES NUMBER OF UNITS SOLD. THAT'S A FUNCTION YOU'D LIKE TO MAXIMIZE. AND JUST TO REMINDS YOU WHAT THESE FUNCTIONS LOOK LIKE, LET ME USE THE ONE I DID OVER THERE, PRICE OF METAL, GOING TO BE P-TIME 100 MINUS TWO P-. USE THE SAME DEMANDS CURVE, I DID OVER THERE SO THIS IS 100 P-MINUS TWO P-SQUARED. WHAT IS THE SHAPE THIS FUNCTION? WHAT'S THE GRAPH?

IT'S AN UPSIDE DOWN PARABOLA. SO IT LOOK LIKE THIS. (ON BOARD). AND THERE'S AN OPTIMAL POINT ON THAT WHERE YOU

STUDENT: YOU FACTORED OUT F-PRIME OF P-

PROFESSOR: I FACTORED THAT OUT OF F-. -- LET ME JUST WRITE THIS DOWN AGAIN. (ON BOARD). SO THERE’S THREE DIFFERENT THINGS THIS COULD BE. IT COULD BE LESS THAN ZERO. EQUAL TO ZERO. OR GREATER THAN ZERO. THOSE ARE THE THREE INTERESTING SITUATIONS, WHERE IT'S INCREASING, WHERE IT'S FLAT, THAT'S THE MAXIMUM OR DECREASING AGAIN. THOSE ARE THE THREE INTERESTING
SITUATIONS. AM I HERE, IN THE MIDDLE OR OVER THERE. AND I CAN TELL EXACTLY WHICH SITUATION I'M IN BY LOOKING AT THE VALUE OF E-OF P-. SO WHEN AM I AT ZERO? WHEN E-OF P-EQUALS ONE. TO MAKE THIS ZERO THAT BETTER BE EQUAL TO ONE. HOW DO I KNOW WHEN I'M IN THE MODE TO INCREASE THE PRICE, E-OF P-BETTER BE LESS THAN ONE. I KNOW TO SUBTRACT SOMETHING LESS THAN ONE SO THIS THING IS POSITIVE. AND THEN THE ONLY POSSIBILITY LEAVE IS IF E-OF P-IS GREATER THAN ONE, THIS IS NEGATIVE. AND REVENUE GOES DOWN. THIS IS THE MAX, THIS IS, THE REVENUE IS STILL GOING UP AND THERE'S THE REVENUE'S GOING DOWN. SO INCREASING REVENUE MAX, AND REVENUE INCREASING. THOSE ARE THE SITUATIONS.

STUDENT: THAT DERIVATIVE R-OF P-

PROFESSOR: SORRY. THIS IS P-. THANK YOU. I WAS WRITING TOO FAST. JUST THE SAME THING AS ON THE BOARD RIGHT ABOVE. THE DERIVATIVE OF THE REVENUE WITH RESPECT TO THE PRICE. AND THIS IS THE SAME EXPRESSION I HAD BEFORE, SO WE CAN TELL THESE THREE DIFFERENT SITUATIONS BY LOOKING AT THIS ELASTICITY I DEFINED, E-OF P-. SO ECONOMISTS LIKE, THEY'VE GIVEN A NAME TO THESE TWO SITUATIONS. I'LL TELL YOU WHAT THEY ARE. IN THIS CASE WHERE ELASTICITY IS LESS THAN ONE, THEY SAY THE ELASTICITY DEMAND IS, THIS IS THEIR WORDS, INELASTIC. THAT MEANS YOU RAISE THE PRICE YOU STILL SELL ENOUGH TO MAKE A PROFIT. YOU JUST SELL MORE. JUST A SYNONYM FOR THAT. AND HERE, WHERE RAISING THE PRICE MEANS PEOPLE AREN'T GOING TO BUY IT ANYMORE AND REVENUE GOES DOWN, THE WORD FOR THAT IS ELASTIC. JUST WORDS IN ECONOMICS. BUT SIMPLY A MATTERS IS ELASTICITY LESS THAN ONE OR GREATER THAN ONE. RIGHT IN THE MIDDLE IS WHEN YOU'RE AT THE TOP OF CURVE AND MAXIMIZE REVENUE. SO FOR OUR EXAMPLE, THE ONE THAT WE'VE BEEN DOING WITH THIS METAL STUFF, FOR WHICH P-IS MAX? ANYBODY WANT TO TELL ME? SO IT LOOKS LIKE THAT. WHEN IT ELASTICITY EQUAL TO
ONE OR WHEN IS, WHEN ARE WE AT THE TOP OF THAT CURVE? P-EQUALS ...

STUDENT: A HUNDRED DIVIDED BY FOUR.

PROFESSOR: A HUNDRED DIVIDED BY FOUR. 25 BUCKS A POUND DOES THE TRICK. WELL, THERE ARE AT LEAST TWO WAYS TO DO IT. DID I ERASE THE FORMULA FOR ELASTICITY? WHERE WAS IT? E-OF P-WAS TWO P-OVER 100 MINUS TWO P, WAS THAT IT? SO SET THAT EQUAL TO ONE. AND SOLVE, YOU GET TWO P-EQUAL 100 MINUS TWO P-. AND SOLVE FOR P-EQUAL 25. ANY QUESTIONS ABOUT HOW TO RUN A BUSINESS? ONLY PRETENDING TO GIVE YOU HINTS ABOUT THAT.

STUDENT: WHICH WAY IS BETTER TO DO IT.

PROFESSOR: YOU COULD HAVE GOTTEN THE SAME ANSWER. LET'S DO IT BOTH WAYS. YOU GET THE SAME ANSWER. SAYING I CAN ALSO COMPUTE 24

R-PRIME OF P AND COULD HAVE GOTTEN 100 MINUS FOUR P-AND SAY WHAT IS THAT ZERO AND THE ANSWER STILL 25. SO YOU CAN GET THE TOP OF THE PARABOLA TWO DIFFERENT WAYS. THIS IS AN IDENTITY. THIS IS EQUAL THAT. THE ONLY WAY THIS TO BE ZERO IS FOR THIS TO BE ZERO. I'M DEFINING THIS WAY BECAUSE IT'S SOMETHING COMMON IN ECONOMICS CLASSES AND COMES STRAIGHT OUT OF CALCULUS. ANY QUESTIONS ABOUT ECONOMICS? BEFORE I GO ON TO SOMETHING DIFFERENT?

OKAY. THAT WAS END OF SECTION FIVE .3. WE'RE OFF TO SECTION 5.4. WHERE I'M GOING TO DO SLIGHTLY MORE INTERESTING AND GENERAL EXPONENTIAL MODELS. SO LET ME AGAIN START WITH SOME EXAMPLES. SO MANY PHENOMENA ARE MODELED BY SAYING THAT, BY SAYING HOW FAST THEY APPROACH AN ASYMPTOTE. OKAY. SO LET ME GIVE TWO EXAMPLES HERE. A PARACHUTIST JUMPS OUT OF A PLAIN. THEY ACCELERATE AND GO FASTER AND FASTER WITH GRAVITY. THEY ACCELERATE DOWNWARD. DO THEY KEEP GOING FASTER AND FASTER FOREVER? THEY EVENTUALLY STOP ACCELERATING, WHY IS THAT? THE
AIR RESISTANCE BUILDS UP AS THEY GO FASTER. UNTIL THEY APPROACH THE FINAL SPEED CALLED THE TERMINAL VELOCITY. WHICH THEY NEVER QUITE REACH BUT IT'S BASICALLY THE FINAL, 120 MILES PER HOUR, SO LET ME DRAW A PICTURE OF THAT. SO IF YOU PLOT VERSE TIME HOW LONG IT'S BEEN SINCE WE JUMPED OUT OF THE PLAIN, VELOCITY DOWNWARD, HOW FAST ARE YOU GOING DOWNWARD, THAT'S AN ASYMPTOTE WHICH IS YOUR TERMINAL VELOCITY, YOU'RE NOT GOING TO GO FASTER THAN THAT BECAUSE OF AIR RESISTANCE. YOU'RE NOT FALLING BUT -- TO DESCRIBE THIS CURVE YOU HAVE TO KNOW, WHAT'S THE ASYMPTOTE, WHAT'S THE TERMINAL VELOCITY. AND HOW STEEPLY ARE YOU APPROACHING. WE'RE GOING TO WRITE DOWN A VERY SIMPLE FORMULA.

, IN FACT, LET ME WRITE DOWN THE FORMULA NOW BECAUSE IT'S GOING TO BE THE SAME FORMULA FOR THE REST OF THIS SECTION. LET ME GIVE A NAME FOR THIS. TERMINAL VELOCITY. CALL IT M. THAT FORMULAS IS GOING TO DESCRIBE THIS CURVE. K-IS SOME POSITIVE CONSTANTS SYSTEM AS T-GETS BIGGER AND BIGGER WHAT HAPPENS TO E-TO THE MINUS K-T? IT GETS SMALLER AND SMALLER, SO EVENTUALLY THIS GETS AS CLOSE TO ZERO AS YOU LIKE AND THE WHOLE THING GETS AS CLOSE TO M AS YOU LIKE. IF K-IS REALLY BIG, THIS GETS SMALL REALLY FAST. AND SO IT MIGHT LOOK LIKE THIS. THIS IS FOR BIG K-. OR IT MIGHT APPROACH VERY SLOWLY, SMALL K-. IF YOU TELL ME HOW FAST IT APPROACHES, TELL ME WHERE IT ENDS UP, IT'S GOING TO LOOK LIKE THIS, LIKE THIS CURVE. TO FIGURE IT OUT WE HAVE TO NUMBERS TO F-OUT, M THE ASYMPTOTE AND K, HOW FAST IT GETS. LET ME GIVE ANOTHER EXAMPLE WHERE IT'S THE SAME FORMULA FOR SOMETHING TOTALLY DIFFERENT. WHICH IS HOW FAST CAN YOU LEARN? OKAY. SO HERE'S A PSYCHOLOGY EXPERIMENT WHICH HAS BEEN DONE MANY TIMES. WHICH IS GIVEN A CERTAIN AMOUNT OF STUDY TIME, T-MINUTES SAY, HOW LONG A LIST OF NONSENSE SYLLABLES CAN LEARN? SO THERE'S NO
PATTERN, YOU JUST HAVE TO MEMORIZE THESE THINGS. AND SO THE
LONGER YOU GET THE MORE YOU CAN MEMORIZE BUT GUESS WHAT? IT THE
LIST IS LONG ENOUGH, DOESN'T MATTER HOW LONG IT IS, YOUR BRAIN
FILLS UP, YOUR MEMORY BANK FILL UP AND YOU CAN'T LEARN ANYMORE.
SO HERE IS THE AMOUNT OF TIME YOU GET TO LEARN THE LIST. HERE
IS THE LIMIT OF YOUR BRAIN. AND THIS IS WHAT THE CURVE LOOKS
LIKE. THE SAME KIND OF CURVE. SO M IN THIS CASE IS THE LIMIT
OF HOW MANY WORDS YOU CAN LEARN. HOW FAST IT APPROACHES THAT.
IT'S THE SAME THING. THE NUMBER OF WORDS YOU CAN LEARN IS THE
LIMIT TIME ONE MINUS E-TO THE MINUS K-T-. THIS IS THE
PHENOMENON. SAME SIMPLE CURVE COMES UP OVER AGAIN.
STUDENT: YOU SAID THE LIST IS LONG ENOUGH YOU GET AS MUCH AS
TIME AS YOU WANT.
PROFESSOR: THERE'S A CERTAIN MAXIMUM THAT'S DUE TO THE WAY
PEOPLE'S BRAINS LEARN. THERE'S A HARD LIMIT THERE. AS TO WHAT
YOU CAN LEARN, THERE'S NO PATTERN. IF THERE'S A PATTERN YOU
COULD MEMORIZE THE BOOK. BUT IT'S NONSENSE SYMBOLS.
STUDENT: THAT APPLICABLE TO MATH.
PROFESSOR: NO THERE'S A PATTERN TO MATH. YOU CAN LEARN AN
INFINITE AMOUNT OF MATH. THAT'S WHY I SAID NONSENSE SYLLABLES.
IT TURNS OUT THAT BOTH OF THESE, IT'S THE SAME FORMULA IN BOTH
CASES. THEY COME FROM A DIFFERENTIAL EQUATION. THIS FUNCTION,
LET ME GIVE IT A NAME, SO IF YOU HAVE Y-OF T-AND IT'S M TIME ONE
MINUS E-TO THE MINUS K-T-IS A SOLUTION FOR A DIFFERENTIAL
EQUATION WHICH GOVERNS ALL THIS STUFF, AND SO WHAT IS THE
DIFFERENTIAL EQUATION? IT SAYS THE DERIVATIVE Y-PRIME IS K-TIMES
M MINUS Y-OF T-. (ON BOARD). SO LET'S SEE WHAT THIS SAYS,
THIS SAYS AS Y-GETS CLOSER AND CLOSER TO M, THIS GETS CLOSER AND

PROFESSOR: NEGATIVE M AND THEN WHEN I DIFFERENTIATE THIS I BRING DOWN NEGATIVE K-. SO THE NEGATIVE'S CANCELING AND I GET M TIMES K-AND E-TO THE MINUS K-T-. NOW I'M GOING TO WRITE THIS A FUNNY WAY, FACTOR OUT THE K-AND M MINUS M IS ZERO. NEGATIVE AND NEGATIVE IS POSITIVE. WHY HAVE I WRITTEN IT THIS WAY. YOU HAVE TO RECOGNIZE THAT THIS THING IS Y-OF T-

STUDENT: HOW DID YOU GET THE FIRST STEP TO THE SECOND STEP.

PROFESSOR: FROM HERE TO HERE IN WHAT'S M MINUS M. ZERO, AND NEGATIVE NEGATIVE IS POSITIVE. SO THIS EQUALS THAT. BECAUSE I KNEW THAT MY GOAL WAS TO GET TO K-TIMES M MINUS Y-. I GET FIDDLE WITH THIS UNTIL I GET A Y-OUT OF IT. OKAY. SO THIS IS MY GOAL IS TO SHOW THIS FUNCTION HERE SATISFIES THAT DIFFERENTIAL EQUATION WHICH I GET FLATTER THE CLOSER I GET TO M. AND THAT IS
ALL I WANT TO DO TODAY. NEXT TIME WE WILL USE THIS AND LOTS OF OTHER EXAMPLES.