MATH 16A LECTURE. OCTOBER 30, 2008.

PROFESSOR: WELCOME BACK. I'LL WAIT UNTIL EVERYBODY IS SEATED.
AND QUIET. THERE WILL BE REVIEW ON MONDAY. I'LL WAIT UNTIL
EVERYBODY'S QUIET. OKAY. THANK YOU. SO WE DEFINED LOGARITHMS
LAST TIME. AND SO WE DID IT THIS WAY. IF WE HAVE TWO NUMBERS,
Y-AND X, THEN X-IS CALLED THE NATURAL LOGARITHM OF Y. AND THE
WAY YOU WRITE IT AS X-EQUALS L N-Y. WE JUST, SAY Y-IN
PARENTHESES. SO WHAT ARE THE TWO MOST IMPORTANT PROPERTIES.
THEY FOLLOW IMMEDIATELY FROM THAT DEFINITION. D-D-X-AND THIS BE
TALK LOG RIGHT AWAY YOU GET X-BACK. AND THE OTHER PROPERTY SAYS
IF YOU TAKE THE NATURAL LOG OF A NUMBER, AND THE EXPONENTIATE IT,
YOU GET THE NUMBER BACK. OPPOSITE FUNCTIONS OF ONE ANOTHER.
THEY SORT OF CANCELING IN A CERTAIN SENSE., OF COURSE, THE
LOGARITHM IS ONLY DEFINED IF Y-IS GREATER THAN ZERO, OTHERWISE WE
CAN'T TALK ABOUT THEM. SO THE OTHER FACT WAS THAT SINCE, PICK A
NUMBER B, B- WE NEED THE LOG OF B, YOU CAN TAKE B-TO ANY POWER,
called X-BY TAKING E-TO THE POWER LOG OF E-TIMES X. SO ANY
FUNCTION B-TO BE X-CAN BE INTERPRETED TO SOME EXPONENTIAL. IF WE
UNDERSTAND E-TO THE X-WE UNDERSTAND B-TO THE X-FOR ANY OLD B-THAT
WE PICK.

JUST A COUPLE MORE IMPORTANT FACT FROM LAST TIME TO REVIEW.
WE ALSO DEFINED THE LOG BASE TEN, REMINDS YOU OF WHAT IT WAS, LOG
BAY TEN SATISFIES IF YOU TAKE TEN TO THAT POWER, YOU GET X-BACK
AGAIN. AND ANALOGOUSLY IF YOU TAKE THE LOG BASE TEN OF TEN TO
ANY NUMBER, YOU GET THE NUMBER BACK. THEY CANCEL ONE ANOTHER.
SO NATURAL LOG, AND TAKING TO THE POWER E, CANCEL ONE ANOTHER AND 
TEN TO THE POWER, LOG BASE TEN CANCEL ONE ANOTHER. SO THIS IS 
PROBABLY SOMETHING YOU'VE SEEN BEFORE BUT IT'S THE SAME KIND OF 
OBJECT. AND THEY'RE VERY CLOSELY RELATED. HERE'S OTHER 
IMPORTANT FACT. IF YOU KNOW HOW TO COMPUTE THE LOG BASE, THEN 
THIS ONE IS JUST A MULTIPLE OF IT. WE KNOW THAT'S TRUE. THAT'S 
JUST E-AND LOG N-CANCELING ONE ANOTHER. WE ALSO KNOW THAT THIS 
IS TRUE (ON BOARD). SO FAR THE SAME REASON. LET ME JUST WRITE 
TEN AS E-TO THE NATURAL LOG OF TEN. BECAUSE THAT'S ANOTHER WAY 
TO WRITE TEN OBVIOUSLY. (ON BOARD). HERE I HAVE E-TO THE 
NATURAL LOG OF X. IS E-TO THIS OTHER THING. SO THESE TWO 
THINGS UP HERE HAVE TO BE EQUAL. SINCE E-TO THE L N-X-EQUALS 
E-TO THE L N-LOG BASE TEN OF X-IT MUST BE THAT THIS IS THE 
RELATIONSHIP. THE EXPONENTS MUST BE EQUAL. (ON BOARD). OR 
SOLVING FOR THIS, (ON BOARD). SO IT'S EASY TO GO FROM THE LOG 
BASE E, NATURAL LOG TO LOG BASE TEN. AND SO THAT IS A REVIEW OF 
LAST TIME TOO.

STUDENT: I HAVE A QUESTION. HOW DID YOU GET FROM THE SENSE, 
THE HE THIRD EQUAL ONE, TO THE ONE UP. TWO TO THE RIGHT. 

PROFESSOR: SO I REWROTE TEN AS E-TO THE LOG TEN. IS THAT OKAY? 
AND THEN THE LOG EXPONENTS SAYS I JUST MULTIPLY TWO EXPONENTS 
TOGETHER. SO THIS IS E-TO THAT POWER AND THEN TAKE THAT TO THAT 
POWER AND MULTIPLY THE EXPONENTS AND THAT'S THE LAW OF EXPONENTS. 
AND THE SAME IDEA FOR ANY LOG FOR ANY BASE B-NOT JUST TEN, 
ANYTHING I SAID WORKS. SO X-IS DEFINE AS B-TO LOG BASE B-OF X. 

OR Y-IS THE LOG BASE B-OF B-TO THE Y. SO B-WOULD BE TEN, OR E, 
B-COULD BE ANYTHING, ANY OLD BASE. AND THE LOG BASE B-OF 
X-EQUALS ONE OVER NATURAL LOG OF B-TIMES NATURAL LOG OF X.
YOU CAN DO LOGARITHMS DO ANY BASE YOU LIKE AND THEY'RE ALL RELATED THIS WAY. (ON BOARD). LOG BASE TEN IS PROBABLY FAMILIAR BUT ANY OLD LOG WILL WORK. MATHEMATICIANS LIKE E-TO THE BASE BECAUSE IT'S MAKE EVERYTHING SIMPLER. BECAUSE OF DERIVATIVE OF E-TO THE X-IS JUST E-TO THE X-. WHAT COULD BE SIMPLER THAN THAT. SO WE'RE GOING TO STICK MOSTLY WITH E. BUT ALL THE OTHER LOGS YOU'VE SEEN, THEY'RE ALL RELATED. SO THAT'S A REVIEWER OF LAST TIME. ANY QUESTIONS.

STUDENT: JUST THE POINT JUST SO WE CAN EXPRESS EXPONENTIAL FUNCTIONS.

PROFESSOR: YES EXPONENTIAL FUNCTIONS AND LOGARITHMS. IN BUSINESS, IT'S ONE PLUS OF INTEREST RATE, WE USED THAT EXAMPLE LAST TIME. THERE ARE LOTS OF BASES THAT COME UP IN PRACTICE. YOU WANT IT FIGURE OUT HOW LONG IT WILL TAKE IT PAY OFF LOANS, LOG BASE ONE PLUS INTEREST RATE OF, TO UNDO THAT. THAT'S WHAT CALCULATORS DO.

STUDENT: HOW ARE YOU ABLE TO GO FROM L N-X-OR, THE, THAT LINE.

PROFESSOR: SO I JUST TOOK THIS AND DIVIDED BOTH SIDE BY THAT CONSTANTS, NATURAL LOG OF TEN. SO JUST DIVIDED BY NATURAL LOG OF TEN AND GET LOG BASE TEN OF X, THIS TIMES, DIVIDED BY THAT. THIS IS JUST A NUMBER. 2.3 SOMETHING.

SO LET ME DO AN EXAMPLE. AND USE SOME OF THIS STUFF. PLOT

THE FUNCTION Y-EQUALS MINUS FIVE X-PLUS E-TO THE X. (ON BOARD). AND LET'S SEE IF WE CAN, DO USUAL TECHNOLOGY WHICH SAYS WE CAN DO DERIVATIVES, DERIVATIVES WITH ZERO. FIGURE OUT IF THAT'S A MIN OR MAX. D-Y-D-X-IS GOING TO BE MINUS FIVE PLUS E-TO THE X. E-TO THE X-IS REALLY SIMPLE TO DIFFERENTIATE. SO THERE IT IS. AND SO
Let's find the critical point. Which means I'm going to set $D - y - d_x$ to be zero. Or $e^x$ is five or $x = \log_e 5$. That's a nice simple way it solve for $x$. So just take the log of both sides. So the next question is, am I concave up or down? And so I'm going to compute the second derivative. There's the first derivative. Differentiate again the minus five goes away. And I have $e^x$. That's the really easy simple second derivative. So is the second derivative positive or negative or zero? It's always positive. No matter which value of $x$ you have in the pick. That means the function is always concave up. That means when I plot it, it's going to have a minimum at log of five. So there's the minimum. And what is the value there? So what is $y$ of natural log of five? It's going to be minus five log five plus $e^\log_e 5$. $e^\log_e 5$, just turns into five. And so I can factor out a five here. I get one minus natural log of five. 5

So there's the value of the function. And that is negative. Some negative value. There it is. Fifteen times one minus natural log of five. And since the function is always concave up, we know it has to look like this. That much more detail. It's concave up so it look like that. There's the single, you know. Any questions about that example?

Student: So the $y$ is negative value.

Professor: Right. So five is bigger than $e$, log of five is a
LITTLE BIT BIGGER THAN ONE. I DIDN'T DO THE ARITHMETIC, IF YOU DO THE ARITHMETIC IT'S A NEGATIVE NUMBER. I DIDN'T BOTHER TO ACTUALLY DO THE COMPUTATION BUT IT DOES HAPPEN TO BE NEGATIVE. SO WHAT YOU CAN TELL, THIS IS THE MINIMUM AND IT'S CONCAVE UP EVERYWHERE. I WILL NOT ASK YOU TO DO DIFFICULT ARITHMETIC WHICH WOULD REQUIRE A CALCULATOR ON IT. I JUST USED THIS AS AN EXAMPLE.

STUDENT: ON THE TEST, IF YOU HAVE A PROBLEM LIKE THIS WOULD YOU WANT US TO SIMPLIFY IT.
PROFESSOR: I WOULD WANT YOU TO SIMPLIFY E TO THE LOGS AND LOGS OF E-AND STUFF LIKE THAT.
STUDENT: INSTEAD OF HAVING FIVE TIMES ONE MINUS.
PROFESSOR: YOU COULD HAVE LET IT HERE TOO. I'M NOT GOING TO WORRY ABOUT THAT. JUST ONE MORE STEP (INAUDIBLE). SO THE NEXT, SO WE HAVEN'T ACTUALLY DO ANY CALCULUS WITH LOGARITHMS YET. AND THAT'S THE NEXT SECTION. 4.5. DIFFERENTIATING LOGARITHMS. AND SO APPLICATION OF CHAIN RULE, GOING TO BE VERY EASY. I'M GOING TO USE THAT FACT THAT EXPONENTIAL AND LOG CANCELING ONE ANOTHER. AND I'M GOING TO WRITE THIS AS, DIFFERENTIATE THAT EQUATION. JUST GOING TO DIFFERENTIATE WHAT YOU SEE THERE. BUT I'M GOING TO WRITE THIS AS F-OF G-OF X-EQUALS X. F-IS GOING TO BE E-TO THE X-BECAUSE THAT'S THE OUTSIDE FUNCTION AND INSIDE FUNCTION IS GOING TO BE G OF X. SO THIS IS JUST F-OF G-OF X-FOR THAT PARTICULAR F-AND G, SO LET ME USE THE CHAIN RULE. JUST DIFFERENTIATE BOTH SIDES OF THAT. AND D-D-OF X-IS ONE. THAT'S EASY. SO NOW I HAVE TO USE CHAIN RULE. SO THIS SIDE EQUALS ONE. WHAT IS THIS SIDE? LET ME DO THE CHAIN RULE WITHOUT FILLING IT
IN YET. THAT'S JUST THE CHAIN RULE. SO NOW WHAT IS F-PRIME?
WHAT'S F-PRIME OF X? E-TO THE X. THAT'S MEAN TO BE E. AND
WHAT'S G-PRIME OF X. THAT'S WHAT WE'D LIKE TO KNOW. WE DON'T
KNOW WHAT G-PRIME OF X-IS. WE'RE GOING TO SOLVE THIS EQUATION
FORCE G-PRIME OF X. SO PLUG IN F-PRIME OF G-OF X-IS JUST E-TO
THE G-OF X-TIMES G-PRIME OF X. AND WHAT'S E-TO THE POWER G-OF X?
E-TO THE LOG X-SO. THIS IS E-TO THE LOG X-TIMES G-PRIME OF
X-EQUALS ONE. PLUG IN WHAT G-WAS. AND NOW WHAT'S E-TO THE LOG?
X. FINALLY I GET X-TIMES G-PRIME OF X-EQUALS ONE. PLEASE SOLVE
THAT FOR G-PRIME? WHAT IS IT? ONE OVER X-. THERE YOU GO. SO
WE JUST FIGURED OUT D-D-X-TO THE NATURAL LOG OF X-IS ONE OVER X.
(ON BOARD). THAT'S PRETTY EASY.

THAT SHOULD BE EASY TO REMEMBER. LET'S JUST DO A FEW
EXAMPLES.

STUDENT: QUESTION. D-D-X-OF LOG X-ALSO BE ONE OVER X-

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PROFESSOR: SO D-D-X-OF LOG, ONLY THIS LOGARITHM, YOU'RE ASKING
ABOUT THE LOG BASE TEN.

STUDENT: YEAH.

PROFESSOR: WE'LL SEE. WE'LL DO THAT AS AN EXAMPLE IN A MOMENT.

SHOULD I DO THAT NOW? SURE. THAT'S EASY. (ON BOARD). OKAY.

SO YOU JUST HAD A FORMULA FOR CHANGING LOG BASE TEN INTO NATURAL
LOG. WHICH I JUST ERASED. DO YOU REMEMBER WHAT IT WAS? IT WAS
SOMETHING TIMES NATURAL LOG OF X. WHAT WENT HERE? (ON BOARD).

SO IT'S, SO THE FUNCTION IS JUST A CONSTANT. SO LET ME DO THE
CONSTANT. (ON BOARD). THERE WE GO. SO IT'S NOT ONE OVER X, YOU
GET THAT FACTOR OUT. BECAUSE THESE TWO FUNCTIONS ARE JUST, ONE
IS CONSTANT MULTIPLE OF THE OTHER. SO THE DERIVATIVES IS EASY.
JUST FACTOR OUT THE CONSTANT.
DO SOME MORE EXAMPLES. SO THIS IS JUST THE GENERALIZED POWER RULE. SO LET'S JUST TRY IT. THAT TELLS US THAT WE TAKE FIVE TIMES IT FUNCTION TO THIS POWER FIVE MINUS ONE. AND THEN WE DIFFERENTIATE THE FUNCTION ITSELF WHICH WE JUST FIGURED OUT HOW TO DO. SO I GET FIVE NATURAL LOG OF X-ALL TO THE FOURTH POWER. MULTIPLIED BY ONE OVER X. MANY PEOPLE ARE STILL LOOKING AT THIS BOARD. I'M HAPPY TO KEEP TALKING ABOUT IT IF THERE'S A QUESTION. SO THIS IS JUST A GENERALIZED POWER RULE. SO WITH THE LOG FUNCTION. DO ANOTHER EXAMPLE.


STUDENT: QUESTION. IN THE FIRST EXAMPLE ON THE THIRD STEP DID YOU USE THE (INAUDIBLE).

PROFESSOR: SO THIS IS A CONSTANT AND THE RULE IS WHEN YOU DIFFERENTIATE A CONSTANT TIMES A FUNCTION YOU FACTOR OUT THE CONSTANT. LOG TEN IS JUST A NUMBER.

OKAY. DO A FEW MORE EXAMPLES. (ON BOARD) THIS IS GOING TO BE THE CHAIN RULE. SO FIRST I DO THE FUNCTION, SO THIS IS F-OF G-OF X. WHERE F-OF X-EQUALS LOG X AND G-OF X-EQUALS X-SQUARED PLUS ONE. SO IT'S JUST THE CHAIN RULE APPLICATION. SO IT'S
GOING TO BE WRITTEN IN CHAIN RULE LANGUAGE. AND WHAT’S F-PRIME?
IT’S ONE OVER X. AND WHAT’S G-PRIME? IT’S JUST TWO X. SO IT’S
GOING TO BE ONE OVER G-OF X, LET ME DO IT STEP-BY-STEP, TIMES
G-PRIME OF X, WHICH IS ONE OVER X-SQUARED PLUS ONE TIMES TWO X.

STUDENT: DOING THESE DO YOU ALWAYS WANT US TO DEFINE --

PROFESSOR: I’M GOING TO FOR, FOR CLARITY IF YOU’RE DOING IT ON A
MIDTERM ON FINAL OR HOMEWORK, THEN YOU CAN WRITE CHAIN RULE. OR
JUST DO IT AND, THE PURPOSE OF WRITING DOWN A LITTLE BIT EXTRA
LIKE CHAIN RULE IS IN CASE YOU MESS UP THE ALGEBRA YOU COULD GET
PARTIAL CREDIT BECAUSE WE KNOW YOU HAVE THE RIGHT IDEAS. SO

JUST DO THE CHAIN RULE, MAYBE YOU MESSED UP THE ALGEBRA.

KEEP GOING.

STUDENT: QUESTION. HOW DID YOU GET ONE OVER G-OF X---

PROFESSOR: BECAUSE F-PRIME IS TAKE THE RECIPROCAL IS ONE OVER
X-FUNCTION. THIS SAYS TAKE RECIPROCAL OF G-OF X.

STUDENT: OKAY.

PROFESSOR: IN FACT, LET ME EXPAND ON THAT EXAMPLE. JUST AN
APPLICATION OF CHAIN RULE. BUT I’M NOT GOING TO TELL YOU WHAT
G-OF X-IS, IT’S JUST GOING TO BE A GENERAL FUNCTION. SO THIS IS
THE SAME IDEA AS OVER HERE, IT’S GOING TO BE THE SAME FUNCTION,
SAME F-PRIME FUNCTION. SO F-PRIME OF G-OF X-TIMES G-PRIME OF X.
SO IT’S GOING TO BE G-PRIME OF X-DIVIDED BY G-OF X. THAT’S TRUE
FOR ANY FUNCTION G. YOU TAKE THE LOG OF FUNCTION AND ASK FOR ITS
DERIVATIVE, IT’S THE DERIVATIVE DIVIDED BY THE ORIGINAL FUNCTION.
IT’S JUST THE CHAIN RULE.

STUDENT: HOW DID YOU GET ONE OVER X-OF THE DERIVATIVE OF L N-X-

PROFESSOR: THAT WAS A COUPLE OF BOARDS AGO. I PROVED IT.
MAYBE IT’S STILL UP THERE. THIS IS THE, THAT WAS THE CONCLUSION
FROM HERE. JUST USING THAT FACTOR. EACH OF THESE EXAMPLES IF YOU DIFFERENTIATE THE LOG YOU GET ONE OVER X.

STUDENT: THAT'S LIKE HOW IT WORKS, THAT ONE.

PROFESSOR: THIS IS ALWAYS TRUE. THIS IS A FORMULA THAT IS TRUE FOR ANY FUNCTION G. I USED IT FOR A SPECIAL FUNCTION HERE, X-SQUARED PLUS ONE BUT THE FORMULA IS TRUE FOR ANY G-THAT I CAN TAKE THE DERIVATIVE OF. I DID A SPECIAL CASE WHERE G-OF X-WAS 10


STUDENT: HOW DID YOU SIMPLIFY THAT ONE.

PROFESSOR: SO THIS.
STUDENT: UNDER HOW DID YOU GET TO THE RIGHT SIDE.

PROFESSOR: THERE'S ONE OVER X SQUARED HERE AND ONE OVER X-SQUARED THERE. SO I PUT OUT A COMMON FACTOR THE ONE OVER X-SQUARED AND THEN I'M LEFT WITH ONE MINUS LOG X. IS THAT OKAY?

STUDENT: WHICH PROPERTY -- (INAUDIBLE).

PROFESSOR: SO THE TWO PROPERTIES ARE, THAT WE'RE USING IS THAT THE LOG OF X-IS EQUAL TO Y-THEN X-IS EQUAL TO E-TO THE Y-. THESE ARE THE SAME THINGS. SO PLUG IN Y-EQUALS ONE. AND E-TO THE ONE EQUALS E-. MULTIPLY THIS BY ITSELF ONCE. SO WE HAVE A CRITICAL POINT. AND NOW WE HAVE TO ASK IS IT MINIMUM OR MAX OR INCREASING OR DECREASING. WE COULD COMPUTE THE SECOND DERIVATIVE BUT LET ME REASON IT DIFFERENTLY FOR SAKE OF ARGUMENT. SO LET'S TRY TO COMPUTE, LET'S TRY TO DECIDE WHERE THE DERIVATIVE IS POSITIVE, NEGATIVE AND WE KNOW EXACTLY WHERE IT'S ZERO. ZERO IS JUST THAT ONE PLACE. SO WHERE IS THE FUNCTION INCREASING?

WHERE IS THE DERIVATIVE POSITIVE AND WHERE IS IT DECREASING. SO LET'S THINK ABOUT THAT. SO HERE'S A WAY TO DO IT. SO HERE'S THE FUNCTION WHICH IS THE LOGARITHM. AND REMEMBER WHAT THE PLOT OF A LOGARITHM IS? IT LOOKS LIKE THAT. GOES DOWN TOWARDS MINUS INFINITY AND THEN KEEPS GOING UP. CROSSES THE AXIS AT ONE COMMA ZERO. THAT IS WHAT WE DID THE OTHER DAY. WHAT DOES THE NEGATIVE LOG PLOTS LOOK LIKE. SO THIS IS LOG. SO NOW LET'S JUST IN OUR MIND DRAW THIS PICTURE. SO WHAT'S NEGATIVE LOG?

IT'S JUST UPSIDE DOWN. JUST FLIP IT. LOOK LIKE THIS. SO HERE I HAVE NEGATIVE LOG X. AND NOW WHAT ABOUT ONE PLUS THAT? IT'S JUST MOVES UP BY ONE. SO MOVE IT UP BY ONE. HERE IS ONE MINUS LOG X. NOW THE QUESTION IS WHERE IS THIS POSITIVE? AND WHERE IS IT NEGATIVE? WELL, CLEARLY HERE IT'S POSITIVE. HERE IT'S
NEGATIVE. AND IT'S EXACTLY ZERO AT E. YOU FIGURED THAT OUT. IT'S ZERO AT E. SO WITHOUT DOING ONLY REMEMBERING WHAT THE PLOTS

OF LOG FUNCTION IS, I TRIED TO FIGURE OUT WHERE IS THIS NUMERATOR POSITIVE ZERO AND NEGATIVE. JUST BY REMEMBERING THE PICTURE.

OKAY. SO NOW I KNOW WHERE THE NUMERATOR IS POSITIVE, NEGATIVE AND ZERO. WHAT ABOUT THE WHOLE FRACTION? NOW I'M GOING TO TAKE THIS AND GOING TO DIVIDE BY SOMETHING WHICH IS ALWAYS POSITIVE. IS THAT GOING TO CHANGE WHERE THE FRACTION IS POSITIVE, NEGATIVE OR ZERO? NO. SO FINALLY I KNOW, LET ME DRAW IT THIS WAY THAT AT E-THE FUNCTION, THE DERIVATIVE Y-PRIME OF IS ZERO. OVER HERE Y-PRIME IS NEGATIVE. AND HERE Y PRIME IS POSITION. BECAUSE I JUST DIVIDED THIS NUMBER BY A POSITIVE, IT DOESN'T CHANGE ITS SUM. THIS IS ALL I NEED TO KNOW, TO KNOW WHERE IS MY FUNCTION INCREASING. WHERE IS IT FLAT. AND WHERE IS IT DECREASING. SO IS THIS POINTS HERE GOING TO BE A RELATIVE MAX OR A RELATIVE MINIMUM. IT'S A MAX. THIS IS ALL WE NEED. FINALLY WHEN I PLOT THIS THING, LOG X-OVER X-I'M GOING TO GET A FUNCTION THAT LOOK LIKE THIS. (ON BOARD). INCREASING, AND THEN IT'S GOING TO BE FLAT RIGHT THERE. AND THEN IT'S GOING TO BE DECREASING.

STUDENT: WHERE DID YOU GET THE Y-VALUE.

PROFESSOR: SO HERE'S MY FUNCTION. AND ALL I KNOW FROM THIS, SO HERE I'M CLAIMING THIS IS THE PLOT OF LOG X-OVER X. WHAT I FIGURED OUT IS IT'S GOING TO GO FLAT RIGHT THERE. INCREASE UP TO THAT POINTS, DECREASE AFTERWARDS. THEN YOU CAN ASK, WHY DOES IT GO DOWN HERE? LOOK AT THIS FUNCTION, IT'S GOING DOWN TO NEGATIVE INFINITY. IF I DIVIDE BY SOMETHING CLOSE TO ZERO IT'S JUST GOING TO KEEP GOING DOWN TO NEGATIVE INFINITY. SO IT KEEPS GOING DOWN
TO NEGATIVE INFINITY. OVER HERE IT HAS IT DECREASE. AND WHERE, IT'S ALL POSITIVE BECAUSE IT'S A POSITIVE NUMBER DIVIDED BY A POSITIVE NUMBER. SO IT'S A DECREASING THING THAT STAYS POSITIVE. SO THAT WAS THE INTUITION THAT SAYS ROUGHLY THAT IS WHETHER THE FUNCTION LOOK LIKE. I'M NOT TRYING TO PRETEND I'M A CALCULATOR THAT GOT EVERY POINT RIGHT BUT I'M TRYING TO DO A LITTLE ALGEBRA IN CALCULUS AS POSSIBLE. AND TRY TO REASON OUT WHAT THE PICTURE LOOK LIKE. BECAUSE I'M LAZY AND DON'T WANT TO HAVE TO DO ANY CALCULUS. REASON BY DRAWING PICTURES UPSIDE DOWN UNSTUFF LIKE THAT. ANY QUESTIONS ABOUT THAT BEFORE I GO ON? THERE ARE MANY WAYS TO DO THAT PROBLEM. I COULD HAVE COMPUTED THE SECOND DERIVATIVE AND FIGURED OUT INFLECTION POINT. THERE ARE MORE THAN ONE WAYS TO SOLVE IT. I JUST CHOSE THAT ONE.

SO WE KNOW WE'RE OTHER ALLOWED TO TAKE LOGARITHMS OF POSITIVE NUMBER. SO LET'S TRY THE LOG OF THE ABSOLUTE VALUE. THAT'S GOING TO WORK WHEN X-IS NEGATIVE I CAN STILL DO THAT. LET'S TRY TO FIGURE OUT HOW TO DIFFERENTIATE THIS. AND WHAT WE LEARNED BEFORE IS IF YOU HAVE A FUNCTION, TWO FORMULAS, FORMULA WHEN X-IS POSITIVE AND FORMULA WHEN X-IS NEGATIVE. WE'RE GOING TO HAVE TWO CASES. JUST DO IT TWICE USING TWO DIFFERENT FORMULAS. SO HERE IT'S GOING TO BE D-D-X-OF NATURAL LOG OF THE ABSOLUTE VALUE OF X-WHICH IS, HOW DOES THAT SIMPLIFY WHEN X-IS POSITIVE? JUST X. BECAUSE THAT'S -- WHAT DO I GET DOWN HERE? D-D-X-OF LOG OF NEGATIVE X. LET'S JUST DO THOSE TWO CASES. THIS ONE WE KNOW IS JUST ONE OVER X-. NOTHING TO DO THERE.

THIS ONE IS MORE INTERESTING. I DO THE CHAIN RULE. AND LET'S BE

THAT IS JUST THE PRODUCT RULE. SO HERE I NEED TO APPLY THE CHAIN RULE BECAUSE IT'S A FUNCTION OF A FUNCTION. THE OUTER FUNCTION IS THE LOG. THE INNER FUNCTION IS X-TO THE POWER A. SO LET ME, SO HERE I'M GOING TO HAVE F-OF X-IS LOG OF X-G-OF X-IS X-TO THE A. JUST GOING TO DO THE CHAIN RULE. AND LET ME SEE IF WE CAN DO IT WITHOUT WRITING DOWN EVERY INTERMEDIATE ONE. SO THERE WE GO. SO IT'S GOING TO BE, SO F-OF THIS, F, THIS IS GOING TO BE F-PRIME OF G-OF X, THAT'S GOING TO BE THE RECIPROCAL OF G-OF X. So WHAT'S THAT GOING TO BE? ONE OVER X-TO THE A, TIMES.

STUDENT: A-X-


STUDENT: A-OVER X-

PROFESSOR: A-OVER X, THANK YOU. BECAUSE A-TO THE X-CANCEL THAT X-TO THE A. AND I'M LEFT TO X-TO THE MINUS ONE, DENOMINATOR.


DENOMINATOR. SO X-TO SOME POWER. THERE I HAVE X. AND HERE I'M GOING TO DO THE SAME THING. OKAY. WHAT POWER DO I GET DOWN HERE? X-TO WHAT POWER IN THE DENOMINATOR, ONE PLUS B. AND WHAT DO I GET HERE? PUT THIS GOES GUY DOWN NOT DENOMINATOR. TO MIC IT A SIMPLER FRACTION. TURNS INTO X-TO THE POWER ONE PLUS B. SO THERE'S A COMMON FACTOR. (ON BOARD). LET ME PUT OUT THAT COMMON
FACT.

STUDENT: SHOULD THAT BE L N-X-TO THE A-OR --

PROFESSOR: SORRY. DID I -- YES. THANK YOU. I COPIED IT WRONG. THERE'S AN A-THERE. I HOPE THAT'S THE ONLY MISTAKE SO FAR. FINALLY I'M GOING TO GET A-MINUS B-TIMES LOG OF X-TO THE A-DIVIDED BY X-TO THE ONE PLUS B. SO THERE IS AFTER SOME ALGEBRA THE DERIVATIVE, AND IT'S NOT THAT COMPLICATED.

STUDENT: YOU SAY YOU SIMPLIFIED A-OVER X-

PROFESSOR: SO HERE'S AN X-TO THE A. THERE'S AN X-TO THE A. I JUST, ONE IN THE NUMERATOR AND ONES IN THE DENOMINATOR AND I JUST CANCELED. AND I GOT X-TO THE MINUS ONE WHICH PUTS IT DOWN HERE.

STUDENT: YOU SPLIT THEM UP.

PROFESSOR: SO STEP-BY-STEP. (ON BOARD). I SKIPPED SOME STEPS. OKAY. SO FAR SOME GOOD. NOT ALWAYS CLEAR WHICH ARE THE RIGHT STEPS TO SKIP. SO FEEL FREE TO ASK. I WANTED TO FIND THE CRITICAL POINT SO I HAVE TO FIND OUT WHEN THIS THING IS EQUAL TO ZERO? THE DENOMINATOR IS THAT GOING TO HELP ME MAKE IT TO ZERO. ALL YOU CARE ABOUT IS WHETHER THE NUMERATOR IS, I HAVE TO SOLVE A-MINUS B-TIME LOG X-TO THE A-EQUAL ZERO. SO A-EQUAL B-TIMES LOG

OF X-TO THE A. A-DIVIDED BY B-EQUALS LOG OF X-TO THE A. SO NOW WHAT AM I GOING TO DO?

STUDENT: THE A-ROUTE.

PROFESSOR: TAKE THE EXPONENTIAL OF BOTH SIDE. SO (ON BOARD).

SO I HAVE E-TO THE A-OVER B-EQUALS WHAT? X-TO THE A.

STUDENT: WHAT ALLOWS US TO DO THAT? WHAT ALLOWS ME TO DO WHAT? IF R-EQUALS S-THEN E-TO THE R-EQUALS E-TO THE S. ANY FUNCTION OF R-EQUALS ANY FUNCTION OF S-BECAUSE IT'S THE SAME NUMBER.
WHATEVER YOU WANT TO DO. R-AND S. IT'S THE SAME ANSWER. SO NOW I'M NOT DONE. I HAVE TO SOLVE FOR X. WHAT DO I GET OVER HERE? E-TO THE WHAT POWER? E-TO THE ONE OVER B. JUST DEPENDS ON B. (ON BOARD). A-S CANCEL. ANY QUESTIONS ABOUT THAT? LET ME DO SOME PROPERTIES, SO THAT'S THE ENDS OF 4.5. DIFFERENTIATEING. I WANT TO DO SOME MORE ALGEBRA PROPERTIES OF LOGARITHMS TO HELP YOU SIMPLIFY THESE CONDITIONS.

4.6 PROPERTIES OF LOGARITHMS. WE STARTED WITH CHAPTER BY DOING LAWS OF EXPONENTS. WELL FOR EVERYONE LAW OF AN EXPONENTS WE HAVE A LAW OF LOGARITHMS. LET ME JUST WRITE DOWN THESE NAMES AGAIN. BECAUSE THESE PROPERTIES ARE GOING TO APPLY TO EVERY LOGARITHM. APPLY TO THE COMMON LOG. AND THESE PROPERTIES WILL WORK FOR ANY OLD LOG BASE B. LOG BASE B. OKAY. SO ALL THESE LAWS THEY DON'T CARE WHAT THE BASE IS. MAKE LIFE SIMPLER. SO I'M GOING TO WRITE THE WORD LOG WITHOUT SAYING WHAT THE BASE IS BECAUSE THEY'RE ALL TRUE NO MATTER WHAT THE BASE IS.

SO THE LOG OF X-TIME LOG, THIS LOG CAN BE ANY OF THOSE LOGS.

DON'T CARE WHICH, IS THE LOG OF X-PLUS THE LOG OF Y-IF X-IS GREATER THAN ZERO AND Y-IS GREATER THAN ZERO. SO THAT'S MAY BE FAMILIAR BUT WE'RE GOING TO PROVE IT CAREFULLY. BUT JUST FALLS IN THE LAW OF EXPONENTS. THEN WE'RE GOING TO PROVE THAT THE LOG OF ONE OVER X-WHICH IS EQUAL TO MINUS LOG X. SO THAT MAKE IT EASY. AND IF YOU PUT THOSE TWO RULES TOGETHER, YOU GET LOG OF X-OVER Y-IS LOG X-MINUS LOG Y. AND SOMETHING THAT WOULD HAVE MADE THE LAST PROBLEM A LITTLE BIT EASIER. YOU TAKE LOG OF A-TO THE A-POWER YOU CAN JUST PULL THE A-OUT FRONT. THAT'S TRUE FOR ANY OF THESE. SO JUST CURIOUS HOW MANY PEOPLE DID LOGS IN HIGH SCHOOL AND LEARNED THESE THINGS ABOUT THEM? OKAY. FINE. SO
Let's us then, I'm glad it's familiar. Let me go through and explain why they're all true because they're all just identical to the laws of exponents.

Let me explain why they're all true. I'm going to do did, I need it use the fact that here's some dates. So I'm going to do it all for base B. So here's why one is true. So I can (on board). So if I take B-to the log B-of anything, I get that. That's how exponents and logs work, they just cancel. But what's x? x-can be written as B-to the log base B-of x. And y-can be written as B-to the log base B-of y. Just that equals that and that equals that. And now I'm going to use the law of exponents. How do I multiply these two together? What do I put, do with these two? I add them. That's the whole proof. So B-to this equal B-to that, the exponents, the thing up here, I can do the same. Okay that's all there is to it. Law the exponents. Let us do the same thing, is there a question? I just, use the law of exponents to get there. Here's the next one. B-to the power log B-again they canceling so I get one over x. That's the definition of B-to the log B, they undo one another and X-I can write the way I did here as B-to the log B-of x. Now what's the law of exponents say? How do I write this fraction as B-to some exponent? What do I do? I just, I just negate. And I'm done. If B-to this power equals B-to that power, these have to be the same. (On board). So that's the second one.

The thirds one is, follows from the first two. So let me just do that. Log of x-over y-is the same thing as log of x-times one over y. And so now I have log of product of two

STUDENT: HOW DID YOU GET THE LAST PART OF PROOF.

PROFESSOR: OVER HERE? SO WHAT I SAID IS IF B-TO THAT EQUALS
B-to that, these two guys have to be equal. So I said log basic
b-of x-to the a is a-times log base b-of a.

Student: Could you say how you on the e-to the a-over b-equal
x-to the a-how you simplify that.

Professor: Here? e-to the log of anything, these just cancel.
So I get x-to the a-am. And then I took the a-th root of both
sides. Took one over a-power. And so the law of exponents says
that's x-to the a-over x-or x-to the one or x. Over here I get
E-to the power, (on board). The just use the law of exponents
frequently. Let's apply these a few times. Let me leave these
up.

Simplify a few formulas. Are there anymore questions about
that? Let's practice with them a little bit. I think I did this
one before. Let's simplify this as best we can. So two log
three I'm going to put the two inside. Just move the two up.
That's one of my rules. This guy and that guy are equal.
Because two come out front. So I'm going to put it back up. So
threes squared is nine. And so this is natural log of five times
nine. Or the natural log of 45. So it all (inaudible). Let me
do another example. (On board). Let's use the law of exponents,
this hats law again but move the factor a half up not exponents.
So it's going to be log of 14 to the one-half power minus log
t-squared plus one. So what's one-half power mean? That's the
square root, right? So it's going to be, I'll write it this way,
square root of four time the square root of t. So it's two time
the square root of t. And pulling it all together it's going to
be the log of a fraction, and the numerator is two squared t and
THE DENOMINATOR IS T-SQUARED PLUS ONE. SO GETS ONE LOG. WHICH IS EASIER TO UNDERSTAND.

OKAY. LET'S DIFFERENTIATE, NOW WE'RE GOING TO DO A FEW MORE PROBLEMS WHERE YOUR LOG ARE YOUR FRIEND, ACTUALLY MAKE IT EASIER TO DIFFERENTIATE. NOW I COULD GO AHEAD AND USE THE PRODUCT RULE BUT IT COULD BE TEDIOUS. BECAUSE I HAVE THREE THINGS TO MULTIPLY TOGETHER. SO I'M GOING TO USE LOGARITHM DID MAKE IT EASIER ACTUALLY. SO ACTUALLY LET ME, I'M GETTING AHEAD OF MYSELF, SORRY. I'LL DO THAT IN A MOMENT. LET ME DIFFERENTIATE THE LOG OF ALL THAT. NOW AS I SAID I COULD USE THE CHAIN RULE RIGHT AWAY. AND BUT THEN I HAVE TO DIFFERENTIATE EVERYTHING ON THE INSIDE AND THEN IT WOULD BE MESSY. SO LET ME USE THE FACT THAT I CAN WRITE IT THIS WAY. (ON BOARD). INVERT THE LOG OF PRODUCT WHICH IS THE SUM OF THE LOGS. AND SO IF I WANT TO DO F-PRIME I HAVE TO DIFFERENTIATE EACH TERM AND ADDS THEM UP. SO WHAT'S THE DERIVATIVE OF LOG X? ONE OVER X. WHAT IS THE DERIVATIVE OF THIS GUY? ONE OVER X-PLUS ONE. TIME THE DERIVATIVE OF X-PLUS ONE WHICH IS ONE. (ON BOARD). THAT'S A VERY SIMPLE FORMULA. EASIER THAN USING, YOU WOULD HAVE GOTTEN THE SAME ANSWER BUT MUCH MESSIER IF YOU USE THE PRODUCT RULE A BUNCH OF TIMES. SPREAD IT OUT USING LOGARITHM.

SO NOW LET ME DO THE FOLLOWING. SUPPOSE G-OF X-IS JUST THE STUFF ON THE INSIDE. WITHOUT THE LOGARITHM. JUST ALL THOSE THINGS. AND I'M TO DIFFERENTIATE THAT. SO CAN I USE LOGARITHM TO GET G-PRIME OF X-EASILY? SO I DON'T WANT, I WANT TO AVOID USING THE PRODUCT RULE BECAUSE BIG MESS. I WANT TO USE THE FACT THERE WAS SOMETHING REALLY SIMPLE WOULD HAPPEN RIGHT THERE TO THE
LOG TO MAKE IT EASY TO DIFFERENTIATE THIS GUY.

ANY CALCULUS. CAN I SIMPLIFY THAT ANYMORE? LOG OF SOMETHING TO
A POWER? WHAT HAPPENS? I JUST PULL OUT THAT TWO. (ON BOARD) SO

THIS IS THE LOG OF TWO X-PLUS FIVE, WELL TO THE MINUS ONE. SO I
PULL OUT A MINUS ONE. NOW I JUST HAVE TO DIFFERENTIATE IT ONE
TERM AT A TIME. FACTOR OF TWO I CAN PULL OUT FRONT. (ON BOARD).
FACTOR MINUS ONE, I CAN PULL OUT FRONT. STILL HAVEN'T DONE ANY
INCLUSION. JUST ALGEBRA. NOW I HAVE THE LOG OF SOMETHING. AND
I HAVE TO DIFFERENTIATE IT. SO I PUT THE SOMETHING IN THE
DENOMINATOR. AND WHERE DID I PUT THE NUMERATOR, THE DERIVATIVE
OF THE THING ON THE INSIDE. WHICH IS TWO X. HERE I GET X-CUBED
MINUS THREE. AND WHAT GOES IN THE NUMERATOR? THREE X-SQUARED.
AND THEN I GET TWO X-PLUS FIVE, AND THE NUMERATOR IS TWO. AND
THAT'S ABOUT AS LITTLE ALGEBRA I CAN IMAGINE DOING TO
DIFFERENTIATE THAT MESS THAT I STARTED UP THERE. HERE WAS THE
STARTING POINT. I STILL HAVE THAT FACTOR RIGHT THERE. I'M NOT
GOING TO BOTHER WRITING IT OUT. THIS IS GOOD ENOUGH. AND YOU
WOULD HAVE GOTTEN THE SAME ANSWER WITH THE PRODUCT RULE. BUT
THIS IS EASIER.

STUDENT: NEVER MIND.

PROFESSOR: OKAY. AGAIN I TOOK, I SKIPPED A FEW STEPS BUT ...

ONE MORE ILLUSTRATION HERE WE LEARNED THIS BACK IN CHAPTER ONE.

SO LET'S SEE WHERE IT COME FROM. D-D-X-OF X-TO THE R-IS R-X-TO
THE R-MINUS ONE. USING THAT FOR A LONG TIME. FINALLY WE CAN
ACTUALLY PROVE IT.

STUDENT: HOW DID YOU GET THAT TWO --

PROFESSOR: BECAUSE IT WAS THE LOG OF THIS THING SQUARED AND

SINCE IT'S A LOG I JUST PULLED IT OUT FRONT. THE SAME THING WITH
THE MINUS ONE THERE.

SO LET ME USE THAT SAME TRICK THAT I DID BEFORE. I WANT TO
DIFFERENTIATE G-OF X-WHERE X-IS X-TO THE R. I'M GOING TO USE
THAT SAME IDEA I HAD WHICH, G-PRIME OF X-EQUALS G-OF X-TIMES
THIS IS, SO I NEED TO WRITE DOWN WHAT IS THE LOG OF G-OF X? THE
LOG OF THIS IS, THE THE LOG OF X-TO THE R-WHICH IS R-TIMES LOG X.
HERE I GET TIMES D-D-X-OF R-TIMES LOG OF X. (ON BOARD).

STUDENT: WHAT DOES IT SAY DOWN THERE, G-OF X-

PROFESSOR: THE LOG OF G-OF X-IS THE LOG OF X-TO THE R-WHICH, AND
LOG OF X-TO THE R-IS R-LOG X. BECAUSE THAT'S ONE OF OUR LAWS OF
LOGARITHMS. SO NOW I HAVE THE LOG OF A CONSTANT TIMES LOG X. I
CAN FACTOR OUT CONSTANT. AND THE DERIVATIVE OF LOG X-IS ONE OVER
X. AND SO I SIMPLIFY. AND I GET R-TIMES X-TO THE R-MINUS ONE.
WHICH IS THE POWER LAW. COMES FROM, WHICH WE WROTE DOWN AND
WE'VE USED DOLLARS FOR A LONG TIME WITHOUT EVER SAYING WHY IS IT
TRUE. WE ACTUALLY EXPLAINED WHY IT WAS TRUE WHEN R-WAS THE
INTEGER BUT WE DIDN'T EXPLAIN WHY IT WAS TRUE WHERE R-WAS IN ELSE
LIKE A HALF OR ANY OTHER NUMBER. THAT'S WHY IT'S TRUE.

A FEW MORE EXAMPLES TO ILLUSTRATE HOW THIS STUFF WORK.

OKAY. LET ME DO D-D-X-OF F-OF X-(ON BOARD). OKAY. THAT LOOKS
LIKE A BIG MESS BUT LET'S JUST USE THE LAW OF EXPONENTS AND LOGS
TO FIGURE IT OUT. SO THIS IS THE PRODUCT OF THREE THINGS. E-TO
THE X-E-SQUARE ROOT OF X-AND X-PLUS ONE, THIS IS DENOMINATOR, SO
IT'S THE MINUS SAMETH POWER. LOG OF E-TO THE X-JUST TURNS INTO

X. THIS IS LOG OF X-TO THE ONE-HALF SO LOG OF X-TO THE ONE-HALF
IS JUST ONE-HALF LOG X. AND THERE I JUST FACTOR OUT THE MINUS SIX. SO THERE'S ANY FUNCTION ONCE I'VE SIMPLIFIED IT.

STUDENT: COULDN'T YOU HAVE DONE L N-E-TO THE X---

PROFESSOR: THAT WOULD WORK TOO. THERE ARE SEVERAL WAYS TO PROCEED. YES. BECAUSE THE, THAT'S WHAT I'VE DONE HERE, SORT OF MINUS THE LOG. SO NOW IF THIS IS F-OF X-I CAN DO F-PRIME EASILY. ONE PLUS A HALF OF X-MINUS SIX OVER X-PLUS ONE. THAT'S AWFULLY SIMPLE EXPRESSION THAT YOU GET AT THE ENDS ONCE I'VE GOTTEN DOWN TO THIS SUM.


STUDENT: YOU'RE SAYING X-TO THE X-THE DERIVATIVE OF THAT'S ONE. THE DERIVATIVE OF X---

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PROFESSOR: LET'S SEE, SO WHAT I WANT TO COMPUTE IS THE DERIVATIVE OF G-OF X. WHERE G-OF X-IS X-TO THE X. AND FINALLY GOT NEXT TO THE X. TIMES SOME OTHER STUFF. I'M JUST GOING TO DO ONE MORE HERE AND THEN CALL IT Quits. UNLESS YOU HAVE MORE
QUESTIONS ABOUT THAT ONE.