MATH 16 A-LECTURE. OCTOBER 2, 2008.

PROFESSOR: WELCOME. WELCOME BACK. I'D LIKE TO START WITH A BELIEF ANNOUNCEMENT.

STUDENT: MY NAME IS BRIAN. I'M A FIFTH YEAR STUDENT AT CAL.

I'M RECRUITER. I WANTED TO LET YOU KNOW BOTH THE WALT DISNEY WORLD RESORTS ARE LOOKING FOR INTERNS SUCH AS YOURSELF TO INTERN IN EITHER RESORTS. IN ADDITION TO THE INTERNSHIP DISNEY OFFERS A PROGRAM WHERE THEY DO PROFESSIONAL DEVELOPMENT. AND IN ADDITION YOU GET TO MEET SOME OF THE EXECUTIVES THE COMPANY. THE PROGRAM IS CALLED THE DISNEY RESORTS COLLEGE PROGRAM. I HAVE FLIERS I'LL PASS OUT. GO TO DISNEY COLLEGE PROGRAM.COM. AND PRESENTATION ON-LINE ABOUT WHAT EXACTLY YOU WOULD DO AS AN INTERN AND PAY RATE. IF YOU HAVE ANY QUESTIONS AT ALL YOU CAN GO AHEAD AND ACCEPTED AN E-MAIL, AT DREAM COME TRUE @BERKELEY.EDU.

PART OF INTERNSHIP NEXT YEAR YOU'LL GET PARKS, CALLED WHAT
WILL YOU CELEBRATE. COME FOR FREE, ON THEIR BIRTHDAY. IF YOU HAVE ANY QUESTIONS GO AHEAD AND E-MAIL ME. YOU DO ALSO GET FREE ADMISSION TO BOTH PARKS. THANK YOU.

PROFESSOR: OKAY. SO MAKE THAT HOMEWORK ASSIGNMENT I GAVE YOU ABOUT WHAT HAPPENS TO A ROCKET THAT YOU SHOOT UP WOULD BE APPROPRIATE PREPARATION FOR THAT INTERNSHIP. THEY DO A LOT OF FIRE WORK THERE. FIGURE OUT WHERE THEY LAND. OKAY. SO WHAT I'D LIED TO DO TODAY IS KEEP TALKING ABOUT PLOTTING GRAPHS. AND I'LL WAIT UNTIL I HAVE YOUR UNDIVIDED ATTENTION. SO I'M GOING TO TALK ABOUT USING CALCULUS TO PLOT GRAPHS A BIT MORE. AND I'D LIKE TO REVIEW FROM LAST TIME SINCE I PUT UP AN ALL OF LOT OF DEFINITIONS, SO LET ME REMINDS YOU OF WHAT THEY ALL ARE. SO. I'M GOING TO COMBINE GOING UP AND GOING DOWN IN ONE DEFINITION IT MAKE IT EASIER. A FUNCTION, I'LL WRITE IT THIS WAY IS...
ON AN INTERVAL, X-ONE COMMA X-TWO I-E- IS GOING UP IF ITS DERIVATIVE IS POSITIVE ON THE INTERVAL. SO FOR X-BETWEEN X-ONE AND X-TWO AND THE OTHER COULD YOU REPEAT THE PART TO THAT IS DECREASING ON THAT INTERVAL, GOING DOWN IF ITS DERIVATIVE IS NEGATIVE. GOING UP AND GOING DOWN ARE THE TWO MOST OBVIOUS QUESTIONS TO ASK ABOUT A GRAPH. SO WE'RE GOING IT FIGURE WHAT THAT IS BY ASKING WHEN IS A DERIVATIVE POSITIVE OR NEGATIVE.

SO THE NEXT DEFINITION BY WAY OF REVIEW IS LOOKING FOR WHEN THE FUNCTION REACHES A MAXIMUM OR WHEN IT REACHES A MINIMUM. SO A FUNCTION F-OF X-HAS A RELATIVE MAXIMUM AT A POINT X-IF IT'S THERE. SO FOR THE REST OF TODAY UNTIL I TELL YOU OTHERWISE, I'M GOING TO ASSUME MY FUNCTION DOESN'T HAVE ANY CORNERS. SO FUNCTION CAN LOOK LIKE THAT. SO I WANT TO HAVE DERIVATIVES EVERYWHERE. SO IT HAS TO BE FLAT THERE. AND, WELL THERE ARE TWO WAYS TO DO IT. F-PRIME OF X-IS SO IF IT'S A MAXIMUM, MEANS IT'S INCREASING ON ONE SIDE, ALL RIGHT, WRITE IT THIS WAY. F-PRIME GOES FROM POSITIVE, INCREASING TO NEGATIVE, AT THAT POINT, SO IT GOES FROM POSITIVE TO NEGATIVE, THAT'S ONE WAY
SAY IT. OR YOU CAN ASK WHAT IS THE SECOND DERIVATIVE THERE.

AND

THE SECOND DERIVATIVE HERE HAS TO BE NEGATIVE. THAT MEANS

IT'S

TURNING AROUND AND GOING DOWN. THAT'S ONE THING BY WAIVE

REVIEW

3

FROM LAST TIME. THAT'S WHAT IT HAS A RELATIVE MAXIMUM.

AND THE

OTHER THING WE TALKED ABOUT IS WHEN DOES IT HAVE A RELATIVE

MINIMUM ON POINT X. SO AGAIN HAS TO BE FLAT THERE. AND F-

PRIME

LOOKS IS NOW GOING TO GO FROM NEGATIVE TO POSITIVE, THAT MEANS IT

WE LIKE THAT, IT GOES DOWN AND BACK UP AGAIN, AND THE OTHER WAY

WAS TALKED ABOUT RECOGNIZING THAT WAS IF THE SECOND DERIVATIVE

POSITIVE AT THIS POINT X. THERE'S X. SO THAT WAS JUST LAST

TIME'S LECTURE IN ONE BOARD. TO A LARGE EXTENT.

OTHER MAKE SURE I DIDN'T SKIP ANYTHING. SO AND THERE'S ONE

DEFINITION FROM LAST TIME TO REVIEW. WE SAY THAT F-OF X-IS

YOU CONCAVE UPWARDS, IN OTHER WORDS, IT LOOKS LIKE A SMILE IF

DERIVATIVE IS LIKE, THAT'S CONCAVE UPWARDS, ON AN INTERVAL IF THE
AS A FUNCTION IS INCREASING ON THAT INTERVAL. SO IT GOES FROM SLOPING DOWN, NEGATIVE DERIVATIVE TO SLOPING UP A POSITIVE DERIVATIVE. SO THE F-PRIME IS INCREASING ALONG THERE. OR ANOTHER WAY TO SAY IT, IS THAT IF F-PRIME OF X-IS POSITIVE ON THE INTERVAL, THAT'S ANOTHER WAY WE DESCRIBED IT, OR THE CURVE, WHEREVER YOU DRAW A TANGENT LINE IT LIES ABOVE, F-OF X-IS ABOVE ANY TANGENT LINE. SO HERE I'VE DRAWN TWO TANGENT LINES AND YOU SEE CURVED LINES COMPLETELY ABOVE IT, THAT'S CONCAVE UPWARDS. AND ALONG WITH THAT GOES JUST SORT OF FLIP IT UPSIDE DOWN, IS LOOKS LIKE A FROWN INSTEAD. IT'S CONCAVE DOWNWARDS ON THAT INTERVAL. THAT'S GOING TO BE THE CASE IF F-OF X-IS DECREASING GOES SO, SLOPE HERE IS POSITIVE, THEN IT TURNS INTO ZERO AND THEN NEGATIVE THAT MEANS SLOPE IS DECREASING. ANOTHER WAY TO 4 RECOGNIZE IS THAT IS IF THE SECOND DERIVATIVE IS NEGATIVE. AND THE OTHER WAY YOU CAN SEE FROM THIS SPICT THAT F-OF X-IS BELOW
ANY TAKEN LINE. ANY TANGENT LINE I DRAW, THE CURVE IS ALWAYS BLER. SO ALL OF THOSE ARE KIND OF EQUIVALENT TO EACH OTHER. AND WE ALSO ALONG THE SAME SIDE, ALONG THE SAME IDEA OF BEING CONCAVE UP AND CONCAVE DOWN. WE ASK WHERE DOES THE FUNCTION CHANGE UP AND CONCAVE DOWN. THOSE POINTS ARE INTERESTING. WHERE IT CHANGES FROM ONE TO THE OTHER. SO IF F, IF THE SECOND DERIVATIVE IS ZERO, THAT'S THIS POINT HERE, AND F-DOUBLE PRIME OF A-IS ZERO AND F-DOUBLE PRIME OF X-IS LESS THAN ZERO ON ONE SIDE OF A, AND GREATER THAN ZERO ON THE OTHER ZERO ON ONE SIDE OF A, AND GREATER THAN ZERO ON THE OTHER ZERO. SO THAT'S WHAT'S HAPPENING HERE. THERE'S, IT'S SWITCHING FROM CONCAVE UP TO CONCAVE DOWN. THOSE POINTS ARE CALLED AN INFECTION POINT. SO THERE WHERE IT SWITCH FROM A SMILE TO A FROWN OR THE OTHER WAY AROUND. SO I THINK I HAVE INDEED SUMMARIZED...
EVERYTHING FROM THE LAST LECTURE.

STUDENT: FOR THE CONCAVE UP, DOWN, WHAT DO YOU MEAN IF F-OF X IS ABOVE OR BELOW.

PROFESSOR: HERE'S THE FUNCTION F-OF X, THAT CURVE. AND HERE'S A TANGENT LINE, THE THE FUNCTION IS BELOW. BUT EVERYWHERE ELSE IT'S UNDERNEATH OR BLOASM LESS THAN, AND FOR THIS ONE NO WHERE I DRAW A TANGENT LINE THE FUNCTION F-OF X-IS ABOVE, THAN THE TANGENT LINE, ABOVE AND BELOW. OTHER QUESTIONS?

SO THESE ARE THE MOST IMPORTANT FACT ALL FROM LAST TIME. SO LET'S KEEP GOING NOW. SO WHAT ARE THE MOST IMPORTANT, I MADE A LIST HERE, THE MOST IMPORTANT FEATURES OF THE GRAPH TO UNDERSTAND HOW IT BEHAVES? OR ANOTHER WAY TO ASK THE QUESTION IS, WHAT ARE THE MOST IMPORTANT PROPERTIES OF X-OF THE FUNCTION F-OF X, SORRY, PROPERTIES OF F-OF X-THAT YOU NEED TO KNOW TO GRAPH IT. OKAY. SO LET'S, WE'VE DONE A BUNCH OF THEM BUT
LET ME, THERE ARE A FEW MORE, LET ME MAKE SURE WE HAVE THEM ALL. WHERE IT GOES UP AND WHERE IT GOES DOWN. AND ALL OF THAT.

BUT LET ME BE MORE SPECIFIC. SAY A FEW MORE. OTHER THINGS TO KNOW ARE THE X-INTERCEPTS. WHAT DOES THAT MEAN? THAT MEANS F-OF X-EQUAL ZERO, WHERE IT CROSSES THE X-AXIS. THOSE ARE IMPORTANT ONES. YOU CAN IMAGINE IN THIS FUNCTION IS YOUR PROFIT, YOU WANT TO KNOW WHERE THE PROFIT IS ZERO, THE ORIGINAL NATURAL PLACE ASK ABOUT IS WHERE THE FUNCTION CROSSES THE Y-AXIS.

SO THAT'S JUST SIMPLY F-OF ZERO. THAT'S PRETTY EASY TO FIGURE OUT. WHERE IT CROSSES THE Y-AXIS. HOW MANY Y-INTERCEPTS ARE THERE IN A FUNCTION? ONE. HOW MANY X-INTERCEPT ARE THERE? SO THERE COULD BE AS MANY AS YOU WANT. HERE THERE'S ONLY ONE. THOSE ARE SLIGHTLY DIFFERENT. SO THE NEXT IMPORTANT PROPERTY TO ASK ABOUT A FUNCTION IS DOES IT HAVE ANY ASYMPTOTES. AND THERE ARE TWO KINDS OF ASYMPTOTES. HORIZONTAL, AND VERTICAL. OKAY. YOU HAVE TO KNOW WHERE THOSE ARE TO BE ABLE TO PLOT A FUNCTION LET'S
Do an example who's graph question know VSM y-equals one over x-

which is a hyperbola. So there it has the horizontal asymptote.

at y-equals zero and it has a vertical asymptote at x-equals zero.

And so the general question you'd ask is to find out if there's an asymptote is is there a limit, you take the limit as x-

the limit as x-goes to infinity of f-of x. If there's a limit, like here, what's the limit as x-goes to infinity of that function, it's zero. Is there that limit? If so, then there is a horizontal asymptote.

And where is that horizontal asymptote, what's of horizontal line? L. So here L is zero. The function's approaching zero so, but it could be something else. So if I give you another example.

So if I do the function y-equals one over x-plus one, I just take that function and shift it up by one. So horizontal line.
AND HERE'S WHAT THIS FUNCTION LOOKS LIKE. SO NOW THE
HORIZONTAL
ASYMPTOTE IS ONE. SO IT'S, THAT'S EASY TO DECIPHER. SO
THAT'S
HOW YOU DO HORIZONTAL ASYMPTOTE. YOU ASK IS THERE A LIMIT?
WHAT ARE VERTICAL ASYMPTOTE. HOW DO YOU FIND THEM. TO
FIND A
VERTICAL ASYMPTOTE YOU ASK IS THERE A POINT WHERE YOU DIVIDE
BY
ZERO. SOMETHING NONE ZERO BY ZERO. THERE'S WHERE YOU GET
AN
ASYMPTOTE. SO HERE IN THIS FUNCTION, X-EQUAL ZERO I'M
DIVIDED BY
ZERO. THAT'S WHERE YOUR VERTICAL ASYMPTOTE IS.
(INAUDIBLE).
SO THAT'S PRETTY EASY TO SEE TOO. LET'S DO ANOTHER EXAMPLE
HERE.
FIRST
(ON BOARD). LET'S DO THAT FUNCTION WHICH IS LET'S ASK
IS THERE, SO THIS IS GOING TO BE F-OF X. SO LET ME TAKE THE
LIMIT AS X-GOES TO INFINITY OF F-OF X DOES THAT THING HAVE A
7
THINGS,
LIMIT. REMEMBER HOW WE DID THESE BEFORE. TO DO THESE
NUMERATOR BY
X-AND GET THAT AND IF I DIVIDED NUMERATOR BY X-AND GET WHICH
NOW IT'S EASY TO SEE WHAT THAT IS. IF X-GETS BIG THE

NUMERATOR

GET -- SO I HAVE A HORIZONTAL. SO THAT MEANS I HAVE A

HORIZONTAL ASYMPTOTE AT ONE. AND NOW WHAT ABOUT'S VERTICAL

ASYMPTOTE. I ASK IS THERE SOME VALUE OF X-IF I PLUG IF I

DIVIDE

BY ZERO, I DIVIDE SOMETHING \NONE\NON ZERO BY ZERO. AND

THAT'S

OBVIOUS. WHERE THE VERTICAL ASYMPTOTE.

STUDENT: TWO.

PROFESSOR: TWO. OKAY. (ON BOARD). SO THAT MEANS THAT LET

ME

DRAW A HORIZONTAL ON THE ONE. A VERTICAL LINE AT TWO. AND

SO

SOMEHOW MY FUNCTION IS GOING TO GO TO INFINITY WHEN I GET

CLOSE

IT THAT LINE AND WHEN I GET OUT HERE IT'S GOING DO GET CLOSE

to

ONE

THese TWO LINES. NOW LET ME GIVE YOU A TRICK TO PLOT THIS

FIGURE

THAT WORK HERE. SO I COULD ASK YOU NOW TO DIFFERENTIATE.

LEARN

OUT WHERE IT'S INCREASING AND DECREASING. BUT WE HAVEN'T

HOW TO DIFFERENTIATE THESE THINGS YET. IN THE NEXT CHAPTER

TO

WE'LL LEARN HOW TO DIFFERENTIATE THAT. BUT THIS ONE IS EASY

GOING TO

PLOT. SO LET ME EXPLAIN IT. SO HERE IT IS. WHAT I'M
DO IS CHANGE THIS INTO A FUNCTION WE ALREADY RECOGNIZE.

IT'S ALREADY ON THE BOARD. SO LET ME START WITH THIS. AND DO

THE DEVICE. I'M GOING TO DO LONG DEVICE. DIVIDE THE

NUMERATOR BY

THE DENOMINATOR. AND THE WAY I DO THAT IS BY WRITING IT

OVER THAT WAY. SO NOW I GET X-MINUS TWO OVER X-MINUS TWO PLUS ONE

OVER 8

X-MINUS TWO AND THAT'S ONE PLUS ONE OVER X-MINUS TWO. SO

NOW THIS IS A LITTLE EASIER TO THINK ABOUT. SO.

STUDENT: HOW GET VERTICAL ASYMPTOTE TO BE TWO.

PROFESSOR: SO WHEN I PLUG IN THE VALUE X-EQUALS TO TWO TO

THIS FUNCTION I HAVE ONE DIVIDED BY ZERO THAT MEANS THE FUNCTION

BLOWS UP. I CAN'T EVALUATE IT THERE. BUT IF X-IS REALLY REALLY

CLOSE TO TWO I'M GOING TO HAVE COMG CLOSE TO ONE DIVIDED BY REALLY

TINY NUMBER OF THAT CLOSES UP. ALL YOU HAVE TO LOOK WHERE THE

DENOMINATOR IS ZERO AND MAKE SURE THE NUMERATOR IS NOT ZERO

AND THAT'S THE VERTICAL ASYMPTOTE. DOES THAT MAKE SENSE?
DIVIDED BY

ZERO MAKE YOU GO TO INFINITY.

STUDENT: (INAUDIBLE).

PROFESSOR: HERE WAS MY STARTING POINT. AND I WANT TO DO

LONG

DIVISION. BUT I WANT TO DO IS CHEAP. SO I'M GOING TO WRITE

THE

NUMERATOR AS X-MINUS TWO PLUS ONE. AND THEN I BREAK INTO

THESE

TWO PIECES. SO X-MINUS TWO PLUS ONE. THAT PART CANCELS.

AND

I'M LEFT WITH THIS. SO I'M NOT DONE YET. I STILL HAVE TO

PLOT

THIS BUT THAT'S EASIER TO SEE WHAT'S GOING ON. SO LET ME

KEEP

GOING.

THIS FUNCTION HERE IS NOT VERY DIFFERENT FROM WHAT WE

PLOTTED THERE. IT'S NOT VERY DIFFERENT FROM ONE OVER X-.

IT'S

KINDS OF LIKE ONE OVER X. SO LET'S JUST START WITH THIS

PLOT.

WE KNOW THAT. NOW LET ME THAT I CAN FUNCTION AND CHANGE IT

JUST

A LITTLE BIT. AND ASK, WE KNOW WHAT THIS LOOKS LIKE. WHAT

IT

YOU DO IS SUBTRACT TWO FROM X-. WHAT HAPPENS TO THE

PICTURE. I
KNOW IT'S GOING TO HAVE A VERTICAL ASYMPTOTE HERE AT TWO. 

WHAT HAPPENS IF I TAKE, IF THIS IS THE PICTURE AND I JUST 

TIME I SEE X-I SUBTRACT TWO, WHAT DOES THIS PICK DO? IT 

SIDeways. SO THIS IS THE SAME PICTURE AS THIS EXCEPT IT 

SIDeways BY TWO. SO IT LOOKS LIKE THIS. ALL I HAD STOOD 

MOVE IT SIDEWAYS BY TWO. THAT DIDN'T TAKE MUCH WORK. AND 

TAKE THAT ONE, AND I ADD ONE TO IT. IT MOVES THE PICTURE UP 

ONE. OKAY. SO THAT'S AN EASIER WAY TO DRAW THE PICTURE. 

JUST, EMPHASIZE THIS AGAIN, SO SUPPOSE I HAVE ANY FUNCTION. 

HAS SOME WIGGLE WILL I LOOKING THING. SUPPOSE IT LOOKS 

THAT. AND NOW I SAY PLEASE TELL ME WHAT THE PLOT OF F-OF X- 

ONE IS. HOW DO I GET THAT PICTURE? WHAT HAPPENS TO THIS 

PICTURE? IT JUST MOVES SIDEWAYS BY ONE. SO IT WILL LOOK 

THIS. OTHERWISE IT'S THE SAME. SHIFT IT OVER BY ONE OR 

WHATEVER I SUBTRACT. SO THAT MAKES IT EASY TO PLOTS 

FUNCTIONS 

SOMETIMES. JUST SHIFT ONE. SO LET ME WRITE THAT DOWN.
SAME

GRAPH SHIFTED RIGHT BY C. WHATEVER C-IS, IT'S SHIFTED RIGHT BY C. SO WHAT, I'VE BEEN MAKING THIS LIST UP. SO LET ME FINISH THE LIST.

STUDENT: HOW DID YOU DO THAT ONE. THAT PLUS ONE RIGHT THERE ON THE RIGHT SIDE.

PROFESSOR: SO WHAT THIS SAYS IS TAKE WHATEVER VALUE YOU HAD AND ADD ONE TO IT. SO I TAKE EVERY POINT ON THIS PLOT AND ADD AND THE WHOLE THING GOES UP BY ONE. SO THAT MAKES IT EASY TO SEE.

10

SO THERE ARE THOSE THREE THINGS. WHERE DOES IT HIT THE X-AXIS. Y-AXIS. WHERE DOES IT HAVE HORIZONTAL ASYMPTOTES AND VERTICAL ASYMPTOTES. AND THE OTHER ONES I THINK I ALREADY MENTIONED BUT I'LL JUST PUT DOWN HERE FOR THE LIST. WHERE ARE THE INTERVALS WHERE F-OF X-IS INCREASING OR DECREASING, DEPENDING, EITHER WAY, THESE ARE INTERESTING INTERVALS.
Talked about how to figure that out. And we also talked about finding relative maxima or minima. So that's where the derivative is zero and either \( f'(x) < 0 \) or \( f''(x) > 0 \). I've written that down already. And then the last interesting place I almost have enough space is six, that was finding intervals where \( f \) is concave, either up or down. And if you know all those different things then you can draw the function without much extra work.

So let me, so that's the whole list. So let me now draw a function and identify all those different features.

So I'm going to draw a function here. Let's see if I can do this. Okay. So there I've drawn a function and what we're going to do now by eyeball is ask what are the interesting points? I'll give them some names. \( p_1, x_1, y_1 \). That's interesting.

Which features is that of the one I talked about? That's the \( y \)-intercept. Okay. There's an interesting point. And there's, here's some more interesting points. Those are the
x-intercepts. so that looks interesting visually. let me give some names to these things. p-two, p-three, p-four, i way i

11

draw this i hope you recognize what i meant there. p-five, p-six, p-seven, p-eight, p-nine, p-ten. okay. so let's just label all the different things that are going to go on here. so p-two, p-four. let me label those. p-two, p-four, p-eight, and p-ten goes with the x-intercepts. p-one, that was a y-intercept. let me erase this. so what else is interesting? how about relative minimum. what point to relative momentum. p-three and p-nine. what about relative max? p-six. okay. so now let me draw some lines here. there's going to be some intervals where something happens. so let's just say what happens to f-prime of x-in all these intervals. so what happens to f-prime of x here? i'm asking what's the slope of tangent line? what about f-prime of x on this side? it's slaning down so
WHAT'S F-PRIME OF X-HERE. NEGATIVE. HERE THE FUNCTION IS INCREASING. SO WHAT'S F-PRIME? POSITIVE. HERE I HAVE ANOTHER FLAT TAKEN LINE SO THE DERIVATIVE IS ZERO AGAIN. HERE THE FUNCTION IS DOES HE CREASING. SO F-PRIME IS ZERO, IS LESS ZERO AGAIN. HERE'S ANOTHER FLATS PLACE. THE DERIVATIVE IS AND OVER HERE THE FUNCTION'S INCREASING SO IT GOES BACK TO POSITIVE. SO THAT'S, JUST LOOK AT IS THAT PICTURE AND THIS IS WHAT IT TELLS YOU ABOUT F-PRIME AND F-PRIME TELLS NEW TURN DOWN UP DOWN, YOU KNOW EXACTLY WHERE THAT HAPPENED. LET'S THE SAME KINDS OF NUMBER LINE FOR F-DOUBLE PRIME. AND REALLY WAY I DRAW IT, I WAS HOPING YOU'D RECOGNIZE THERE'S TWO WHERE SOMETHING HAPPENS. OTHER. SO JUST BY EYEBALL, IT'S 12 CONCAVE UP THERE. HERE IT'S CONCAVE DOWN. AND HERE IT'S CONCAVE UP AGAIN. CONCAVE UP. SO WHAT HAPPENS AT THIS POINT? IT GOES FROM CONCAVE UP TO CONCAVE DOWN. SO WHAT HAPPENS IN THE
SECOND DERIVATIVE AT THIS POINT? IT’S ZERO. THAT’S INLECTION POINT.

AND THE SAME THING HERE IT GOES FROM CONCAVE DOWN TO CONCAVE UP.

THAT’S ANOTHER INFLATION POINT. SO INFLATION POINTS. SO

INFLATION POINTS ARE WHICH POINT? FIVE, AND P-SEVEN. P-

AND P-SEVEN. SO WHAT IS THE SECOND DERIVATIVE OVER HERE?

IT’S CONCAVE UP? POSITIVE. THE DERIVATIVE IS INCREASING.

THE SECOND DERIVATIVE IS POSITIVE. WHAT ABOUT FOR THIS INTERVAL WHERE IT’S A FROWN? I’VE ERASED IT SO YOU HAVE TO REMEMBER OR LOOK AT YOUR NOTES. HERE IT’S NEGATIVE, THE SLOPE STARTS POSITIVE, GOES TO ZERO AND GOES NEGATIVE SO THE SLOPE IS DECREASING. AND HERE, IT’S CONCAVE UP. AND TO THE SECOND DERIVATIVE IS POSITIVE. SO THAT’S WHAT HAPPENS TO THE SECOND DERIVATIVE ALONG ALL THESE LITTLE INTERVALS THERE. OKAY.

THAT’S.

STUDENT: HOW DID YOU GET THE INFLATION POINT AGAIN.

PROFESSOR: SO THE INFLATION POINT IS WHERE THE FUNCTION CHANGE FROM BEING CONCAVE UP TO CONCAVE DOWN. AND IT’S A LITTLE
HARD TO SEE BUT I'M LETTING YOU KNOW THE WAY I DRAW IT YOU MENT IT UP HERE. THERE IT'S CONCAVE DOWN AND THERE'S IT UP. SO THIS POINT IS THE INFLECTION POINT BECAUSE IT GOES FROM CONCAVE UP TO CONCAVE DOWN. THAT'S A LITTLE HARD TO SEE FROM THE WAY I DRAW IT BECAUSE MY CHALK IS WIDE BUT WHAT YOU WOULD DO IN PRACTICE IS TAKE THE FUNCTION F, COMPUTE TWO DERIVATIVES AND THEN FIGURE OUT EXACTLY WHERE IT WAS ZERO. IT WOULD TELL YOU WHERE THAT IS. DOES THAT MAKE SENSE?

STUDENT: I GET THE INFLECTION POINTS BUT I DON'T GET WHY ON THE FIRST DERIVATIVE IT'S GREATER THAN ZERO AND THE SECOND IT'S EQUAL.

PROFESSOR: YOU'RE ASKING ABOUT THIS ONE OR THIS ONE.

STUDENT: I JUST DON'T GET THE DISTINCTION.

PROFESSOR: F-PRIME, WHAT F-PRIME TELLS YOU IS THE FUNCTION GOING DOWN WHEN IT'S NEGATIVE OR IS IT GOING UP, THAT'S WHEN IT'S
GOING POSITIVE. SO HERE IT'S GOING DOWN. HERE'S IT SWITCHES TO
AND UP SO IT CHANGE SINE. IT'S GOING TO GO ZERO IN THE MIDDLE
CHANGE SINE. THAT'S WHERE FLAT. THAT'S, WHAT I, THE CURVY
NECESSARY. IS CURVED UP OR -- TWO DIFFERENT FACTS. SO
THESE FUNCTIONS OF COURSE ARE RELATED BUT THIS WOULD BE A BIG ZERO
ANY WR. SO DRAW, WHERE IS THIS EQUAL TO ZERO AND THIS EQUAL TO
02 DIFFERENT QUESTIONS. YOU HAVE TO DIFFERENTIATE AGAIN, GETS
THIS FUNCTION, SET IT EQUAL TO ZERO AND SOLVE. AND THAT AND
THAT AND ACTUALLY FIND IT. SO DOING IT SO WE HAVE'S CONNECTION
BETWEEN THE ALGEBRA AND THE PICTURE. ALGEBRA AND GEOMETRY.
LET ME KEEP DOING A FEW MORE EXAMPLES. LET'S TAKE AN
SAME EXAMPLE WE'VE DONE MANY TIME. Y-EQUALS X-AND MAKE THAT
KINDS OF PICTURE. SO --
STUDENT: HOW WOULD YOU DETERMINE ASYMPTOTES FOR THAT.
PROFESSOR: THAT GUY UP THERE THE WAY I DRAW IT DOESN'T HAVE
ANY ASYMPTOTES. IT'S, AND YEAH, SO THE ANSWER TO THIS ONE IS
THERE ARE NO ASYMPTOTES. THE WAY I DREW DID IT GOES OFF THAT WAY.

AND GOES OFF AT ANGLE THAT WAY. SO IT NEVER IS HORIZONTAL,

NEVER BLOWS UP ANYWHERE. SO BUT MY NEXT EXAMPLE HAS ASYMPTOTES.

WE'VE SEEN THAT BEFORE. SO I'M NOT GOING TO TALK ABOUT

ASYMPTOTES AGAIN BUT WHAT I WANT IT TALK ABOUT IS F-PRIME

AND F-DOUBLE PRIME. IS SO IF THAT'S THE FUNCTION WHAT'S THE

DERIVATIVE.

STUDENT: NEGATIVE ONE OVER (ON BOARD). AND Y-DOUBLE PRIME?

PROFESSOR: NEVER ONE TIME X-TO THE MINUS TWO SO YOU TAKE

THE MINUS TWO. SO WHAT IS IT? (ON BOARD). OKAY. SO LET'S

ASK WHEN THOSE ARE POSITIVE AND NEGATIVE. SO WHAT I WANT TO DO

IS TAKE THE FUNCTION NEGATIVE ONE OVER X-SQUARED, AND I WANT TO

KNOW WHEN IS THAT POSITIVE AND WHEN IS THAT NEGATIVE? WHICH X-IS

F-PRIME WHICH IS NEGATIVE ONE OVER X-SQUARED, WHEN IS THAT

POSITIVE. CAN YOU FIND ANY X-FOR WHICH THAT'S POSITIVE? NO

BECAUSE IT'S NEGATIVE ONE DIVIDED BY A POSITIVE NUMBER. SO

THAT'S NEVER GREATER THAN ZERO. IT'S ALWAYS NEGATIVE,

RIGHT?

ALWAYS LESS THAN ZERO BECAUSE IT'S NEGATIVE ONE DIVIDED BY A
POSITIVE NUMBER. AND IF YOU LOOK AT IN THIS FUNCTION IS

DECREASING, ISN'T IT? WHENEVER THERE'S A DRIFT IT'S GOING

BOTH SIDES OF ASYMPTOTES IT'S GOING DOWN. SO F-OF X, SO

THE DERIVATIVE IS ALWAYS NEGATIVE, IT'S ALWAYS DECREASING ON

SIDE OF ASYMPTOTE. SO THIS IS A LITTLE BIT FUNNY BECAUSE IT

LOOKS LIKE WHEN I GO FROM THERE TO THERE I'VE INCREASED.

TO HOOK AT THIS SIDE ALL BY ITSELF, IT'S DECREASING. THIS

ALL BY ITSELF, IT'S DECREASE. SO YOU CAN'T GO ACROSS

AND ASK THAT QUESTION. SO THAT MAKES SENSE. SO WHAT

LET'S ASK FOR INFLECTION POINTS. FOR WHICH X-IS F-DOUBLE

OF X, LET'S SAY LESS THAN ZERO. THAT'S WHEN IT WOULD BE

RIGHT, OKAY, SO HERE ARE THE TWO QUESTIONS. WHAT IS IT

DOWN? AND WHEN IS IT CONCAVE UP? SO WHAT I WANT TO KNOW IS

IS TWO OVER X-CUBED LESS THAN ZERO? SO FOR WHICH X-IS TWO
X-CUBED LESS THAN ZERO? WHEN X-IS -- SO WHEN X-IS POSITIVE, X-CUBED IS POSITIVE. AND SO TWO OVER X-CUBED IS POSITIVE. THAT DOESN'T WORK. WHAT HAPPENS WHEN X-IS NEGATIVE? X-CUBED IS NEGATIVE. AND SO THIS IS WHEN X-IS NEGATIVE. THAT'S EXACTLY WHEN THE SECOND DERIVATIVE IS NEGATIVE. HOW ABOUT THIS GUY? WE'RE ASKING WHICH IS TWO OVER X-CUBED POSITIVE? THAT'S EXACTLY WHEN WHICH IS IS POSITIVE. THAT SAYS YOU'RE GOING TO GO CONCAVE DOWN OVER HERE. AND THAT'S EXACTLY WHAT THE FUNCTION LOOK LIKE. IT'S CONCAVE DOWN. BECAUSE THE SECOND DERIVATIVE IS NEGATIVE. ON THIS SIDE, YOU'RE CONCAVE UP BECAUSE THE SECOND DERIVATIVE IS POSITIVE. IT'S CONCAVE UP. SO OWN EITHER SIDE OF ASYMPTOTE IT'S DIFFERENT, THAT'S OKAY. THIS IS THE SAME RULES BUT NOW APPLIED TO ASS TOTED. STUDENT: ARE GENERIC (INAUDIBLE). CUBIC LIKE X-CUBED (INAUDIBLE). PROFESSOR: I WAS JUST ABOUT TO DO A CUBIC. BUT LET ME DRAW A PICK IT MAKE SURE I'M GOING TO DO THE RIGHT QUESTION. YOU
THE FUNCTION \( y = x^3 \). SO THAT LOOK LIKE THIS.

AND AT WHICH POINT \( x = 0, 0 \). SO (ON BOARD). AT THIS POINT, \( y = 0, y' = 0 \). THE TANGENT IS HORIZONTAL. AND \( y'' = 0 \). IT'S INLEKX POINT. HERE IT'S CONCAVE DOWN AND THERE'S IT'S CONCAVE UP. BUT IT'S NOT A RELATIVE MAXIMUM OR RELATIVE MINIMUM IS IT? YOU DON'T HAVE TO HAVE EITHER. SO LET ME, YOU'VE MOTIVATED SOMETHING HE WAS ABOUT TO DEFINE WHICH IS A POINT \( x \)-WHERE \( f' \) OF \( x \) IS ZERO, AND THAT'S ALL WE KNOW ABOUT IT IS CALL A CRITICAL POINT.

JUST A WORD FOR IT. AND SO THAT'S A CRITICAL POINT. IT ISN'T A RELATIVE MINIMUM OR RELATIVE MAX. THAT'S A CRITICAL POINT.

A RELATIVE MINIMUM. AND THERE'S ONE. SO IF ALL YOU CARE ABOUT IS THE DERIVATIVE IS ZERO, THAT'S CALLED A CRITICAL ONE.

THAT'S WHAT YOU WERE ASKING. THAT'S A DEFINITION THAT I WAS GOING TO GET TO IN A MOMENT ANYWAY.
STUDENT: So that's how define relative minimums and max.

PROFESSOR: What I set last time if you want it find minimums and matches you have to find all the the critical point.

And then ask are they maxima or minima or maybe neither.

The way you did that, unless I've erased it, you look at the second derivative. And here, at this point, this is x. So f-prime of x is zero, what's f-double prime here? If the function is curve up like this, relative minimum. So that's point. So it's a relative minimum if f-double prime is greater than zero here's another point where f-prime of x equals 17.

Where flat horizontal taken line. But here f-double prime of x is greater than, sorry less than zero. And over here, I have f-prime of x is zero. But f-double prime of x is also zero. So it's neither. It's neither relative or relative max.

STUDENT: (inaudible).

PROFESSOR: Point much inflection you ask is second derivative.
ZERO. THIS ONLY ASKS IS THE FIRST DERIVATIVE ZERO AND THEY DON'T REALLY HAVE ANYTHING TO DO WITH ONE ANOTHER. HERE, HAPPEN TO BE THE SAME. HERE F-PRIME AND F-DOUBLE PRIME HAPPEN TO BE ZERO AT THE SAME PLACE. CEMENT WILL IT'S A CRITICAL POINT --

STUDENT: SO I UNDERSTAND THAT TO FIND RELATIVE MAX SET EQUAL TO ZERO BUT DO YOU HAVE TO CONSIDER THE SECOND DERIVATIVE OR CAN YOU JUST USE --

PROFESSOR: YOU DON'T HAVE TO. SO HERE YOU'VE SET F-PRIME ZERO AND FIND THAT POINT. IF F-PRIME IS LESS THAN ZERO ON ONE SIDE AND BIGGER ON THE OTHER THEN YOU KNOW IT'S GOT TO GO DOWN AND BACK UP AND THAT MEANS IT'S A MINIMUM. SO YOU DON'T ACTUALLY NEED THE SECOND DERIVATIVE. IN FACT, WELL, I WILL HAVE EXAMPLES WHERE THE -- LET ME WAIT AND GIVE YOU AN EXAMPLE WHERE YOU CAN HAVE A RELATIVELY MINIMUM BUT YOU REALLY HAVE TO LOOK AT T-PRIME. F-DOUBLE PRIME DOESN'T TELL YOU THAT. LET ME GET THERE.

SO THIS IS AN EXAMPLE THAT I MAY HAVE DONE A SLIGHTLY
DIFFERENT VERSION FROM LAST TIME. BUT LET ME DO IT AGAIN.

HERE'S A CUBIC. HERE'S F-OF X. F-PRIME OF X-IS X-SQUARED
MINUS THREE X-PLUS TWO. AND F-DOUBLE PRIME IS TWO X-MINUS THREE.
OKAY. SO I'VE WRITTEN THIS SO IT'S EASY TO IF FIGURE OUT
WHAT THE CRITICAL POINT ARE WHERE F-PRIME IS ZERO. SO THIS FACT
ZERO. X-MINUS ONE, X-MINUS TWO SO YOU CAN SEE EASILY WHERE IS IT
OR AT X-MINUS ONE -- ALSO EASY TO SEE WHETHER THIS IS POSITIVE
LETF ME NEGATIVE. WHEN EITHER SIDE OF X-EQUALS THREE HALVES. SO
DRAW EVERYTHING I KNOW ABOUT THIS FUNCTION. SO ONE, TWO AND
SO THERE'S X-EQUALS ONE, X-EQUALS TWO, THIS IS A CRITICAL
CRIRL APPOINTMENT. I KNOW THE FUNCTION IS FLAT. AND THERE'S A
IT'S CRITICAL POINT. I THINK THE FUNCTION IS FLAT. F-PRIME,
OR ZERO, IT'S ZERO, WHAT ABOUT ON THIS SIDE, IS F-PRIME BIGGER
EQUALS LESS THAN ZERO? SO WHEN I TAKE X-TO BE, LET ME PLUG IN X-
ZERO, THEN F-PRIME OF ZERO IS TWO. OVER HERE ON THIS SIDE
F-prime is greater than zero, that means the function is increasing: so this is a graph of a parabola. So I really only have to understand where it's zero and that moons it's going to be positive here and negative in the middle. Because it's a parabola. This function is a parabola. And so that's what the derivative's doing. Here it's increasing and it's flats decreasing and increasing again. That's what I know from that.

And let's me now also draw a little line. Say what happens with f-double prime? So I'm going to find the inflection points. The inflection point is when this equals zero. And that's when x-equals three halves. Right here in the middle. Make these dotted lines. So right here at three halves, I have f-prime is zero. And when x-is less than three halves, is positive or negative? x-is smaller than three halves. This is going to be less than zero, it's a straight line. So it's
THAN ZERO THERE AND GREATER THAN ZERO THERE. SO THERE'S AN
INFLECTION POINTS. AND IS THE FUNCTION CONCAVE UP OR
CONCAVE DOWN ON THIS SIDE? CONCAVE DOWN. AND ON THIS SIDE IT'S
UP. OKAY. SO I KNOW A LOT ABOUT THE FUNCTION NOW. I CAN
ALMOST PLOT IT. I REALLY ONLY NEED TO EVALUATE THIS FUNCTION AT
TWO PLACES. I'M GOING TO ASK WHAT IS ITS VALUE HERE AND WHAT IS
VALUE THERE AND EVERYTHING ELSE I'LL JUST FILL IN WITH THIS
INFORMATION. SO ALL I REALLY WANT TO NO WHAT IS F-ONE AND
F-TWO. SO I JUST PLUG IT IN. F-OF ONE, YOU DO THE A RIDGES MA
\TICK\TIC. IT'S MINUS 16TH. F-MUCH TWO YOU JUST PLUG IN
YOU GET MINUS ONE THIRD. SO THERE'S MINUS ONE, SIXTH. AND
THERE'S MINE ONE THIRD. SO INCREASING ON THIS SIDE, IT GOES TO THAT
POINT. DECREASING IN HERE. GOES THROUGH THAT POINT. AND
THE INCREASING OVER HERE. THAT'S WHAT THE F-PRIME TELLS YOU.
AND F-DOUBLE PRIME TELLS YOU IN THIS RANGE IT'S CONCAVE DOWN.
THIS TELLS YOU ON THIS RANGE IT'S CONCAVE UP. SO AT THIS
POINT YOU SHOULD BE ABLE IT DRAW IT IN YOUR HEAD. SO IT'S GOING
INCREASE, AND THEN DECREASE. CONCAVE DOWN THE WHOLE TIME.

AND

HERE IT'S GOING TO BE DECREASE, HIT THIS POINT AND INCREASE

AND

BEING CONCAVE UP. SO THERE'S THE FUNCTION. AND I ONLY HAD

TO

PLOT AT TWO PHOTON FILL IN THE REST. IT HAS TO LOOK LOOK

THAT.

20

YOU HAVE NO CHOICE. ALL I USED WAS WHERE IT WAS INCREASING,

WHERE IT WAS DECREASING AND WHERE IT WAS INCREASING. AND

WHERE

IT WAS CONCAVE DOWN AND WHERE IT WAS CONCAVE UP. SO ANY

QUESTIONS ABOUT THAT EXAMPLE? IS.

STUDENT: HOW DID YOU FINE THE POINT OF INFLECTION.

PROFESSOR: I SOLVE FOR F-DOUBLE PRIME IS ZERO AND X-EQUALS

THREE

HALVES.

STUDENT: ALSO GET THE Y-INTERCEPT.

PROFESSOR: OKAY. SO IN THAT CASE THAT WOULD MAKE THE PLOT

EVEN

MORE ACCURATE. SO WHAT IS F-OF ZERO? MINUS ONE. SO WE

KNOW

THIS POINT HERE IS MINUS ONE. NOW WHAT'S HARD IS TO FIGURE

OUT

THAT POINT. BECAUSE THAT SOLVED THE CUBIC. FIND THE ROOTS
OF A CUBIC. AND THERE'S A FORMULA FOR THAT. IT'S GENERALLY YOU DON'T DO IT BY THE FORMULA. YOU DO IT WITH SOME NUMERICAL. YOU ASK THE COMPUTER TO DO THAT. I DIDN'T DO THAT. ACTUALLY SO LET ME MAKE A COMMENT ON THAT. DOES ANYBODY HAVE AN H-P-CALCULATOR? YOU, IF YOU HAVE A CALCULATOR I GUESS IT'S PROBABLY T-I. THANK YOU BUY AN H-P-CALCULATOR, THERE'S A LITTLE BUTTON IN THE BOTTOM OF IT THAT SAYS SOLVE. IT WOULD SOLVE THIS PROBLEM FOR YOU. YOU WOULD TYPE IN, PROGRAM IN THIS FUNCTION. AND HIT SOLVE. THIS BUTTON WHICH WAS ON MOST H-P-CALCULATORS WAS INVENTED AT BERKELEY. BY PROFESSOR W-CAN K-A-H A N. HE WAS'S CONSULTANT FOR H-P-AND HE BUILT THAT BUTTON. NOT THAT PLASTIC BUTTON BUT WHAT HAPPENS WHEN YOU PUSH IT.

OKAY. LET ME JUST THINK HERE.

STUDENT: I HAVE A QUESTION. I'M SORRY. I'M LOST. WHERE YOU
YOU GETTING THE ZEROS. HOW DO YOU KNOW WHICH ONE IS LESS THAN ZERO AND EQUAL TO ZERO BASE THE ON THAT QUESTION.

STUDENT: LOOKING AT THE FUNCTION F-PRIME. THAT TELLS ME IT'S GOING UP OR DO YOU THINK.

A IF IT'S POSITIVE IT'S GOING UP. SO HERE'S THE FUNCTION.

AND I PLAYED IT NICE AND SIMPLE SO YOU CAN FACTOR IS IT. SO ASKING WHEN IS THIS FUNCTION POSITIVE, NEGATIVE AND ZERO.

IT'S EASE HE'S TO SEE WHETHER IT'S ZERO, DO. ALL I HAVE TO DO IS FILL IN THIS BETWEEN. IF X-IS BIGGER TWO, IT'S POSITIVE TIME POSITIVE. THAT TELLS ME I'M POSITIVE OUT THERE. IF X-IS LESS THAN ONE, IT'S NEGATIVE TIMES NEGATIVE. THAT'S POSITIVE.

SO OVER HERE I'M POSITIVE. BUT IF X-IS BETWEEN ONE AND TWO, IT'S POSITIVE TIMES NEGATIVE, AND SO I'M NEGATIVE HERE AND IT'S DECREASING. THAT'S ANOTHER WAY. THE OTHER WAY YOU CAN SAY THIS IS A QUADRATIC AND ALL PLOTS OF QUADRATIC LOOK THE SAME.

THEY'RE PARABOLAS. SO LONG IT'S A PLUS SIGN IT GOES UP. AND SO IT LOOKS LIKE THIS AT ONE, AND TWO, SO IT'S GOING TO BE NEGATIVE IN BETWEEN, AND POSITIVE ON EITHER SIDE. THAT'S THE OTHER WAY
YOU CAN THINK OF THAT. BECAUSE ALL PARABOLAS LOOK ALIKE.

SO LET ME SAY A LITTLE BIT MORE ABOUT FINDING, DECIDING ABOUT RELATIVE MINIMAS AND MAXIMA. SOMEBODY HAD ASKED ABOUT THAT. HOW TO TELL IF A CRITICAL POINT, SO IF $f$'S PRIME OF $x$ IS EQUAL TO ZERO, IS A RELATIVE MINIMUM, A RELATIVE MAXIMUM, OR POSSIBLY NEITHER. YOU'VE SEEN ALL THREE POSSIBILITIES THERE ON THE BOARD RIGHT NEXT TO IT. SO LET ME GIVE YOU SOME WAYS, I'M GOING TO SUMMARIZE WHAT I SAID BEFORE. SO IT'S A RELATIVE MAX IF, LET ME DO IT THIS WAY, RELATIVE MAX AT A PARTICULAR POINT, $a$, $x$-EQUALS $a$ IF THE FOLLOWING HOLD. WELL FOR ONE THING IT'S A CRITICAL POINT. AND THE OTHER THING YOU CAN SAY IS $f$-OF $x$ IS INCREASING, THESE ARE DIFFERENT WAYS OF SAYING THE SAME THING, BUT WITH A LITTLE DIFFERENCES I CAN TELL YOU THAT, INCREASING ON ONE SIDE, ON THE LEFT, AND DECREASING ON THE OTHER SIDE. THAT'S ONE WAY TO SAY IT. THE OTHER WAY TO SAY IT IS $f$-PRIME OF $x$-
GREATER THAN ZERO ON THE LEFT AND F-PRIME OF X-IS LESS THAN
AND THE RIGHT. A-IS EVERYWHERE. THOSE ARE JUST SYNONOMOUS.

OTHER WAY TO SAY IT IS THAT F-PRIME OF X, EXCUSE ME, IS THAT
CONCAVE DOWNWARDS AT X-EQUALS A, SO CONCAVE DOWNWARDS AT X-
A, ANOTHER WAY TO SAY IT. AND THE OTHER WAY TO SAY IT IS

THE SECOND DERIVATIVE IS THAT F-DOUBLE PRIME OF A-IS

THAT'S ANOTHER WAY TO, WE TALKED ABOUT HOW DO YOU RECOGNIZE
IT'S CONCAVE DOWNWARDS. NOW LET ME ALSO SAY, IT'S ALMOST

THE SAME, HOW ABOUT RECOGNIZING IF YOU HAVE A RELATIVE MINIMUM
AT X-EQUAL TO A-AGAIN IT'S GOT TO BE A CRITICAL POINT. FLAT
TAKEN LINE THERE. AND THESE RULE ARE REVERSES, IT'S DECREASING ON

THE LEFT AND INCREASING ON THE RIGHT OR EQUIVALENTLY ALMOST F-
PRIME OF X-IS LESS THAN ZERO ON THE LEFT AND F-PRIME OF X-IS

ON THE RIGHT OR F-IS CONCAVE UPWARDS, SO IT LOOK LIKE A

SMILE, OR F-DOUBLE PRIME OF A-IS GREATER THAN ZERO. THOSE ARE, THOSE,
THESE ARE FOUR SIMILAR BUT NOT QUITE THE SAME WAYS OF SAYING
MINIMUM MA. I WANT, WHY AM I TELLING YOU FOUR DIFFERENT
WAYS TO
SAY THE SAME THING? BECAUSE THEY'RE NOT QUITE THE SAME
THING.
I WANT TO GIVE YOU AN EXAMPLE WHERE ONE RULE WORKS AND THE
OTHER
NOT.
HERE'S AN EXAMPLE WHERE RULE ONE WORKS BUT FOUR DOES
NOT.
FOUR DOESN'T TELL YOU ANYTHING. OKAY. LET'S LOOK AT THE
FUNCTION. F-OF X-EQUALS X-TO THE FOURTH. AND YOU HAVE A
HOMEWORK ASSIGNMENT THAT'S SORT OF GENERALIZATION OF THAT
QUESTION. PLEASE PAY ATTENTION. SO WHAT DOES THIS
FUNCTION
LOOK LIKE? IT'S X-SQUARED SQUARED, LOOK AT IT THAT WAY. SO
IT'S
LIKE A BIT OF A STEEPER PARABOLA. SORT OF LOOKS LIKE THAT.
IT
STILL GOES THROUGH THE POINTS ONE, ONE, ONE MINUS ONE, ONE,
KINDS OF LOOKS LIKE THAT. SO F-PRIME OF X-IS FOUR X-CUBED,
F-DOUBLE PRIME OF X-IS WHAT? TWELVE X-SQUARED. SO WHERE IS
THE
CRITICAL POINT? IT'S AT ZERO. PLUG IN ZERO, HERE THE
ZERO.
POINT, RIGHT AT THE ORIGIN. BUT F-DOUBLE PRIME IS ALSO
Plug zero in there and the function is, stays at zero. So you can't tell by looking at the second derivative. But obviously this is a minimum. Over here on this side, f-prime of x, four x-cubed, that's bigger than zero. On this side it's increasing. And here f-prime of x is less than zero. So decreasing to increasing. Rule one tells you or rule to either one, rule one or rule, sorry. Rule one or rule two tells you this is a minima. Goes from decreasing to increasing. But you can't tell by looking at the second derivative it's zero.

 Doesn't have any information in it. You have to use rule one or two but four doesn't tell you anything.

 So let me say something similar here about inflection points. What was inflection point at x equals a if the second derivative was zero but in particular, the way we said it in words was that f-changed from concave up to concave down or well from down to up. Either way, it does to change. That
WHAT INFLECTION POINT WAS. SO LET'S WRITE DOWN THE RULES IN SORT OF SAME WAY. SO THE SECONDS DERIVATIVE HAS TO BE A. AND EITHER F-DOUBLE PRIME OF X-IS LESS THAN ZERO IF X-IS LESS THAN A. AND F-DOUBLE PRIME OF X-IS GREATER THAN ZERO IF X-IS GREATER THAN. THAT'S ONE WAY TO DO IT. OR YOU CAN DO IT THE OTHER WAY. (ON BOARD). THAT'S JUST IN LANGUAGE WHAT THIS MEANS. I HAVEN'T SAID ANYTHING DIFFERENT. BUT NOW I WANT TO SAY YOU ANOTHER WAYS TO RECOGNIZE IT BY LOOKING AT ONE FUNCTION. SO HERE, I DISTINGUISHED, I LOOKED AT THE NEXT DERIVATIVE, SO HERE'S, LET ME WRITE DOWN, F-TRIPLE PRIME IS NOT ZERO. SO THE THIRD DERIVATIVE IS NOT ZERO. GUARANTEES THAT YOU HAVE INFLICTION POINTS. SO WHY IS THAT? SO LET'S SUPPOSE F-PRIME GOES FROM NEGATIVE TO ZERO TO POSITIVE. SO IF I PLOT IT, IF I PLOT F-DOUBLE PRIME OF X-IT GOES THROUGH ZERO AND IT GOES FROM NEGATIVE TO ZERO TO POSITIVE. THAT'S THE PICTURE OF F-DOUBLE PRIME. HOW DO I RECOGNIZE THAT? THAT IT'S GOING
FROM NEGATIVE TO ZERO TO POSITIVE? I CAN LOOK AT THE TANGENT LINE OF THAT CURVE. THE TAKEN LINE TO F-DOUBLE PRIME, WHAT IS THE SLOPE 25 TANGENT LINE IF IT GOES FROM NEGATIVE TO POSITIVE? IT'S SLOPING UP. SO IT'S POSITIVE. WHAT'S THE SLOPE OF TANGENT LINE? IF THE CURVE IS THE CUB OF F-DOUBLE PRIME, WHAT THE THE SLOPE OF TANGENT LINE? DOUBLE PRIME. IT'S F-TRIPLE PRIME. YOU HAVE TO DIFFERENTIATE THIS FUNCTION. THAT'S ONE MORE DERIVATIVE. THAT'S WHY I NOW HAVE THREE INSTEAD OF TWO. THAT TELLS ME THE FUNCTION IS INCREASING. BUT I CAN ALSO HAVE IT, I CAN ALSO HAVE F DOUBLE PRIME LOOK LIKE THIS. IT GOES FROM POSITIVE TO ZERO TO NEGATIVE. I RECOGNIZE THAT BECAUSE THE TANGENT LINE SLOPES DOWN AND THAT'S THE CASE F-TRIPLE PRIME IS LESS THAN ZERO. THIS IS THE CASE F PRIME PULL PRIME IS GREATER THAN ZERO. SO EITHER WAY
AS LONG AS IT'S POSITIVE OUR NEGATIVE IT'S GOING TO CHANGE SIGN.

AND SO THAT’S WHAT THIS RULE IS. SO THAT’S ANOTHER WAY.

JUST HAVE TO COMPUTE ONE MORE DERIVATIVE AND LOOK AT IT AS ONE VALUE.

JUST MAKE SURE IT'S NOT ZERO. AND THEN YOU HAVE AN INFLECTION POINTS. LET ME JUST DO AN EXAMPLE.

IT’S ONE WE’VE HAD BEFORE. F-OF X-EQUALS X-CUBED. F-
PRIME OF X-IS THREE X-SQUARED. LET ME DRAW A FAMILIAR ONE. THERE IT IS. F-DOUBLE PRIME OF X-EQUALS SIX X-AND SO THE CANDIDATE IS AN INFLECTION POINT IS WHEN F-DOUBLE PRIME IS ZERO. SO WHEN THIS FUNCTION ZERO? SO THERE’S THE CANDIDATE FOR THE INFLECTION POINTS. AND WHAT I NEED TO DO IS LOOK AT THE THIRD DERIVATIVE. AND THE THIRD DERIVATIVE IS SIX WHICH IS CERTAINLY NON ZERO THAT GUARANTEES THAT IS AN INFLECTION POINTS BECAUSE IT GOES CONCAVE DOWN TO CONCAVE UP. SO THAT’S AN EXAMPLE USING THAT RULE.
LET ME TALK A LITTLE BIT MORE ABOUT ASYMPTOTES. LET ME LOOK
AT THIS FUNCTION. AND I'M GOING TO MAKE A SLIGHTLY MORE
COMPlicated FUNCTION IN A SECOND. I WANT TO KNOW DOES THIS
THING HAVE ASYMPTOTES. SO THERE'S HORIZONTAL. AND I DO THAT BY
TAKE THE LIMIT AS X-GOES TO INFINITE, (ON BOARD). AND HOW ABOUT
VERTICAL? WHEN DO I DIVIDE BY ZERO? AT X-EQUALS ZERO. NO
MYSTERY. AND SO I KNOW MY FUNCTION IS GOING TO BLOW UP
HERE.
AND IT'S GOING TO GO OFF TO EITHER SIDE. BUT LET ME LOOK
AT, LET ME APPLY MY RULES IT FIGURE OUT WHAT THE RESTED OF IT LOOK
LIKE.
SO F-PRIME, THAT'S, SO WHAT IS F-PRIME? IF X-ONE OVER X-
SQUARED. MINUS TWO OVER X-CUBED. THE QUESTION IS WHEN IS THIS
POSITIVE AND WHEN IS THIS NEGATIVE? SO GOING TO BE POSITIVE IF, SO I
HAVE TO ASK THE QUESTION WHEN IS NEGATIVE TWO OVER X-CUBED? WHEN
X-IS NEGATIVE. AND THERE THAT'S NOT SO HARD AM WHEN X-IS
POSITIVE.
SO THIS TELL ME ALL THE WAY OVER HERE IN THIS ENTIRE RAINING
ON THIS SIDE OF ASYMPTOTES, F-IS DECREASE IS. AND ON THIS
IT'S INCREASING. I CAN ALSO LOOK AT THIS AND SAY ONE OVER X-SQUARED IS ALWAYS POSITIVE. THAT TELLS ME IT'S GOING TO BE VERY CLOSE TO ZERO AND ALWAYS GOING TO INCREASING BLOW UP THERE.

ALL I NEED TO KNOW. HAS HORIZONTAL ASYMPTOTE AT ZERO.

INCREASING, IT HAS A LERL ASYMPTOTE THERE SO IT'S NO CHOICE BUT TO GO LIKE THAT. ON THIS SIDE, IT'S ALWAYS DECREASING.

START UP HERE. AND IT GOES DOWN LIKE THAT. NOW, ACTUALLY, TO SIMPLIFY MATTERS, LET'S SUPPOSE I DRAWN THIS, AND I WANT TO BE LAZY AND DO NO WORK AND DRAW THIS FUNCTION. WHAT HAPPENS IF I CHANGE X-AT THE NEGATIVE X-IN THIS FUNCTION, DOES THAT CHANGE OF VALUE OF FUNCTION? NO. IT'S A MIRROR IMAGE ON EACH SIDE.

SO SINCE F-OF X-IS THE SAME AS F-OF MINUS X, IT DOESN'T CHANGE, THE GRAPH IS A MIRROR IMAGE ACROSS, IT'S A MIRROR IMAGE ACROSS X-EQUALS ZERO. THAT MAKE IT EASY TO PLOT. THAT'S THAT LET ME MAKE IT GO ONE MORE STEP. MAKE IT A LITTLE BIT MORE
INTERESTING.

LET ME NOW TRY PLOTTING F-OF X-IS ONE OVER X-SQUARED

PLUS X.

SO JUST GOING TO BE SIMILAR TO THAT. SO LET ME GET THE

INFORMATION OUT THAT I INSIDE FOR DERIVATIVES. IS F-PRIME

IS

GOING TO BE, THE FIRST TERM I'VE ALREADY DONE AND SECOND ONE

I

JUST ADD ONE TO IT. AND THEN I HAVE F-DOUBLE PRIME, THE ONE

GOES

AWAY. AND I'M LEFT WITH THE SECOND DERIVATIVE THAT I HAD

BEFORE

WHICH IS, I DIDN'T BUT THE IT, I'LL PUT IT NOW THIS TIME.

SO

WHAT IS THE SECOND DERIVATIVE OF THAT? SIX OVER X-TO THE

FOURTH.

DO I THAT RIGHT? THIS IS, LET'SASK IS THERE OF ZERO ARE

THERE

ANY INFLECTION POINTS. SIX OVER X-TO THE FOURTH OF EQUAL

ZERO?

IT'S

NO. SO THIS IS NEVER ZERO. IS IT POSITIVE OR NEGATIVE?

ALWAYS POSITIVE. SO WHAT CAN I SAY ABOUT MY FUNCTION, IT'S

GLAD

ALWAYS CONCAVE UP. SO IT'S, WHEREVER IT'S CONCAVE UP. I'M

WE ALL REMEMBER THAT NOW. YES. SO LET, LET'SASK ABOUT

ASYMPTOTES.

SO I HAVE TWO FUNCTIONS HERE. AND ADDING THEM.

THERE'S A
FUNCTION I JUST PLOTTED. AND THEN THERE'S X-WHICH IS PRETTY EASY

THING TO PLOT. SO WHEN I ADD THOSE TWO FUNCTIONS TOGETHER WE CAN DO IT BY EYEBALL. JUST THINK ABOUT IT. SO LET'S SUPPOSE

X-IS VERY LARGE. WE JUST DECIDED THIS THING HAS ASYMPTOTES AM THAT MEANS ONE OVER X-SQUARED IS SMALL. SO WHAT CAN I SAY ABOUT ONE, AND IT ALSO MEANS THAT X-IS LARGE. SO WHAT IS THIS VERY CLOSE TO? X. OKAY. NOW WHAT IF X-IS SMALL? THAT IMPLIES ONE OVER X-SQUARED IS LARGE. SO WHAT IS THAT APPROXIMATELY EQUAL TO? ADDING SOMETHING LARGE IT SOMETHING SMALL. I CAN'T CARE ABOUT THE SMALL STUFF, RIGHT? SO THAT'S ABOUT ONE OVER X-SQUARED THAT TELL ME WHEN X-IS SMALL, THAT MEANS MERE HERE, MY PLOT IS GOING TO BASICALLY LOOK LIKE THIS. SO THIS RANGE, THAT'S WHAT THE FUNCTIONS GOING TO LOOK LIKE THERE. THAT'S WHEN X-IS SMALL. BUT WHEN X-LARGE, THE FUNCTION IS GOING TO LOOK LIKE THIS.
TO LOOK LIKE THAT. SO NEAR HERE I KNOW WHAT IT LOOK LIKE.

HERE I KNOW WHAT IT LOOK LIKE. I JUST HAVE IT FILL IN THIS BETWEEN. SO I HAVE TO SAY WHEN NEITHER ONE IS BIG OR SMALL. WHAT HAPPENS IN THERE. BUT I WANT THE END POINTS. OKAY.

SO LET'S GO LOOK AT THIS AND ASK WHEN IS THIS INCREASING OR DECREASING? WELL, LET'S SOLVE IT TO BE ZERO. LET'S LOOK FOR CRITICAL POINT. SO WHEN IS THIS EQUAL TO ZERO? IT'S WHEN EQUAL TWO OVER X-CUBED, WHO CAN SOLVE THAT FOR X? MULTIPLY THROUGH BY X-CUBED, WHAT IS X? I GET X CUBED EQUALS TWO SO X-EQUALS THE CUBED ROOT OF TWO. SO THAT TELLS ME THERE'S SOME POINT HERE, WHERE THE CUBED ROOT OF TWO, IT'S FLAT, HORIZONTAL TANGENT.

STUDENT: WHAT ABOUT THE NEGATIVE SIGN.

PROFESSOR: SO I SAID SET THIS EQUAL TO ZERO AND I MOVED OVER TO THAT SIDE. AND I GOT, IS THAT WHAT YOU'RE ASKING?

I GOT ONE EQUALS TWO OVER X-CUBED. I MULTIPLY THROUGH BY X-CUBED
X-equals two and I get, this tells me x-cubed equals two and that tells me -- (on board). So sorry I'm almost out of time here.

So let me come over here and finish, so we need to figure outs when it is the function decreasing, and when is it increasing?

These are the two thing I need it figure out. So I need to solve that negative two over x cubed plus one is less than zero.

Okay. So let me try to figure that out. That means that one is less than two over x cubed. So am I doing this right? Yes.

Okay, so that says that if x-is positive I gets that x cubed is less than two where x-less than two in that range. That's when it's decreasing. And how about when it's positive? So that's asking when is one greater than two over x-cubed. That's (on board).

Is this going too fast. I'm trying to get the algebra down. So there's two cases. If x-is negative, that means two over x-is negative. And that implies that one is bigger than its negative number. So the function is going to be increasing.
Here. It's going to be increasing over here. And in this tiny little range in there it's going to be decreasing. And so if my function look like $x$-down here and one over $x$-squared up always increasing, it look like this. That's what it has to look like. Always increasing. Over here, it decreases. Until it gets to this points. Cubed root of two and then increases again and gets closer and closer to the line $x$. So down here it's close to $x$-. Up there it's close to $x$- and there's what it look like. Okay. Sorry the algebra went too fast.