016A Homework 6 Solution

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• 2.1 *4 Which functions have the property that the slope always decreases as $x$ increases?

  Solution (a), (e). You can find these answers by finding the graphs which is convex down everywhere. For (b),(d),(c), the slope increases as $x$ increases. For (e), we have an inflection point.

• 2.1 *6 Describe the graph. (See p.150)

  Solution It has a maximum at $a \approx -0.5$ Increasing for $x < a \approx -0.5$, relative maximum at $x = a \approx -0.5$ with maximum value $\alpha \approx 5.2$. decreasing for $x > a$. Concave down for $x < 3$ and inflection point at $x = 3$, Concave up for $3 < x$. $x$-intercept at $(-3.5,0)$, $y$-intercept at $(0,5.1)$. ($y = 0$ is an asymptote? Since we don’t have an information for $8 < x$, this is not clear. You may say we have an asymptote or not. Either way should be fine.) (Every value other than $x = 3$ is an approximate)

• 2.1 *10 Describe the graph. (see. p. 150)

  Solution Increasing for all $x$. no relative maximum or minimum. Concave down for $x < 3$, inflection point at $x = 3$. Concave up for $3 < x$. $x$-intercept at $(-5,0)$ (approximate), $y$-intercept at $(0,1)$. Defined for all $x$ and no asymptote.

• 2.1 *12 Describe the graph. (see. p. 150)

  Solution Increasing for $x < a \approx -1.5$, relative maximum at $x = a \approx -1.5$ with maximum value $\approx 3.4$, decreasing for $a \approx -1.5 < x <
\[ b \approx 2, \text{ relative minimum at } x = b \approx 2 \text{ with minimum value } \approx -1.5, \]
increasing for \( b(\approx 2) < x < c \approx 5.5, \) relative maximum at \( x = c \approx 5.5 \) with maximum value \( \approx 3.3, \) decreasing for \( c(\approx 5.5) < x. \) Concave down for \( x < 0, \) inflection point at \((0,1), \) concave up for \( 0 < x < 4, \) inflection point at \((4,1), \) concave down for \( 4 < x. \) \text{ } x- \text{ intercepts are } (-2.8,0),(0.5,0),(3.5,0),(6.8,0) \text{ } y- \text{ intercept is } (0,1).

**2.1 *18** See p. 151 Figure 20.

\[ \text{Solution (a) At which labeled points is the function decreasing? A,E (b) At which labeled points is the graph concave down? Assuming the points C,E are the inflection points, the answer is D. (c) Which labeled point has the most negative slope (that is, negative and with the greatest magnitude)? At A,E, we have negative slopes. By inspecting, the point E has the most negative slope.} \]

**2.1 *28** One method of determining the level of blood flow through the brain requires the person to inhale air containing a fixed concentration of \( \text{N}_2\text{O}, \) nitrous oxide. During the first minute, the concentration of \( \text{N}_2\text{O} \) in the jugular vein grows at an increasing rate to a level of .25%. Thereafter it grows at a decreasing rate and reaches a concentration of about 4% after 10 minutes. Draw a possible graph of the concentration of \( \text{N}_2\text{O} \) in the vein as a function of time.

\[ \text{Solution Look at other file.} \]

**2.1 *30** Figure 22 gives the US electrical energy production in trillion kilowatt-hours from 1935 \((t = 0)\) to 1995 \((t = 60)\) with projections. In what year was the level of production growing at the greatest rate?

\[ \text{Solution The growing rate of the level of production is the slope of the tangent line of the given graph. The slope is increasing for } 0 < t < 40, \text{ and attains its maximum at } t = 40, \text{ which is the inflection point and is decreasing after that point. Since } t = 40 \text{ is equivalent to 1975, we conclude that the level of production was growing at the greatest rate in 1975.} \]

**2.1 *40** Suppose the function \( f(x) \) has a relative minimum at \( x = a \) and a relative maximum at \( x = b. \) Must \( f(a) \) be less than \( f(b)? \)

\[ \text{Solution No, it doesn’t have to be that way. Look at other file.} \]

**Problem 1** Suppose that \( f(x) \) is differentiable and increases when \( x < 8, \) decreases when \( 8 < x < 10, \) has an inflection point at \( x=9 \)
and increases when $x > 10$. Does $f(x)$ have any relative maxima or minima? Where?

Solution Relative maxima is where the graph changes from increasing to decreasing. So we have a relative maxima at $x = 8$. Similarly, relative minima is where the graph changes from decreasing to increasing. So we have a relative minima at $x = 10$.

- **Problem 2** Let $g(x) = f(x) + 83$, where $f(x)$ was defined in the last question. What properties does $g(x)$ have, i.e. where does it increase, decrease, have any inflection points, relative minima, and relative maxima? Justify your answers.

  Solution All the listed properties are invariant under parallel translation along $y$-axis. So, $g(x)$ increases when $x < 8$, decreases when $8 < x < 10$, has an inflection point at $x = 9$ and increases when $x > 10$. We have a relative maxima at $x = 8$ and a relative minima at $x = 10$.

- **Problem 3** Let $h(x) = -g(x)$, where $g(x)$ was defined in the last question. What properties does $h(x)$ have, i.e. where does it increase, decrease, have any inflection points, relative minima, and relative maxima? Justify your answers.

  Solution The graph of $h(x)$ is the reflection about $x$ axis of the graph of $h(x)$. So, $h(x)$ decreases when $x < 8$, increases when $8 < x < 10$, has an inflection point at $x = 9$ and decreases when $x > 10$. We have a relative minima at $x = 8$ and a relative maxima at $x = 10$.

- **Problem 4** Let $f(x) = (x - 1)^n$, where $n$ is a positive integer.

  For which positive integer values of $n$ does $f(x)$ have a critical point? Where is it?

  Solution The critical point $(a, f(a))$ is where we have $f'(a) = 0$. Since $f'(x) = n(x - 1)^{n-1}$, unless $n = 1$, we have $f'(1) = 0$. So we have a critical point $(1, 0)$ unless $n = 1$.

  For which positive integer values of $n$ does $f(x)$ have a relative extremum? Where is it? Is is a relative maximum or minimum?

  Solution The critical point above are relative extremum when $f'(x)$ changes sign. If $n$ is even, $f'(x)$ changes sign and have a relative minima. If $n$ is odd, $f'(x)$ is nonnegative. So we don’t have a relative extremum.
For which positive integer values of \( n \) does \( f(x) \) have an inflection point? Where is it?

*Solution* The inflection point is where the concavity of the graph changes. We can check it by sketching graph. If \( n \) is even, the graph is concave up. If \( n \) is odd and \( n \neq 1 \), the concavity of the graph changes at \( x = 1 \).

- **2.2 *2** Which function have a negative first derivative for all \( x \)?
  *Solution* We have to find the graphs which is decreasing for all \( x \) and have no critical points. Considering the graphs, we get (b),(c),(f).

- **2.2 *4** Which function have a negative second derivative for all \( x \)?
  *Solution* If the second derivative is negative for all \( x \), the slope is decreasing, so we have concave down. So (f).

- **2.2 *6** Which one of the graph in Fig 17 could represent a function \( f(x) \) for which \( f(a) = 0, f'(a) < 0, f''(a) > 0 \)?
  *Solution* (a) and (c) have \( f(a) = 0 \). But for (a), \( f'(a) = \). For (c), we can check \( f'(a) < 0 \) and \( f''(a) > 0 \).

- **2.2 *10** Sketch the graph with prescribed property.
  *Solution* For example, \( f(x) = \frac{3}{16}(-x^3 + 12x) + 2 \). Look at the other file.(It’s labeled as Number 2.2 30 by mistake)

- **2.2 *30** Explain why \( f(x) \) must be concave down at \( x = 2 \)
  *Solution* The graph of \( f'(x) \) is decreasing. It means the slope of \( f(x) \) is decreasing, which means \( f(x) \) is concave down.

- **2.2 *36** If \( f(0) = 3 \), what is the equation of the tangent line to the graph of \( y = f(x) \) at \( x = 0 \)?
  *Solution* If you read the graph, you can find that \( f'(0) = 1 \). So the equation of the tangent line is \( y = f'(0)(x - 0) + f(0) = x + 3 \).

- **2.2 *42** Match each observation.
  *Solution* (a) Since we have \( f'(3) = 4 \), the slope of the tangent line at \( x = 3 \) is 4 - (C)
  
  *Solution* (b) Since \( (3, 4) \) is on the graph of the \( f(x) \), the value of \( f \) at \( x = 3 \) is 4 - (D)
Solution (c) The point (3, 4) is on the graph of \( f''(x) \). So \( f''(3) = 4 > 0 \), which means that \( f'(x) \) is increasing near \( x = 3 \), in other words, concave up. - (B)

Solution (d) The point (3, 4) is on the graph of \( f''(x) \) and also \( f'(3) = 0 \). So the graph of \( f(x) \) is concave up at \( x = 3 \) and has a horizontal tangent line. So it has a relative minima. - (B),(A)

Solution (e) The point (3, -4) is on the graph of \( f''(x) \) and also \( f'(3) = 0 \). So it is concave down and has a horizontal tangent line. So it has a relative maxima. - (B),(E)