1. (a) $T$, (b) $T$, (c) $T$, (d) $F$, (e) $F$.

2. (a) $f'(x) = \frac{xe^{2x^2}y - e^{2x^2}}{x^2} = \frac{e^{2x^2}(4x^2 - 1)}{x^2} = \frac{e^{2x^2}(2x+1)(2x-1)}{x^2}$.
   Since $e^{2x^2} > 0$, the only critical points for $f$ are $x = \pm \frac{1}{2}$.

   Next, $f'(1) = 3e^2$ and $f'(1) = e^2$, so the tangent line at $x = 1$ has equation $y - e^2 = 3e^2(x-1)$, or $y = 3e^2x - 2e^2$.

   Thus, $3e^2x - 2e^2$ is the linear fcn required.

   (b) $\frac{dy}{dx} = \frac{d}{du} (ue^{-u}) \frac{du}{dx} = (e^{-u} - ue^{-u})(-2x) = -2xe^{-u}(1-u)$
   $= -2xe^{x^2-1} (1-1+x^2) = -2xe^{x^2-1}$.

3. (a) $0 = \frac{d}{dx} 1 = xe^y \frac{dy}{dx} + e^y + 2x \frac{dy}{dx} + 2y \Rightarrow \frac{dy}{dx} = \frac{-e^y + 2y}{x(e^y + 2)}$.
   At $(1, 0)$, $\frac{dy}{dx} = -\frac{1}{2}$, so the tangent line is: $y = \frac{x}{3}(x-1)$.

   (b) Taking $\frac{dt}{dx}$ on the equation $2xy = 1 + \frac{x}{y}$, we get
   $2(xy + yx) = \frac{y^2 - x^2}{y^2}$. Thus, $2xy^2 + xy = y^2 - 2y^2$, which yields the desired expression $\frac{xy y^2}{x(2y^2 + 1)}$.

4. $f(x) = x^2(3-x)$, so $\lim_{x \to -\infty} f(x) = \infty$, $\lim_{x \to \infty} f(x) = -\infty$. Now,
   $f'(x) = 6x - 3x^2 = 3x(2-x)$, so the critical points are 0, 2. And
   $f''(x) = 6 - 6x = 6(1-x) \Rightarrow$ possible inflection point at 1.

   $\begin{array}{|c|c|c|c|c|}
   \hline
   & (-\infty, 0) & 0 & (0, 2) & 2 & (2, \infty) \\
   \hline
   f' & - & 0 & 0 & - \\
   \hline
   f & \nearrow & \text{loc, min.} & \nearrow & \text{loc, max} & \searrow \\
   \hline
   \end{array}$

   $\begin{array}{|c|c|c|}
   \hline
   & (-\infty, 1) & (1, \infty) \\
   \hline
   f'' & + & 0 & - \\
   \hline
   f & \text{CU} & \text{inflection point} & \text{CD} \\
   \hline
   \end{array}$

   $x$-intercepts: 0, 3, $y$-intercept: 0
   loc. min: $x = 0$, loc. max: $x = 2$, global max/min: none.
Suppose the dimensions are \( x, \frac{x}{3}, \) and \( y \) as shown. Then we have the constraint equation: \( x^2y = 36 \). The objective function is

\[
A = 2x^2 + 3xy = 2x^2 + 3x \frac{36}{x^2} = 2x^2 + \frac{108}{x},
\]

with \( A'(x) = 4x - \frac{108}{x^2} = \frac{4x^3 - 108}{x^2} = \frac{4(x^3 - 27)}{x^2} \). Setting \( A'(x) = 0 \), we get a unique critical point \( x = 3 \). The chart shows that \( A(x) \) has a global minimum at \( x = 3 \). For \( x = 3 \), \( y = \frac{36}{x^2} = 4 \), so the required box has dim's 3, 3, 4 (inches).