• Closed Book/Notes; no calculators.
• This Final has one True-False question (#0), ten Multiple Choice questions (#1-#10), and five Free Response questions (#11-#15). Please make sure your exam packet has all sixteen questions.
• Answer freely; there will be no penalty (i.e., no negative points) for wrong answers.

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(0) (21 points) Mark the following statements "True" or "False":
→ (a) The graph of $y = \ln \left( \frac{1}{x} \right)$ is concave up for all $x > 0$.
(b) $\ln (x^2-x) = \ln (x^2) - \ln (x)$ for all $x > 1$.
✓ (c) $\frac{d}{dx} e^{2\ln x} = 2x$.
(d) $\frac{d}{dx} g(f(x)) = f'(g(x)) g'(x)$.
(e) $y = 8 \sqrt[8]{e^x}$ satisfies the differential equation $y' = 8y$.
(f) $\int_1^8 \left( \frac{d}{dx} f(x) \right) \, dx = 7$ if $f(1) = 8$ and $f(8) = 1$.
(g) $\frac{d}{dx} \int_1^8 (x+x^2+x^3+x^4) \, dx = 0$.

Answers ("T" or "F"):

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Multiple Choice Section (100 points total)

#1 to #10 are multiple choice questions. Please circle all your answers in the chart below (for each of the ten questions). Circle no more than one answer for each question; otherwise you get 0 points.

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\[
\lim_{{h \to 0}} \frac{\sqrt{49+h} - 7}{h}
\]

is equal to:

(a) \( \frac{1}{\sqrt{h}} \), (b) 7, (c) 14, (d) \( \frac{1}{14} \), (e) n.o.t.a.

(none of the above)
Please make sure that all of your answers are entered on the chart on p. 2. Only the chart will be graded for all Multiple Choice questions.

(2) What differential equation is satisfied by the function $y = 6 - 6e^{-x/2}$:
   (a) $y' = \frac{3}{2} y$,  (b) $y' = \frac{1}{2} (6 - y)$,  (c) $y' = \frac{1}{2} (y - 6)$,
   (d) $y' = \frac{1}{2} y$,  (e) n.o.t.a.

(3) The graph of $y = x^3 - 6x^2 + 25x - 3$ has:
   (a) two extreme points and no inflection points,  
   (b) two extreme points and one inflection point,  
   (c) no extreme points and no inflection points,
   (d) no extreme points and one inflection point,
   (e) n.o.t.a.
(4) For a differentiable function $f(x)$, which of the following statements is correct?
   (a) If $a$ is a critical point for $f(x)$, then $a$ is an extreme point for $f(x)$.
   (b) At an inflection point of $f(x)$, the tangent line has to be horizontal.
   (c) Local max. points of $f(x)$ cannot be inflection points.
   (d) If $f(x)$ is always increasing, then the graph of $y = f(x)$ is concave up for all $x$.
   (e) n.o.t. a.

(5) At the point $(1, 0)$, the tangent line to the curve $x^2 + y^2 = e^y$ has equation:
   (a) $y = x - 1$,  (b) $y = \frac{1}{2}(x-1)$,  (c) $y = 2x - 1$,
   (d) $y = 2 - 2x$,  (e) n.o.t. a.
(6) \(\frac{d}{dx}[(\ln x)^3 e^{-3x^2}]\) is given by:
(a) \(e^{-3x^2}\left[\frac{3(\ln x)^2}{x} - 6x(\ln x)^3\right]\),
(b) \(-18e^{-3x^2}(\ln x)^2\),
(c) \(e^{-3x^2}\left[\frac{3}{x} - 18x\ln x\right]\),
(d) \(e^{-3x^2}\left[3(\ln x)^2 - 6x(\ln x)^3\right]\),
(e) n.o.t. a.

(7) Where is the graph of \(y = \frac{x^2 - 1}{x^2 + 1}\) increasing?
(a) nowhere, (b) everywhere, (c) \(x > 0\), (d) \(x < 0\),
(e) n.o.t. a.
(8) A company making plastic C.D. cases has marginal profit $100-2x$ (in thousands of dollars), in terms of $x$ = production level (in millions of C.D. cases). Fixed costs are 36 (also in thousands of dollars). In the same units, what is the profit when the production level is 2 (again in millions)?
(a) 96, (b) 164, (c) 196, (d) 200, (e) n.o.t.a.

(9) The finite region bounded by $y=x^2$ and $y=\sqrt{x}$ has (geometric) area:
(a) $\frac{1}{2}$, (b) $\frac{1}{3}$, (c) $\frac{1}{4}$, (d) $\frac{1}{6}$, (e) n.o.t.a.
(10) The radioactive substance Iillium decays with a half-life of 10 years. At time $t=0$, a sample of Iillium has mass 50 grams. When (in years) will there be 40 grams left?

(a) $\frac{10 \ln(5/4)}{\ln 2}$, (b) $10 \ln(5/4)$, (c) $\frac{\ln(5/4)}{10}$

(d) $\frac{10}{\ln 2} \ln(4 \ln 2)$, (e) n.o.t.a.
Free Response Section (119 points total)

(11) (a) (12 points) Compute \( \lim_{n \to \infty} (1 - \frac{2}{3n})^6n \).

(b) (12 points) The Bank Kruptcy pays interest at a certain rate per annum, compounded continuously. A banner at the bank says: “You’ll Double Your Money in Eight Years!” What is the present value of $10,000 in 10 years at this bank? Write down your answer without using logarithms or the number e. (Explain your work, but you need not compute the answer in decimals.)
(12) (24 points) Compute the following (in 4 parts):
(a) \[ \int (x + \frac{1}{x})^2 \, dx = \]

(b) \[ \int \frac{1}{(2-5t)^3} + e^{-4t} \, dt = \]

(c) \[ \frac{d}{dx} \int (e^{x^2} + \ln x) \, dx = \]

(d) \[ \int_0^4 (\sqrt{16-x^2} + \frac{1}{2x+1}) \, dx = \]
(3) (a) (12 points) Compute the equation of the tangent line to the curve \( y = (x^2+1)(e^x-3)(x^2+3x-1) \) at the point where the curve cuts the y-axis.

(b) (12 points) A helicopter is rising straight up in the air. Its velocity at time \( t \) is \( 2t+1 \) feet per second. How high does the helicopter rise during the first 6 seconds?
(14) (a) (12 points) Find the area of the finite region bounded by $y = (1-x)(x-3)$ and the x-axis.

(b) (12 points) Find the geometric area of the finite region bounded by the graphs of $y = x^2 + x$ and $y = 3 - x$. 
(15) (23 points) A wire 10 inches long is to be cut in two pieces. A circle is formed from one piece and a square from the other. How should the wire be cut in order to minimize the total area enclosed by the two figures? (Your answer should give the lengths of the two pieces.)