Two Person Games

- Mathematics
- Problem Solving
- Software Development

Billy has a used car for sale and is asking $2,000. Beth offers him $1,500. So Billy splits the difference and asks $1,750. If Billy and Beth continue in this manner, what common price will then settle on?

Features

- Two person
- Economic
- Fixed strategy
- Iterative
- Terminating?

Extensions

- If Beth wanted to pay $1600, what should her first offer have been?
- Generalize problem and solution (Billy asks $A$, Beth offers $O$)
- Program it!

General Characteristics

- Only 2 players  [Could be relaxed]
- Only thinking skills  [Not physical]
- Full previous information known at all times
- No luck  [Can be exceptions]
- Finishes in a reasonable time
- Little special equipment required

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Adapted from 'Popularizing Mathematics', edited by A J C Begg
**Why Games? Interdisciplinary**

- Sociology
- Criminal Justice
- Philosophy
- Economics
- Biology
- Evolution
- Engineering

**Why Games? Mathematics**

- How to play?
- Best way to play?
- Play to win …
- Strategy for winning ..
- Can always win if?
- What happens if ..
- Game is similar to …
- Game specification …

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**Why Games? Software**

- Easily understood rules
- Intellectually challenging & motivational
- Competitions (pencil & paper)
- Understanding, mathematical analysis, abstraction, reflection before programming
- Object oriented (reuse)
- Competitions (software, networks)

**Prisoners Dilemma**

Cooperation vs Conflict Game
Simultaneous Moves
Prisoner/Player A       Prisoner/Player B

*Four possibilities:*
- A & B both cooperate
- A & B both defect
- A cooperates & B defects
- A defects & B cooperates

**PD Punishment & Rewards**

<table>
<thead>
<tr>
<th></th>
<th>B cooperates</th>
<th>B defects</th>
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<tbody>
<tr>
<td>A cooperates</td>
<td>A gets CC, B gets CC</td>
<td>A gets CD, B gets DC</td>
</tr>
<tr>
<td>A defects</td>
<td>A gets DC, B gets CD</td>
<td>A gets DD, B gets DD</td>
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</tbody>
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DC > CC > DD > CD
CC > (DC + CD)/2

**Iterative PD - Max Rewards Strategies**

- Meanie – always defects
- Sucker – always cooperates
- Spaz – switches randomly
- Fair play – adjusts to count of actions of other player
- Tit for Tat - cooperates on the first round, every subsequent round mimics the other player's previous move
2 D  Prisoners Dilemma

- **is cooperating, did cooperate**
- **is defecting, did defect**
- **is cooperating, did defect**
- **is defecting, did cooperate**

\[ b: \text{advantage for defection when opponent cooperates} \]
\[ p: \text{fraction (0 . . . 1) of defectors in the first round} \]