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Two Person Games

• Mathematics
• Problem Solving
• Software Development
Billy has a used car for sale and is asking $2,000. Beth offers him $1,500. So Billy splits the difference and asks $1,750. If Billy and Beth continue in this manner, what common price will they settle on?
Features

• Two person
• Economic
• Fixed strategy
• Iterative
• Terminating?
Extensions

• If Beth wanted to pay $1600, what should her first offer have been?

• Generalize problem and solution (Billy asks $A$, Beth offers $O$)

• Program it!
General Characteristics

- Only 2 players  
  [Could be relaxed]
- Only thinking skills  
  [Not physical]
- Full previous information known at all times
- No luck  
  [Can be exceptions]
- Finishes in a reasonable time
- Little special equipment required
  
  – Adapted from 'Popularizing Mathematics', edited by A J C Begg
Why Games? Interdisciplinary

- Sociology
- Criminal Justice
- Philosophy
- Economics
- Biology
- Evolution
- Engineering
Why Games? Mathematics

- How to play?
- Best way to play?
- Play to win …
- Strategy for winning ..
- Can always win if?
- What happens if ..
- Game is similar to …
- Game specification …

- Understanding
- Strategy/Optimize
- Analysis/Strategy
- Generalization
- Proof
- Variations
- Isomorphism
- Symbols & Notation

Adapted from 'Popularizing Mathematics', edited by A J C Begg
Why Games? Software

• Easily understood rules
• Intellectually challenging & motivational
• Competitions (pencil & paper)
• Understanding, mathematical analysis, abstraction, reflection before programming
• Object oriented (reuse)
• Competitions (software, networks)
Prisoners Dilemma

Cooperation vs Conflict Game

Simultaneous Moves

Prisoner/Player A

Prisoner/Player B

Four possibilities:

– A & B both cooperate
– A & B both defect
– A cooperates & B defects
– A defects & B cooperates
## PD Punishment & Rewards

<table>
<thead>
<tr>
<th></th>
<th>B cooperates</th>
<th>B defects</th>
</tr>
</thead>
<tbody>
<tr>
<td>A cooperates</td>
<td>A gets CC</td>
<td>A gets CD</td>
</tr>
<tr>
<td></td>
<td>B gets CC</td>
<td>B gets DC</td>
</tr>
<tr>
<td>A defects</td>
<td>A gets DC</td>
<td>A gets DD</td>
</tr>
<tr>
<td></td>
<td>B gets CD</td>
<td>B gets DD</td>
</tr>
</tbody>
</table>

DC > CC > DD > CD  
CC > (DC + CD)/2
Iterative PD - Max Rewards Strategies

- Meanie – *always defects*
- Sucker – *always cooperates*
- Spaz – *switches randomly*
- Fair play – *adjusts to count of actions of other player*
- Tit for Tat - *cooperates on the first round, every subsequent round mimics the other player's previous move*
2D Prisoners Dilemma
<table>
<thead>
<tr>
<th>$O_1$</th>
<th>$O_2$</th>
<th>$O_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_4$</td>
<td>P</td>
<td>$O_5$</td>
</tr>
<tr>
<td>$O_6$</td>
<td>$O_7$</td>
<td></td>
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</tbody>
</table>

Player Cooperates

<table>
<thead>
<tr>
<th>Opponent Cooperates</th>
<th>Opponent Defects</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 1</td>
<td>0, $b$</td>
</tr>
<tr>
<td>$b$, 0</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

Player Defects

- is cooperating, did cooperate
- is defecting, did defect
- is cooperating, did defect
- is defecting, did cooperate

$b$: advantage for defection when opponent cooperates

$p$: fraction (0..1) of defectors in the first round