Everything You Always Wanted To Know about Game Theory* *but were afraid to ask

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What is “Game Theory”? Combinatorial / Computational / Economic

- Combinatorial
  - Sprague and Grundy’s 1939 Mathematics and Games
  - Board (table) games
  - Nim, Domineering
  - Complete info, alternating moves
  - Goal: Last move
- Computational
  - R. Bell and M. Cornelius’ 1988 Board Games around the World
  - Tic-Tac-Toe, Chess
  - Complete info, alternating moves
  - Goal: Varies
- Economic
  - von Neumann and Morgenstern’s 1944 Theory of Games and Economic Behavior
  - Matrix games
  - Prisoner’s dilemma
  - Incomplete info, simultaneous moves
  - Goal: Maximize payoff

Know Your Audience…

- How many have used games pedagogically?
- What is your own comfort level with GT?
  (hands down = none, one hand = ok; two hands = you could be teaching this session)
  - Combinatorial (Berlekamp-ish)
  - Computational (AI, Brute-force solving)
  - Economic (Prisoner’s dilemma, matrix games)

EYAWTKAGT*bwata
Here’s our schedule:

(“GT” = “Game Theory”)

- Dan: Overview, Combinatorial GT basics
- David: Combinatorial GT examples
- Dan: Computational GT
- Peter: Economic GT & Two-person games
- Dan: Summary & Where to go from here
  (All of GT in 75 min? Right!)

Why are games useful pedagogical tools?

- Vast resource of problems
  - Easy to state
  - Colorful, rich
  - Use in lecture or for projects
  - They can Use their projects when they’re done
  - Project Reuse -- just change the games every year!
  - Algorithms, User Interfaces, Artificial Intelligence, Software Engineering

“Every game ever invented by mankind, is a way of making things hard for the fun of it!”
  – John Ciardi

What is a combinatorial game?

- Two players (Left & Right) alternating turns
- No chance, such as dice or shuffled cards
- Both players have perfect information
  - No hidden information, as in Stratego & Magic
- The game is finite – it must eventually end
- There are no draws or ties
- Normal Play: Last to move wins!
Combinatorial Game Theory
The Big Picture

- Whose turn is not part of the game
- SUMS of games
  - You play games \( G_1 + G_2 + G_3 + \ldots \)
  - You decide which game is most important
  - You want the last move (in normal play)
  - Analogy: Eating with a friend, want the last bite

Classification of Games

- Impartial
  - Same moves available to each player
  - Example: Nim
- Partisan
  - The two players have different options
  - Example: Domineering

Nim: The Impartial Game pt. I

- Rules:
  - Several heaps of beans
  - On your turn, select a heap, and remove any positive number of beans from it, maybe all
- Goal
  - Take the last bean
- Example w/4 piles: (2,3,5,7)
- Who knows this game?

Nim: The Impartial Game pt. II

- Dan plays room in (2,3,5,7) Nim
- Ask yourselves:
  - Query:
    - First player win or lose?
    - Perfect strategy?
  - Feedback, theories?
- Every impartial game is equivalent to a (bogus) Nim heap

Nim: The Impartial Game pt. III

- Winning or losing?
  - Binary rep. of heaps
  - Nim Sum = XOR
  - Zero = Losing 2nd P win
- Winning move?
  - Find MSB in Nim Sum
  - Find heap w/1 in that place
  - Invert all heap’s bits from sum to make sum zero

Domineering: A partisan game

- Rules (on your turn):
  - Place a domino on the board
  - Left places them North-South
  - Right places them East-West
  - Goal
  - Place the last domino
- Example game
- Query: Who wins here?
**Combinatorial Game Theory**

**The Basics I - Game definition**

- A game, $G$, between two players, Left and Right, is defined as a pair of sets of games:
  - $G = \{ G^L | G^R \}$
  - $G^L$ is the typical Left option (i.e., a position Left can move to), similarly for Right.
  - $G^L$ need not have a unique value
  - Thus if $G = \{ a, b, c, \ldots | d, e, f, \ldots \}$, $G^L$ means $a$ or $b$ or $c$ or $d$ or $e$ or $f$ or ...

**Combinatorial Game Theory**

**The Basics II - Examples: 0**

- The simplest game, the Endgame, born day 0
  - Neither player has a move, the game is over
  - $\{ \emptyset | \emptyset \} = \emptyset$, we denote by 0 (a number!)
  - Example of $P$, previous/second-player win, losing
  - Examples from games we’ve seen:

<table>
<thead>
<tr>
<th>Nim</th>
<th>Domineering</th>
<th>Game Tree</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>

**Combinatorial Game Theory**

**The Basics II - Examples: 1**

- Another simple game, 1, born day 1
  - Left wins no matter who starts
  - $\{ 0 | 0 \} = 1$, this game is a number
  - Called a Left win. Partisan games only.
  - Examples from games we’ve seen:

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Combinatorial Game Theory
The Basics II - Examples: -1

- Similarly, a game, -1, born day 1
  - Right wins no matter who starts.
  - \{ | \} = -1, this game is a number.
- Called a Right win. Partisan games only.
- Examples from games we've seen:
  - Nim
  - Domineering
  - Game Tree

<table>
<thead>
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<tbody>
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<td>Right wins</td>
<td>Left wins</td>
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Combinatorial Game Theory
The Basics III - Outcome classes

- With normal play, every game belongs to one of four outcome classes (compared to 0):
  - Zero (=)
  - Negative (<)
  - Positive (>)
  - Fuzzy (||), incomparable, confused

<table>
<thead>
<tr>
<th>Zero</th>
<th>Negative</th>
<th>Positive</th>
<th>Fuzzy</th>
</tr>
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<tbody>
<tr>
<td>{</td>
<td>} = 0</td>
<td>{ -1</td>
<td>} = -1</td>
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<table>
<thead>
<tr>
<th>Left starts</th>
<th>Right starts</th>
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</thead>
<tbody>
<tr>
<td>and L has winning strategy</td>
<td>and R has winning strategy</td>
</tr>
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</table>

Combinatorial Game Theory
The Basics IV - Values of games

- What is the value of a fuzzy game?
  - It's neither > 0, < 0 nor = 0, but confused with 0.
  - Its place on the number scale is indeterminate.
  - Often represented as a "cloud".

- Values:
  - Zero
  - Negative
  - Positive
  - Fuzzy

Combinatorial Game Theory
The Basics V - Final thoughts

- There's much more!
  - More values
  - Up, Down, Tiny, etc.
  - How games add
  - Simplicity, Mex rule
  - Dominating options
  - Reversible moves
  - Number avoidance
  - Temperatures

- Normal form games
  - Last to move wins, no ties
  - Whose turn not in game
  - Rich mathematics
  - Key: Sums of games
  - Many (most) games are not normal form!
    - What do we do then?
    - Computational GT!

And now over to David for more Combinatorial examples…

<table>
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<th>0nd now for over to David for more</th>
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Computational Game Theory (for non-normal play games)

- Large games
  - Can theorize strategies, build AI systems to play
  - Can study endgames, smaller version of original
    - Examples: Quick Chess, 9x9 Go, 6x6 Checkers, etc.

- Small-to-medium games
  - Can have computer solve and teach us strategy
  - I wrote a system called GAMESMAN which I use in CS0 (a SIGCSE 2002 Nifty Assignment)

How do you build an AI opponent for large games?

- For each position, create Static Evaluator
- It returns a number: How much is a position better for Left?
  - (+ = good, − = bad)
- Run MINIMAX (or alpha-beta, or A*, or …) to find best move

Use of games in projects (CS0)

Language: Scheme & C

- Every semester...
  - New games chosen
  - Students choose their own graphics & rules (i.e., open-ended)
  - Final Presentation, best project chosen, prizes
- Demonstrated at SIGCSE 2002 Nifty Assignments
And now over to Peter…

- Two player games
- More motivation
- Prisoner’s Dilemma

Summary

- Games are wonderful pedagogic tools
  - Rich, colorful, easy to state problems
  - Useful in lecture or for homework / projects
  - Can demonstrate so many CS concepts
- We’ve tried to give broad theoretical foundations & provided some nuggets…

Resources

- www.cs.berkeley.edu/~ddgarcia/eyawtkagtbwata/
- www.cut-the-knot.org
- R. Bell and M. Cornelius: *Board Games around the World* [1988]
- K. Binmore: *A Text on Game Theory* [1992]