The Point Spread Function

Dan Garcia <ddgarcia@cs.berkeley.edu>

1 Introduction

In this section we discuss the a retinal representation which utilizes geometric, or ray optics to reveal how well parallel light comes to a crisp focus on the retina of the eye. We make use of two standard techniques common for describing the quality of optical systems.

2 Methods

We wish to measure how well the corneas focus light from a single, distant source like a bright star. One standard technique is called the *point spread function* (PSF), which is the retinal energy distribution from a point source [2, 11, 52, 57, 87, 115, 116].

2.1 Point Spread Function

We begin the calculation of the PSF by ray tracing light through the system and onto the retina, as others have done [16, 17, 37, 45, 76, 77, 98, 108]. We assume the point is placed at optical infinity, so that the incoming light rays are parallel. We also assume that the retina is planar, a reasonable approximation given that our entire retinal region of interest (ROI) is 0.64 mm wide and deviates in height from a spherical retinal model by only 3 μm. The PSF is the distribution, or "spread", of light at the retina. Figure 1 illustrates light rays showering a cornea and forming a PSF on the retina.

We generate a *normalized* PSF (denoted $\overline{\text{PSF}}$), which is simply our PSF divided by the number of rays N_r which land:

$$\overline{\text{PSF}}(x,y) = \frac{\text{PSF}(x,y)}{N_r} \tag{1}$$

and typically plot it as a height field for analysis, as in the example from Figure 2. The $\overline{\text{PSF}}$ for an ideal optical system (like a perfect ellipsoid) would have a very small, localized region of height one and zero elsewhere, as we will see in Figure **??**. The *Strehl ratio* is the relative peak intensity of an optical system's PSF with that of an ideal, aberration-free system. As we are only dealing with geometric optics, we define an equivalent concept, the *geometric* Strehl

Figure 1: Light from a point source is refracted to form the PSF.

ratio, which we will call $\overline{\text{Strehl}}_g$. Since we normalize our PSF, it is simply the maximum value of our $\overline{\text{PSF}}$.

2.1.1 Sampling

To create the PSF, we must shower the cornea with light rays and trace them all as they are refracted toward the retina. Ideally, every microscopic patch of the cornea would be pierced with a light ray, but for computation purposes, we must sample the continuous surface into discrete corneal points. We use a crosshatched sampling pattern as shown in Figure 3. We typically sample the corneas at a 100×100 resolution (with an inner 99×99 group), which produces 19,801 samples. As most corneal reconstructions are round, approximately 78% or 15,000 of those samples produce valid rays.

We introduce a *simulated pupil* located at the front surface of the eye. As we reduce the diameter of our pupil, we subsequently reduce the number of rays that are allowed to pass. This is consistent with our model that the samples represent a predefined *flux* of incoming light radiation, and culling the peripheral rays with our pupil reduces that flux accordingly. The simple expression for rays allowed to pass is that their distance from the CT axis be less than our pupil radius. That is, if the ray pierces the $z = 0$ plane with polar coordinates (r, θ), the rays allowed to enter are those satisfying the expression

$$r < r_{pupil}. \tag{2}$$

At each sample in our pupil, we query point and derivative information necessary for determining the refraction into the eye. It is here that we apply our simulated *spectacle correction.* We found that simple aberrations based on astigmatism dominated our distribution, so we compensate by placing a

Figure 2: A height field representation of a sample $\overline{\mathrm{PSF}}$.

Figure 3: The crosshatched sampling pattern used to sample the cornea. Rays from the infinite light source pass through these samples and are refracted into the eye.

simulated infinitely thin contact lens on the cornea to correct for basic cylinder. We search to find the optimal cross-cylinder lens that gives the best retinal focus, which we discuss shortly in Section **??**.

The corrected ray then enters the corneal surface according to Snell's law. After the rays pass through the cornea and eye, they intersect the retinal "plane", which is a variable distance away. This distance is the third and final parameter in our spectacle correction optimization.

We then sample the retinal plane, dividing it into 257×257 square buckets which are 2.5 μm, or roughly one-half minute on a side. This produces a square retinal "patch" which is 0.6425 mm on a side. Figure 4 shows a ray striking the retina and landing within one of the sample buckets.

Every time a ray lands in a sample, it contributes one unit to our histogram. The accumulation of all the rays forms the overall PSF energy distribution.

3 See What You See : Simulating Corneal Visual Acuity

4 Introduction

Our goal with this work is to simulate the corneal contribution to visual acuity. In the previous chapter we showed retinal representations of corneal acuity, but these fail to capture what the patient actually sees. We utilize a modified Snellen eye chart and a sample outdoor scene as our input, and image how they

Figure 4: A refracted ray lands in one of the sample buckets on our retinal plane.

would be seen with different patient's corneas and pupil sizes to achieve a fairly accurate first-person representation of visual acuity.

5 Methods

The use of ray tracing to determine the PSF and the resulting retinal blur for images such as Snellen charts is not new. Section **??** discusses several prominent researchers who have created sophisticated models and optical bench software tools using ray tracing for evaluation of optical performance.

We choose to implement the technique as part of our software suite to provide the final stage of visualization, the simulation of optical acuity through the cornea in question. In this section we'll discuss the process of calculating the $\overline{\text{PSF}}$, calibrating it with an image, and convolving them together to form the final blurred result.

It is important to note the assumptions and limitations of this technique for the simulation of visual acuity. First, it assumes that all incoming light is parallel, having arrived from optical infinity. Thus, we can only simulate what a patient would see while looking at something reasonably far away. Second, the computational model described in Section **??** is very simplistic and does not take other components of the eye into account, like the lens, vitreous humor or corneal layers. This means the many effects they induce (e.g., ciliary bloom and lens glare) are completely ignored. Third, the pupil only expands to large diameters in extremely low light situations, so simulating the aberrations with an 8 mm pupil on a daylight scene is quite artificial. Finally, as all of our $\overline{\text{PSF}}$ construction uses ray tracing and geometric optics, we ignore the important

Figure 5: Simulation of an eye viewing an image exactly as it was seen by the camera. The centers of projection are aligned and the image is scaled so that one pixel on the image maps to one sample on the $\overline{\text{PSF}}$.

effects of diffraction, which is the limiting acuity factor for small pupils.

5.1 Normalized Point Spread Function

We begin with the normalized point spread function, $\overline{\text{PSF}}$, a computed histogram of retinal energy from a distant point source of light as discussed in Section 2.1. This serves as the "impulse response" of the patient's optical system. As we mentioned, we sample the retina at half-minute (2.5 micron) intervals; this will be important in the following section on image calibration.

5.2 Image Calibration

When we wish to apply a filter to an image, it is critical that the parameters of the filter be tuned to the spatial frequencies of the image. Our filter is the $\overline{\text{PSF}}$, which is sampled at half-minute intervals, producing a fine grid of retinal energy distribution.

Again, we are given (or synthetically generate) an input image, and our goal is to create a first-person simulation. There are two alternatives, simulating the eye in the camera's place, or simulating the patient viewing the image from afar. Each of these will affect the calibration differently, and we discuss this in the following sections.

5.2.1 Eye is the Camera

This is perhaps the most effective and convincing simulation, and the one we use to simulate the patient seeing an eye chart in Section 6.1. Here we place the

Figure 6: Simple geometry controls the relationship between an object's distance (d), its image size (h) and the angle it subtends (θ). That relationship is captured by the expression $h = d \tan \theta$.

eye where our camera was when the scene was captured on film as in Figure 5. Before we can accurately simulate the blur the patient would see, we must assure that they are calibrated together, as described below.

A photograph of a scene is taken, or one is synthetically generated. Real or virtual camera information is recorded, e.g., lens, field of view, center of projection, image plane distance, etc. The image is then digitized (or rendered) at the same spatial frequency as our filter.

If we have control over the digitization and camera information, we simply tune our sampling resolution to the correct value. If we do *not* have camera information, but *do* have objects in our scene whose distance to the camera and size we know accurately, we can still adjust our sampling as follows. We want one pixel to be thirty seconds of arc. If an object is at a distance d from our camera, this means (using the simple geometry from Figure 6) that:

$$h_1 = d \tan \theta = d \tan \left(\frac{1}{2} \frac{1}{60} \frac{\pi}{180} \right) \approx d \; 1.45444 \times 10^{-4} \tag{3}$$

where h_1 represents the distance in object space that we need for one pixel. If an object's size is h, then it should be $\frac{h}{h_1}$ pixels high.

If we do not have either camera or object size and distance information, we have no way to estimate how finely to sample our scene. When we do have this information, but the image has already been digitized, then there are three options. If the image is sampled at calibration density, we're done. If the image is sampled finer than that, then an intelligent bicubic interpolation can be used to reduce the image size without significant loss. However, if the image is sampled more coursely than calibration, we're in trouble. We either have to reduce the size of our filter or increase the size of our image. Either technique will result in unacceptable artifacts.

If there is no calibrating camera or object information, and the image has already been digitized, we cannot reliably put the eye in the camera. We could, however, simulate what the patient would see if they were to visualize the digitized photo on a computer monitor; this is discussed next.

Figure 7: Simulation of an eye viewing a projected digitized image from afar. The eye's center of projection is placed at a distance d and the image is scaled so that one pixel on the digital image maps to one sample on the $\overline{\text{PSF}}$.

5.2.2 Patient Views Projected Digital Image

This technique, as shown in Figure 7, allows us to simulate what the patient would see looking at an image from a fixed distance. It is most useful when we do not have object information necessary for eye-is-the-camera calibration.

When images are viewed on monitors in a what-you-see-is-what-you-get (WYSIWYG) fashion, they are represented at 72 dots per inch (DPI). This means one pixel is $\frac{1}{72}$ inch. Plugging in this value for h_1 in Equation 3, we know the distance d must be:

$$d = \frac{h_1}{\tan\left(\frac{1}{2}\frac{1}{60}\frac{\pi}{180}\right)} \approx 95.5 \text{ inches} \approx 8 \text{ feet.} \tag{4}$$

Therefore, if we perform our filtering on an unmodified input image, the result would be what a person would see viewing the picture on a monitor from a distance of approximately 8 feet. This is the technique we employ with our outdoor scene from Section 6.2.

5.3 Measuring Visual Acuity

Visual acuity is the measurement of the eye's ability to resolve the form and detail of an object. It is most commonly determined by testing the ability of the patient to read standard letters at a fixed distance [10, 25].

5.3.1 Snellen Notation

In 1862 Snellen devised a system for measuring acuity which has since become a fundamental clinical technique for acuity assessment [25]. The smallest detail of an object an eye can see is called the *minimum angle of resolution* (MAR) of the eye. The *Snellen fraction* is the reciprocal of the MAR. The test is usually performed at 20 ft (6 m), an acceptable approximation to optical infinity.

Figure 8: A typical Snellen letter.

MAR	*Snellen decimal notation*	*Snellen fraction*
0.5	2	20/10
1	1	20/20
2	1/2	20/40
4	1/4	20/80
8	1/8	20/160
⋮	⋮	⋮

Table 1: The relationship between MAR and Snellen notations.

On a typical chart, letters are scaled so that each will subtend an angle of 5 minutes at a given distance. The details of the letters themselves make an angle of 1 minute of arc, as shown in the classic "E" in Figure 8. The letters are then labeled by this distance, e.g., the "20-foot" letter makes a total angle of 5 minutes at a distance of 20 feet. The classic fraction is recorded as the ratio of the testing distance and the label of the smallest letter a patient can resolve. Thus, if a patient at 20 feet is only able to read the "100 foot" letter, their vision would be classified as "20/100". Table 1 shows the relationship between MAR and Snellen notation. A historical note: one minute of arc became a standard in the days of early astronomers who determined it to be the minimum angular separation for two different stars to be perceived as distinct [25].

5.3.2 Simulated Snellen Eye Chart

Tumbling E charts like the one shown in Figure 9 are often used for preliterate children. We choose to use a modified version of this instead of a standard Snellen chart since rendering the letter "E" to an image requires no anti-aliasing as would letters with curved edges. We are able to create Es down to 5×5 pixels with no loss.

Our test acuity image is shown in Figure 10, which has been calibrated so that each pixel is thirty seconds. This is the spacing between the bars of the smallest E, used for testing 20/10 acuity. We include Es which double in size

Figure 9: A typical acuity chart with tumbling Es used for acuity assessment of preliterate children. This chart is especially useful for us since rasterization requires no anti-aliasing as would a chart with curved-edge letters.

Figure 10: The image we use for our acuity simulation. The size of the Es ranges from 20/10 on the left to 20/160 on the right. The fan pattern on the far right is used to test for astigmatism.

Figure 11: The image we use for our outdoor scene simulation, a photograph of U. C. Berkeley's Campanile courtesy of Paul Debevec.

up to 20/160. We add a fan pattern to test for astigmatism, whose direction is determined by the bars perceived as least blurred.

5.3.3 Outdoor Scene

As shown in Figure 11, the input image is a crisp low dynamic range digital photograph of U. C. Berkeley's Campanile tower, courtesy of Paul Debevec. It has clearly defined edges and a full-range luminance histogram. The simulations we present are what a patient would see when looking at the full 250×500 pixel image on a 72-dpi computer screen at a distance of 8 feet. The blur is identically computed for each of the R, G and B channels in the image, so there is no chromatic aberration in the simulation. There was no digital processing done on the input image; every pixel was simply copied from the original PhotoCD, down to the small smudge on the right of the tower.

Figure 12: Instead of convolving the $\overline{\text{PSF}}$ with our image (shown in grey), we multiply their Fourier transforms and return the inverse Fourier transformation (shown in black).

5.4 Convolution

Now that we have our input images and $\overline{\text{PSF}}$ "impulse response" distribution, we simply need to convolve them together to form the blurred output images. We make use of the *convolution theorem* which tells us the convolution in the spatial domain can be obtained by taking the inverse Fourier transform of the products of the spectra in the frequency domain, as shown in Figure 12. That is,

$$\begin{aligned} \text{Image}_{\text{blur}}(x,y) &= \overline{\text{PSF}}(x,y) * \text{Image}(x,y) \\ &= \mathcal{F}^{-1}\left\{\mathcal{F}\left\{\overline{\text{PSF}}(x,y)\right\} \times \mathcal{F}\left\{\text{Image}(x,y)\right\}\right\} \end{aligned} \tag{5}$$

where $\mathcal{F}$ is the Fourier transform and $\mathcal{F}^{-1}$ is the inverse Fourier transform [34].

6 Results

We compile the results of simulating all of the corneas with pupil sizes of 2, 4 and 8 mm viewing both our test images in Section **??**, and discuss the overall results in the following sections.

6.1 Snellen Eye Chart

As anticipated, the corneas have much better vision with small pupils than with large. The regular corneas (those without PRK, keratoconus or monocular diplopia) have excellent spectacle-corrected vision with no astigmatism even up to 8 mm; acuity is estimated to be between 20/10 and 20/40. The problem eyes have acute loss of contrast and acuity, sometimes even with small pupils. A telltale ghost image forms with our 8 mm monocular diplopia eye, situated about $\frac{4}{3}^{\circ}$ above the primary image. Overall acuity ranges at around 20/80 for the PRK and diplopic eye to worse than 20/160 for our keratoconic eye.

6.2 Outdoor Scene

The aberrations that we witnessed with the simulated Snellen chart were more mild than with our outdoor scene, since our scene did not have a comparable degree of contrast and sharp edges, and thus was more forgiving. In general, it was harder to differentiate the blur from different corneas, as the results all seemed to converge with large pupils. The most striking feature was the "muddying" of the scene, as everything in the interior of the tower blurred to a dark grey. The ghost image was not as distinct for the diplopic eye as it had been for our Snellen chart.

7 Conclusion

We presented a technique for the simulation of first-order visual acuity using a precomputed normalized point spread function of the eye. We utilized two sample input images: a modified tumbling E Snellen chart with an astigmatic test fan, and a sample full-color outdoor scene. Our results showed a fair approximation of visual acuity, with expected increased blur and loss of contrast for larger pupils as peripheral aberrations became more dominant.

References

[1] F. H. Adler. *Physiology of the Eye, 4th edition.* Mosby Company, St. Louis, MO, 1965.

[2] Pablo Artal, Javier Santamaría, and Julian Bescoós. Retrieval of wave aberration of human eyes from actual point-spread-function data. *J. Opt. Soc. Am. A*, 5:1201–1206, 1988.

[3] T. Y. Baker. Ray tracing through non-spherical surfaces. *Proc. Phys. Soc.*, 55:361, 1943.

[4] Brian A. Barsky, Stanley A. Klein, and Daniel D. Garcia. Gaussian power, mean sphere, and cylinder representations for corneal maps with applications to the diagnosis of keratoconus. *Invest. Ophthalmol. Vis. Sci*, 37:558, 1996.

[5] Brian A. Barsky, Stanley A. Klein, and Daniel D. Garcia. Gaussian power with cylinder vector field representation for corneal topography maps. *Optom. Vis. Sci*, 74(11):917–925, 1997.

[6] Brian A. Barsky, Robert B. Mandell, and Stanley A. Klein. Corneal shape illusion in keratoconus. *Invest. Ophthalmol. Vis. Sci*, 36(Suppl.):5308, 1995.

[7] John P. Beale Jr. *No More Glasses (Or Contact Lenses)*. Doctors' Ophthalmic Press, San Francisco, CA, 1986.

[8] James M. Beck, Rida T. Farouki, and John K. Hinds. Surface analysis methods. *IEEE Computer Graphics and Applications*, 6(12):18–36, 1986.

[9] Michael W. Belin, David Litoff, Salvins J. Strods, Steven S. Winn, and Richard S. Smith. The par technology corneal topography system. *Refractive and Corneal Surgery*, 8:88–96, 1992.

[10] Peter D. Bergenske and Lynn E. Konkel. Training staff to measure visual acuity. *Optometric Economics*, pages 46–51, Fall 1996.

[11] Frans J. J. Blommaert, Henny G. M. Heynen, and Jacques A. J. Roufs. Point spread functions and detail detection. *Spatial Vision*, 2(2):99 – 115, 1987.

[12] Max Born and Emil Wolf. *Principles of optics, 7th Edition*. Cambridge University Press, Cambridge, UK, 1980.

[13] Douglas Brenner. Modeling the cornea with the topographic modeling system videokeratoscope. *Optom. Vis. Sci*, 74(11):895–898, 1997.

[14] britannica.com. *Encyclopædia Britannica Online : Optics : Lens Aberrations.* http://www.britannica.com /bcom/eb/article/5/0,5716,115155+29,00.html.

[15] Lucio Buratto. *Corneal Topography : The Clinical Atlas*. SLACK Incorporated, Thorofare, NJ, 1996.

[16] Jon J. Camp, Leo J. Maguire, Bruce M. Cameron, and Richard A. Robb. A computer model for the evaluation of the effect of corneal topography on optical performance. *Am. J. Ophthalmol.*, 109:379–386, 1990.

[17] Jon. J. Camp, Leo J. Maguire, and Richard A. Robb. An efficient ray tracing algorithm for modeling visual performance from corneal topography. In *First Conference on Visualization in Biomedical Computing*, pages 279–285, Atlanta, GA, May 22–25 1990. The Institute of Electrical and Electronics Engineers, Inc., IEEE Computer Society Press.

[18] Charles Campbell. Corneal aberrations, monocular diplopia, and ghost images: Analysis using corneal topographical data. *Optom. Vis. Sci*, 75(3):197–207, 1998.

[19] James Casey. *Exploring Curvature*. Vieweg, Germany, 1996.

[20] Jacob A. Corbin, Stanley A. Klein, and Corina van de Pol. Measuring effects of refractive surgery on corneas using taylor series polynomials. In *Proceedings of Ophthalmic Technologies IX*, San Jose, CA, January 23–29 1999. SPIE International Symposium on Biomedical Optics.

[21] A. M. de Beus and S. E. Brodie. Towards intrinsic representations of the corneal surface. *Invest. Ophthalmol. Vis. Sci*, 35(Suppl):2197, 1994.

[22] Paul E. Debevec and Jitendra Malik. Recovering high dynamic range radiance maps from photographs. In *Proceedings of ACM SIGGRAPH 1997*, pages 369–378, Los Angeles, CA, August 3–8 1997. SIGGRAPH, Association of Computer Machinery, Inc.

[23] Steven A. Dingeldein and Stephen D. Klyce et al. Quantitative descriptors of corneal shape derived from computer-assisted anaylsis of photokeratographs. *Refractive and Corneal Surgery*, 5(6):372–378, 1989.

[24] Manfred P. Docarmo. *Differential Geometry of Curves and Surfaces*. Prentice-Hall, Englewood Cliffs, NJ, 1976.

[25] J. Boyd Eskridge, John F. Amos, and Jimmy D. Bartlett. *Clinical Procedures in Optometry*. J. B. Lippincott Company, Philadelphia, PA, 1991.

[26] Richard P. Feynman, Robert B. Leighton, and Matthew Sands. *The Feynman Lectures on Physics, Volume I*. Addison-Wesley, Reading, MA, 1963.

[27] James D. Foley, Andries van Dam, Steven K. Feiner, and John F. Hughes. *Computer Graphics : Principles and Practice, 2nd Edition*. Addison-Wesley Publishing Co., Reading, MA, 1990.

[28] Grant R. Fowles. *Introduction to Modern Optics*. Dover Publications, Inc., New York, NY, 1989.

[29] Daniel D. Garcia, Brian A. Barksy, and Stanley A. Klein. Cwhatuc: A visual acuity simulator. In *Proceedings of Ophthalmic Technologies VIII, SPIE International Symposium on Biomedical Optics*, pages 290–298, San Jose, CA, January 24–30 1998. SPIE.

[30] Daniel D. Garcia, Corina van de Pol, Stanley A. Klein, and Brian A. Barsky. Wavefront coherence area for predicting visual acuity of post-prk and post-park refractive surgery patients. In *Proceedings of Ophthalmic Technologies IX, SPIE International Symposium on Biomedical Optics*, San Jose, CA, January 23–29 1999. SPIE.

[31] Al Globus and Eric Raible. Fourteen ways to say nothing with scientific visualization. *IEEE Computer*, pages 86–88, July 1994.

[32] Michael J. Goggin, Paul Kenna, and Frank Lavery. Haze following photorefractive and photoastigmatic refractive keratectomy with the nidek ec5000 and the summit excimed uv200. *Journal of Cataract and Refractive Surgery*, 23(1):50–53, 1997.

[33] Michael J. Goggin and Paul Kenna et al. Photoastigmatic refractive keratectomy for compound myopic astigmatism with a nidek laser. *Journal of Refractive Surgery*, 13(2):162–166, 1997.

[34] Rafael C. Gonzalez and Richard E. Woods. *Digital Image Processing.* Addison-Wesley, Reading, MA, 1992.

[35] Joseph W. Goodman. *Introduction to Fourier Optics.* McGraw-Hill, Inc., San Francisco, CA, 1968.

[36] Christine Gorman. Should you get the laser eye fix? *Time Magazine*, 154(15):58–66, October 11 1999.

[37] J. E. Greivenkamp, J. Schweigerling, J. M. Miller, and M. D. Mellinger. Visual acuity modeling using optical raytracing of schematic eyes. *Am. J. Ophthalmol.*, 120:227–240, 1995.

[38] R. W. Gubisch. Optical performance of the human eye. *J. Opt. Soc. Am.*, 57(3):407–415, 1967.

[39] Hans Hagen, Stefanie Hahmann, Thomas Schreiber, Yasuo Nakajima, Bukard Wördenweber, and Petra Hollemann-Grundstedt. Surface interrogation algorithms. *Computer Graphics and Applications*, 12(5):53–60, 1992.

[40] David Halliday and Robert Resnick. *Physics, part two.* John Wiley and Sons, Inc., New York, NY, 1978.

[41] Mark A. Halstead, Brian A. Barsky, Stanley A. Klein, and Robert B. Mandell. A spline surface algorithm for reconstruction of corneal topography from a videokeratograph reflection pattern. *Optom. Vis. Sci*, 72:821–827, 1995.

[42] Mark A. Halstead, Brian A. Barsky, Stanley A. Klein, and Robert B. Mandell. Reconstructing curved surfaces from specular reflection patterns using spline surface fitting of normals. In *Proceedings of ACM SIGGRAPH 1996*, New Orleans, LA, August 4–9 1996. SIGGRAPH, Association of Computer Machinery, Inc.

[43] Mark Arthur Halstead. *Efficient Techniques for Surface Design Using Constrained Optimization.* PhD thesis, Department of Computer Science, University of California at Berkeley, Berkeley, CA, May 1996.

[44] Eugene Hecht. *Optics, Second Edition.* Addison-Wesley Publishing Co., 1990.

[45] Richard P. Hemenger, Alan Tomlinson, and Katherine Oliver. Corneal optics from videokeratographs. *Ophthal. Physiol. Opt.*, 15(1):63–68, 1995.

[46] P. S. Hersh and R. D. Stulting et al. Results of phase iii excimer laser photorefractive keratectomy for myopia. *Ophthalmology*, 104(10):1535–1553, 1997.

[47] H. Hopkins. *Wave theory of aberrations.* Oxford Univ. Press, 1950.

[48] Bradford Howland and Howard C. Howland. Subjective measurement of high-order aberrations of the eye. *Science*, 193:580–582, 1976.

[49] Howard C. Howland, Jan Buettner, and Raymond A. Applegate. Computation of the shapes of normal corneas and their monochromatic aberrations from videokeratometric measurements. In *Vision Science and Its Applications. 1994 Technical Digest Series. Volume 2*, volume 2, pages 54–57. Optical Society of America, Washington, DC, 1994.

[50] Howard C. Howland and Bradford Howland. A subjective method for the measurement of monochromatic aberrations of the eye. *J. Opt. Soc. Am.*, 67(11):1508–1518, 1977.

[51] Howard C. Howland, Richard H. Rand, and Sharon R. Lubkin. A thin-shell model of the cornea and its application to corneal surgery. *Refract. Corneal Surg.*, 8:183–186, 1992.

[52] J. K. Ijspeert, T. J. T. P. Van Den Berg, and H. Spekreijse. An improved mathematical description of the foveal visual point spread function with parameters for age, pupil size and pigmentation. *Vision Res.*, 33(1):15–20, 1993.

[53] Internet Media Services. *The Vision Correction Website : Laser Eye Surgery : rk / prk / lasik*. http://www.lasersite.com/.

[54] Victoria Interrante. Illustrating surface shape in volume data via principal direction-driven 3d line integral convolution. In *Proceedings of ACM SIGGRAPH 1997*, pages 109–116, Los Angeles, CA, August 3–8 1997. SIGGRAPH, Association of Computer Machinery, Inc.

[55] Michael P. Keating. *Geometric, Physical and Visual Optics*. Butterworths-Heinemann, Stoneham, MA, 1988.

[56] Peter R. Keller and Paul P. van Saarloos. Perspectives on corneal topography: a review of videokeratoscopy. *Clin. Exp. Optom*, 80(1):18–30, 1997.

[57] Miles V. Klein. *Optics*. John Wiley & Sons, New York, NY, 1970.

[58] Stanley A. Klein. Axial curvature and the skew ray error in corneal topography. *Optom. Vis. Sci*, 74:931–944, 1997.

[59] Stanley A. Klein. Corneal topography reconstruction algorithm that avoids the skew ray ambiguity and the skew ray error. *Optom. Vis. Sci*, 74:945–962, 1997.

[60] Stanley A. Klein. Optimal corneal ablation for eyes with arbitrary hartmann-shack aberrations. *J. Opt. Soc. Am. A*, 15(9):2580–2588, 1998.

[61] Stanley A. Klein and Daniel D. Garcia. Alternative representations of aberrations of the eye. In *Presented at Vision Science and Its Applications Meeting*, Santa Fe, NM, Februrary 11–14 2000.

[62] Stanley A. Klein and Robert B. Mandell. Shape and refractive powers in corneal topography. *Invest. Ophthalmol. Vis. Sci.*, 36:2096–2109, 1995.

[63] Stanley A. Klein, Robert B. Mandell, and Brian A. Barsky. Representing corneal shape. In *Vision Science and Its Applications*, volume 1, pages 37–40, Washington, DC, 1995. Technical Digest Series, Optical Society of America.

[64] Stephen D. Klyce. Computer-assisted corneal topography, high resolution graphic presentation and analysis of keratoscopy. *Ophthalmol. Vis Sci.*, 25:1426–1435, 1984.

[65] Douglas D. Koch, G. N. Foulks, and T. Moran. The corneal eyesys system: accuracy, analysis and reproducibility of first generation prototype. *Refract. Corneal Surg.*, 5:424–429, 1989.

[66] Craig Kolb, Don Mitchell, and Pat Hanrahan. A realistic camera model for computer graphics. In *Proceedings of ACM SIGGRAPH 1995*, pages 317–324, Los Angeles, CA, August 6–11 1995. SIGGRAPH, Association of Computer Machinery, Inc.

[67] M. Koomen, R. Tousey, and R. Scolnick. The spherical aberration of the eye. *J. Opt. Soc. Am.*, 39(5):370–376, 1949.

[68] J. H. Krachmer, R. S. Feder, and M. W. Belin. Keratoconus and related noninflammatory corneal thinning disorders. *Surv. Ophthalmol.*, 28(4):293–322, 1984.

[69] Roger Kumpf. Multivariate corneal visualization in the eyeview system. Master's thesis, Department of Computer Science, University of California at Berkeley, Berkeley, CA, May 1995.

[70] J. E. A. Landgrave and Jesús R. Moya-Cessa. Generalized coddington equations in ophthalmic lens design. *J. Opt. Soc. Amer.*, 12(8):1637–1644, 1996.

[71] Junzhong Liang. *A New Method to Precisely Measure the Wave Aberrations of the Human Eye with a Hartmann-Shack Wavefront Sensor*. PhD thesis, Department of Mathematics, Universität Heidelberg, Heidelberg, Germany, December 1991.

[72] Norberto López-Gil and Howard C. Howland. Measurement of the eye's near infrared wave-front aberration using the objective crossed-cylinder aberroscope technique. *Vision Research*, 39:2031–2037, 1999.

[73] H. Lubatschowski and O. Kermani et al. A scanning and rotating slit arf excimer laser delivery system for refractive surgery. *Journal of Refractive Surgery*, 14(2 Suppl):s186–191, 1998.

[74] Klaus Ludwig, Peter Schäeffer, Herbert Gross, Theo Lasser, and Anselm Kampik. Mathematical simulation of retinal image contrast after photorefractive keratectomy with a diaphragm mask. *Journal of Refractive Surgery*, 12:248–253, 1996.

[75] Leo. J. Maguire and W. D. Bourne. Corneal topography of early keratoconus. *Am. J. Ophthalmol.*, 108:107–112, 1989.

[76] Leo J. Maguire, Jon J. Camp, and Richard A. Robb. Informing interested parties of changes in the optical performance of the cornea caused by keratorefractive surgery — a ray tracing model that tailors presentation of results to fit the lefel of sophistication of the audience. In *SPIE Vol. 1808 Visualization in Biomedical Computing 1992*, pages 601 – 609, Chapel Hill, NC, October 13 – 16 1992. SPIE — The International Society for Optical Engineering, SPIE.

[77] Leo J. Maguire, Ralph W. Zabel, Paula Parker, and Richard L. Lindstrom. Topography and raytracing analysis of patients with excellent visual acuity 3 months after excimer laser photorefractive keratectomy for myopia. *Refract. Corneal Surg.*, 8:122–128, March/April 1991.

[78] Virendra N. Mahajan. *Optical Imaging and Aberrations*. SPIE Optical Engineering Press, Bellingham, WA, 1998.

[79] Richard J. Mammone, Martin Gersten, Dennis J. Gormley, Richard S. Koplin, and Virginia L. Lubkin. 3-d corneal modeling system. *IEEE Trans. Biomedical Eng.*, 37:66–73, 1990.

[80] Robert B. Mandell. *Contact Lens Practice, 4th Edition*. Charles C. Thomas, Springfield, IL, 1988.

[81] Robert B. Mandell. The enigma of the corneal contour. *the CLAO Journal*, 18(4):267–273, 1992.

[82] Robert B. Mandell, Stanley A. Klein, C. H. Shie, Brian A. Barsky, and Zijiang Yang. Axial and instantaneous radii in videokeratography. *Invest. Ophthalmol. Vis. Sci*, 35(Suppl):2079, 1994.

[83] Marguerite McDonald. Corneal topography: A vital tool for refractive surgery. *Eyecare Technology*, pages 21–22, 24, 65, June 1996.

[84] M. Minnaert. *The nature of light & color in the open air*. Dover Publications, Inc., New York, NY, 1954.

[85] Parry Moon and Domina Eberle Spencer. On the stiles-crawford effect. *J. Opt. Soc. Am.*, 34(6):319–329, 1944.

[86] Hans C. Ohanian. *Physics, vol. 1.* W. W. Norton and Co., New York, NY, 1985.

[87] Donald C. O'Shea. *Elements of Modern Optical Design.* John Wiley & Sons, New York, NY, 1985.

[88] Sudi Patel, John Marshall, and Frederick W. Fitzke. Model for predicting the optical performance of the eye in refractive surgery. *Refract Corneal Surg*, 9:366–375, September / October 1993.

[89] B. Platt and R. V. Shack. Lenticular hartmann-screen. Newsletter 5, 15, Optical Science Center, University of Arizona, 1971.

[90] Ronald B. Rabbetts. *Clinical Visual Optics, third edition.* Butterworth Heinemann Ltd., Oxford, UK, 1998.

[91] Jeff Rabin and James Wicks. Measuring resolution in the contrast domain: the small letter contrast test. *Optom. Vis. Sci*, 73(6):398–403, 1996.

[92] R. H. Rand, S. R. Lubkin, and H. C. Howland. Analytical model of corneal surgery. *Journal of Biomechanical Engineering*, 113:239–241, May 1991.

[93] Cynthia Roberts. The accuracy of power maps to display curvature data in corneal topography systems. *Invest. Ophthalmol. Vis. Sci*, 35:3525–3532, 1994.

[94] Jyrki Rovamo, Juvi Mustonen, and Risto Näsänen. Two simple psychophysical methods for determining the optical modulation transfer function of the human eye. *Vision Res.*, 34(19):2493–2502, 1994.

[95] Thomas O. Salmon and Douglas G. Horner. Comparison of elevation, curvature, and power descriptors for corneal topographic mapping. *Optom. Vis. Sci*, 72(11):800–808, 1995.

[96] Jim Schwiegerling and John E. Greivenkamp. Using corneal height maps and polynomial decomposition to determine corneal aberrations. *Optom. Vis. Sci*, 74(11):906–916, 1997.

[97] Jim Schwiegerling, John E. Greivenkamp, Joseph M. Miller, Robert W. Snyder, and Millicent L. Palmer. Optical modeling of radial keratotomy incision patterns. *American Journal of Ophthalmology*, 122(6):808–817, December 1996.

[98] T. Seiler, W. Reckmann, , and R. K. Maloney. Effective spherical aberration of the cornea as a quantitative descriptor in corneal topography. *Journal of Cataract and Refractive Surgery*, 19(Supplement):155–165, 1993.

[99] Greg Spencer, Peter Shirley, Kurt Zimmerman, and Donald P. Greenberg. Physically-based glare effects for digital images. In *Proceedings of ACM SIGGRAPH 1995*, pages 325–334, Los Angeles, CA, August 6–11 1995. SIGGRAPH, Association of Computer Machinery, Inc.

[100] Orestes N. Stavroudis. Simpler derivation of the formulas for generalized ray tracing. *J. Opt. Soc. Amer.*, 66(12):1330–1333, 1976.

[101] George B. Thomas Jr. and Ross L. Finney. *Calculus and Analytical Geometry.* Addison-Wesley, Reading, MA, 1980.

[102] S. L. Trokel, R. Rrinivasan, and B. Braren. Excimer laser surgery of the cornea. *Am. J. Ophthalmol.*, 96:710–715, 1983.

[103] Edward R. Tufte. *The Visual Display of Quantitative Information.* Graphics Press, Cherhire, CT, 1983.

[104] Edward R. Tufte. *Envisioning Information.* Graphics Press, Cherhire, CT, 1991.

[105] Edward R. Tufte. *Visual Explanations : Images and Quantities, Evidence and Narrative.* Graphics Press, Cherhire, CT, 1997.

[106] Rasik B. Vajpayee, Catherine A. McCarty, Geoffrey F. Aldred, Hugh R. Taylor, and The Excimer Laser Group. Undercorrection after excimer laser refractive surgery. *American Journal of Ophthalmology*, 122(6):801–807, 1996.

[107] Corina van de Pol. *Corneal Shape and Visual Performance after Keratorefractive Surgery.* PhD thesis, Department of Vision Science, University of California at Berkeley, Berkeley, CA, Fall 1999.

[108] Corina van de Pol, Henry H. Tran, Daniel D. Garcia, and Stanley A. Klein. Three-dimensional analysis of corneal image forming properties: A monocular diplopia example. In *Presented at Vision Science and Its Applications Meeting*, Santa Fe, NM, Februrary 6–9 1998.

[109] A. van Meeteren. Calculations on the optical modulation transfer function of the human eye. *Opt. Acta*, 21:395–412, 1974.

[110] Paul P. van Saarloos and Peter R. Keller. Comparing methods to view corneal topography data. *Clin. Exp. Optom*, 79(3):106–111, 1996.

[111] Wayne Verdon, Mark Bullimore, and Robert K. Maloney. Visual performance after photorefractive keratectomy. *Arch. Ophthalmol.*, 114:1465–1472, 1996.

[112] F. M. Vos and H. J. W. Spoelder. Visualization in corneal topography. In *Proceedings of IEEE Visualization*, Research Triangle Park, NC, October 18–23 1998. IEEE, IEEE Computer Society.

[113] G. Walsh, W. N. Charman, and Howard C. Howland. Objective technique for the determination of monochromatic aberrations of the human eye. *J. Opt. Soc. Amer.*, 1(9):987–992, 1984.

[114] Jianyi Wang, David A. Rice, and Stephen D. Klyce. A new reconstruction algorithm for improvement of corneal topographical analysis. *Refract. Corneal Surg.*, 5:379–387, 1989.

[115] W. T. Welford. *Aberrations of Optical Systems.* Adam Hilger, Ltd., Bristol, England, 1986.

[116] Gerald Westheimer and Fergus W. Campbell. Light distribution in the image formed by the living human eye. *J. Opt. Soc. Am.*, 52(9):1040–1045, 1962.

[117] Charles S. Williams and Orville A. Becklund. *Introduction to the Optical Transfer Function.* John Wiley & Sons, New York, NY, 1989.

[118] Steven E. Wilson and Stephen D. Klyce. Advances in the analysis of corneal topography. *Surv. Ophthalmol.*, 35:269–277, 1991.

[119] Steven E. Wilson and Stephen D. Klyce. Quantitative descriptors of corneal topography. a clinical study. *Arch. Ophthalmol.*, 109(3):349–353, 1991.

[120] Patrick Henry Winston. *Artificial Intelligence.* Addison-Wesley, Reading, MA, 1984.

[121] Robert S. Wolff and Larry Yaeger. *Visualization of Natural Phenomena.* The Electronic Library of Science, Santa Clara, CA, 1993.