

CS 70 SPRING 2008 — DISCUSSION #3

LUQMAN HODGKINSON, AARON KLEINMAN, MIN XU

ADMINISTRATION

Luqman's office hours have been moved to 551 Soda.

1. STRONG INDUCTION

Exercise 1. Let the sequence a_0, a_1, a_2, \dots be defined by the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 2$ and $a_0 = 1, a_1 = 2$. Prove that $a_n \leq n + 2$ for all $n \geq 0$. Hint: you may need to modify the inductive hypothesis.

Exercise 2. One day, Prof. Wagner decided to bring to class a giant chocolate bar with $n \times m$ pieces. He would like to share the chocolate with everyone in the class by giving one piece to each. To break the chocolate into pieces, he would first break the large bar into two, then choose a half-bar and break that into half again, and then he would repeatedly choose one contiguous block of chocolate and break it into two until there are only single pieces of chocolate left. Prove that no matter how you break the chocolate, it would always take $nm - 1$ number of moves to break down the entire bar into single pieces.

2. FINDING A PROOF

Exercise 3. Consider a regular 8×8 chessboard with the two white corners removed. Show that this board cannot be tiled by 2×1 dominoes.

3. WELL-ORDERING

Exercise 4. Consider an infinite sheet of graph paper such that each square contains a natural number. Suppose that the number in each square is equal to the average of the numbers in the four neighboring squares. Prove that each square contains the same number.

4. STABLE MARRIAGE

Exercise 5. Consider an instance of the stable marriage problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m . Does every stable solution S for this instance contain the pair (m, w) ?

Exercise 6. In a large group of n men and n women, Bob, one of the men, gets tipped off that he is the second-highest preference on every woman's list. Bob is pretty happy to hear this. Assuming the traditional (male-optimal) algorithm, might Bob be in for a disappointment? In particular, is it possible that he will end up with the woman he prefers the least of all?

Date: February 14, 2008.

The authors gratefully acknowledge the TA's of CS70 Past for the use of their previous notes: Chris Crutchfield, Alex Fabrikant, David Garmire, Assane Gueye, Amir Kamil, Lorenzo Orecchia, Vahab Pournaghshband, Ben Rubinstein. Their notes form the basis for this handout.