CS 70 Spring 2005 — Discrete Mathematics for CS — Clancy/Wagner — HW 10

Due Thursday, April 14

Coverage: This assignment involves topics from the lectures of March 29 and 31, and from Rosen section 5.2.

Administrative reminders: We will accept only unformatted text files or PDF files for homework submission. Include your name, login name, section number, and partner list in your submission. Give the command `submit hw10` to submit your solution to this assignment.

Homework exercises:

1. **(8 pts.) A paradox in conditional probability?**
Here is some on-time arrival data for two airlines, A and B, into the airports of Los Angeles and Chicago. (Predictably, both airlines perform better in LA, which is subject to less flight congestion and less bad weather.)

	Airline A		Airline B	
	# flights	# on time	#flights	# on time
Los Angeles	600	534	200	188
Chicago	250	176	900	685

   (a) Which of the two airlines has a better chance of arriving on time into Los Angeles? What about Chicago?

   (b) Which of the two airlines has a better chance of arriving on time overall?

   (c) Do the results of parts (a) and (b) surprise you? Explain the apparent paradox, and interpret it in terms of conditional probabilities.

2. **(12 pts.) Independence (due to H.W. Lenstra)**
Let Ω be a sample space, and let $A, B \subseteq \Omega$ be two *independent* events. Let $\overline{A} = \Omega - A$ and $\overline{B} = \Omega - B$ (sometimes written $\neg A$ and $\neg B$) denote the complementary events.

   For the purposes of this question, you may use the following definition of independence: Two events A, B are *independent* if $\Pr[A \cap B] = \Pr[A]\Pr[B]$.

   (a) Prove or disprove: $\overline{A}$ and $\overline{B}$ are necessarily independent.

   (b) Prove or disprove: A and $\overline{B}$ are necessarily independent.

   (c) Prove or disprove: A and $\overline{A}$ are necessarily independent.

   (d) Prove or disprove: It is possible that $A = B$.

3. (8 pts.) Correlation

When $\Pr[A|B] > \Pr[A]$, then A and B may be viewed intuitively as being positively correlated. One might wonder whether "being positively correlated" is a symmetric relation. Prove or disprove: If $\Pr[A|B] > \Pr[A]$ holds, then $\Pr[B|A] > \Pr[B]$ must necessarily hold, too. (You may assume that both $\Pr[A|B]$ and $\Pr[B|A]$ are well-defined, i.e., neither $\Pr[A]$ nor $\Pr[B]$ are zero.)

4. (16 pts.) A flippant choice

We have noted that if a fair coin is flipped three times, there are eight equally probable outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT. Two CS 70 students play a game based on coin flipping. Player A selects one of the triplets just listed; player B selects a different one. The coin is then repeatedly flipped until one of the chosen triplets appears as a run and wins the game. For example, if player A chooses HHT and player B chooses THT and the flips are THHHT, player A wins.

Fill in the table below to show player B's best choice of triplet for each possible choice that player A makes, and the probability of player B winning with a best choice. (We suggest you determine this information with a computer program, which you should submit with your solutions.) Then explain why the odds for one player winning are so lopsided.

Player A's choice	Player B's best choice	Player B's probability of winning
HHH		
HHT		
HTH		
HTT		
THH		
THT		
TTH		
TTT		

5. (16 pts.) How to beat the heat

It's a hot summer day in the Central Valley. Three children Alice, Bob, and Carlos decide to cool off by having a three-way duel with water balloons. They start by drawing lots to determine who throws first, second, and third, then take their places at the corners of an equilateral triangle. They agree to throw single water balloons in turn and continue in the same cyclic order until two of them have been soaked. Each player may throw at any other in his or her turn. You should assume the following: all the children have an essentially infinite supply of ammunition; a water balloon explodes on contact, drenching its target (who then leaves the game); when a water balloon misses its target, it explodes far enough away not to get anyone wet.

All three know that Alice always hits her target, Bob is 80% accurate, and Carlos is 50% accurate. Alice's strategy is to aim at Bob if he hasn't been hit yet, and at Carlos if he has. Bob similarly will aim at Alice first to avoid her hitting him. Carlos doesn't throw at anyone on his turn until one of Alice and Bob is hit; he then gets first shot at the survivor, which should give him an advantage.

Thus, here is an example scenario, with Carlos throwing first, then Bob, then Alice. Carlos passes. Bob aims at Alice, but misses. Alice clobbers Bob. Carlos throws a lucky one at Alice, and wins.

What are the survival probabilities for each of Alice, Bob, and Carlos? (Make clear how you got your answer.)