How Peer Effects Influence Energy Consumption

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Introduction to Peer Effects

**Background**

- Social comparisons influence people's behavior:
  - Conform to a standard
  - Receive social acclaim
  - Other people's choices can be informative (recommender systems)

- Network effects in social networks and platforms
  - Positive externalities

- Impact of Peer Effects on energy consumption?¹
  - Various Randomized Controlled Trials (RCTs) to investigate such effects²
  - High consumers reduce most, efficient ones show “boomerang effect”

**Question**

- How can peer effects in energy networks be exploited for profit-maximization of the load serving entity?

**Methodology**

- Develop a game between utility and electricity users, introducing peer effects

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Two-Stage Game-Theoretic Model

Consumers
- Set of consumers $\mathcal{I} = \{1, \ldots, n\}$ with utility function
  \[ u_i = a_i x_i - b_i x_i^2 - p_i x_i + \gamma_i x_i \left( \sum_{j \in \mathcal{I}} w_{ij} x_j - x_i \right). \]
  - Interaction matrix $W \in [0, 1]^{n \times n}$
  - Each user observes price $p_i^*$ and $x_{-i}$ and maximizes utility:
    \[ x_i^* = \arg \max_{x_i \geq 0} u_i(x_i, x_{-i}, \gamma_i, W). \]

Load-Serving Entity
- Profit: $\Pi = \sum_{i \in \mathcal{I}} p_i x_i - c_i x_i^2$
- Utility determines optimal price $p^*$ to maximize $\Pi$
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Subgame-Perfect Equilibrium
- Nash Equilibria of second stage game and first stage game
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Price and Consumption Equilibria

**Perfect Price Discrimination**

\[ p^* = \frac{a}{2} + CZ\frac{a}{2} - W^\top \Gamma Z\frac{a}{4} + \Gamma WZ\frac{a}{4}, \]

\[ x^* = \left( C + B + 2\Gamma - \frac{W^\top \Gamma}{2} - \frac{\Gamma W}{2} \right)^{-1} a, \]

\[ Z = \left[ 2\Gamma + B + C - \left( \frac{W^\top \Gamma}{2} + \frac{\Gamma W}{2} \right) \right]^{-1}. \]

- Complete knowledge of \( a \) and \( b \)
- Incentive for strongly influential users \( W^\top \Gamma \)
- Additional cost for strongly influenced users \( \Gamma W \)

**Single Price, Complete Information**

\[ \tilde{p}_u^* = \left[ 1 - \frac{1^\top A^{-1}1}{2 \cdot 1^\top (A^{-1} + A^{-1}CA^{-1}) 1} \right] \tilde{a}, \]

\[ x^* = A^{-1} \left[ a - \left( 1 - \frac{1^\top A^{-1}1}{2 \cdot 1^\top (A^{-1} + A^{-1}CA^{-1}) 1} \right) \tilde{a} \right], \]

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- Complete knowledge of \( a \) and \( b \)
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**Single Price, Incomplete Information**

\[ \hat{p}_u^* \geq \frac{\mathbb{E}[a]}{2} \left[ 1 + \frac{c}{n} 1^\top [2\Gamma + (2\mathbb{E}[b] + c)I - \Gamma W]^{-1} 1 \right], \]

\[ \mathbb{E}[\hat{x}_i] \geq \frac{\mathbb{E}[a] - \hat{p}_u^*,LB}{n} \cdot 1^\top (2\Gamma + 2\mathbb{E}[b]I - \Gamma W)^{-1} 1. \]

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\[ \mathbb{E}[x_i] \geq \frac{\mathbb{E}[a] - \bar{p}_u^* \cdot \mathbb{L}B}{n} \cdot 1^\top \left[ 2\Gamma + 2\mathbb{E}[b]\mathbf{1} - \Gamma W \right]^{-1} \mathbf{1}. \]

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Theoretical Statements

Theorem (Monotonicity of Consumption Equilibrium)

If \( a_i = a, b_i = b, \) and \( \gamma_i = \gamma \ \forall \ i \in \mathcal{I}, \) then \( x_i^* \) is strictly monotonically decreasing in \( \gamma \) independent of the network topology \( W. \)

Proof Sketch.

Take derivative \( \frac{dx}{d\gamma} = -\frac{1}{4\gamma(b+\gamma)} K^{-1} F^{-1} (a - p) \) and exploit diagonal dominance of \( K \) and \( F. \) Show that all elements \((K^{-1} F^{-1})_{ij}\) are positive.

Theorem (Influence of High Consumer)

Let \( w_{ij} = \left( \sum_{j \in \mathcal{I}} 1_{w_{ij} > 0} \right)^{-1}, b_i = b, \gamma_i = \gamma \) and \( a_i - p_i = \alpha \) for \( \mathcal{N} = \{i \in \mathcal{I} \setminus j\}. \) Let \( j \) be the “high” consumer. If \( a_j - p_j = \bar{\alpha} > n\alpha, \) then for each neighbor \( i \) of \( j, x_i^* \) is initially increasing in \( \gamma, \) whereas \( x_j^* \) is strictly monotonically decreasing in \( \gamma. \)

Proof Sketch.

Evaluate \( \frac{dx}{d\gamma} \) at \( \gamma = 0 \) and use definition of peer effects.
Theoretical Statements

**Theorem (Monotonicity of Consumption Equilibrium)**

If $a_i = a$, $b_i = b$, and $\gamma_i = \gamma \ \forall \ i \in \mathcal{I}$, then $x_i^*$ is strictly monotonically decreasing in $\gamma$ independent of the network topology $\mathcal{W}$.

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**Theorem (Monotonicity of Consumption Equilibrium)**

If \( a_i = a, \ b_i = b, \) and \( \gamma_i = \gamma \ \forall \ i \in I, \) then \( x_i^* \) is strictly monotonically decreasing in \( \gamma \) independent of the network topology \( W. \)

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Evaluate \( \frac{dx}{d\gamma} \) at \( \gamma = 0 \) and use definition of peer effects.
Theoretical Statements (cont’d.)

Theorem (Targeted Peer Effects)

For \( n = 2 \) users, the network effect reduces the sum of their consumptions iff

\[
b_1 \leq \frac{(a_1 - p)(4b_2 + 3\gamma)}{2(a_2 - p)} \quad \text{and} \quad b_2 \leq \frac{(a_2 - p)(4b_1 + 3\gamma)}{2(a_1 - p)}.
\]

This can be generalized to \( n \geq 3 \).

Proof Sketch.

Utility maximizing response of user \( i \) is \( x_i^* = \frac{a_i - p_i + \gamma_i \sum_{j \in I} w_{ij} x_j}{2(b_i + \gamma_i)} \). Result follows.

Theorem (Efficiency)

The consumption equilibrium \( x^* \) is inefficient as the social welfare \( S \) attained is suboptimal. Specifically, \( x_i^* < x_i^o \ \forall \ i \in I \), where \( x^o \) maximizes social welfare:

\[
x^o = \left( C + \frac{B}{2} + \Gamma - \frac{W^\top \Gamma}{2} - \frac{\Gamma W}{2} \right)^{-1} \frac{a}{2}.
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Allocating users per-unit subsidies \( s_i = (b_i + \gamma_i)x_i^2/2 \) can restore the social optimum.
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Network Uncertainty

Unknown Network Structure

- Let $W = W^\top$ and $\Gamma = \gamma I$
- Monopolist only has estimate $\tilde{W}$, where $\tilde{W} = \tilde{W}^\top$
- Lower bound on expected profit $\tilde{\Pi}^*$ under perfect price discrimination:

$$\frac{\tilde{\Pi}^*}{\Pi^*} \geq \frac{\lambda_{\text{min}}(C + B + 2\Gamma - \Gamma W)}{\lambda_{\text{max}}(C + B + 2\Gamma - \Gamma W) + \gamma \|W - \tilde{W}\|_2}$$

- Simulation for $n = 24$ fully connected users:
Network Uncertainty

Unknown Network Structure
- Let $W = W^T$ and $\Gamma = \gamma I$
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\]

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0
0.2
0.4
0.6
0.8
1.0
\|\tilde{W} \square W\|_2
Uncertainty of Interaction Matrix

0.90
0.92
0.94
0.96
0.98
1.00
$\tilde{\Pi}^*/\Pi^*$
Bounds on Monopolist Profit
Profit Ratio
Lower Bound
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User Selection Problem

Profit Maximization with User Selection

- Which users should be exposed to peer effects to maximize profit?
- Assume $p$ is exogenously set (by the Public Utilities Commission)
- Formulate profit-maximizing problem:

$$\text{maximize} \quad \sum_{i=1}^{n} \delta_i x_i - c_i x_i^2$$

subject to

$$x = (B + 2\Delta \Gamma - \Delta \Gamma W)^{-1} (a - p1)$$

$$\sum_{i=1}^{n} \delta_i = m, \quad \delta_i \in \{0, 1\}$$

$$\Delta = \text{diag}(\delta_1, \ldots, \delta_n)$$

- MIQCP cannot be solved analytically
- Use heuristic for targeting: Only expose highest and lowest consumers to effects
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Conclusion

Summary
- Setup of two-stage game-theoretic model for a network of electricity consumers
- Consumers seek to maximize individual utility function and derive utility from peer comparisons
- Investigated profit-maximizing pricing schemes (subgame-perfect equilibria)
- Heuristic approach for profit maximization problem of utility

Future Work
- Extend setting to sequential problem
- Incorporate fluctuating wholesale electricity prices
- Model peer effects in auction settings (incentive compatibility, ...)

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THANK YOU!
QUESTIONS?