On Calculating Power-Aware Connected Dominating Sets for Efficient Routing in Ad Hoc Wireless Networks

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1This work was supported in part by NSF grant CCR 9900646, NSF grant ANI 0073736, NSERC, and REDII.
Abstract

Efficient routing among a set of mobile hosts (also called nodes) is one of the most important functions in ad hoc wireless networks. Routing based on a connected dominating set is a promising approach, where the searching space for a route is reduced to nodes in the set. A set is dominating if all the nodes in the system are either in the set or neighbors of nodes in the set. Wu and Li proposed a simple and efficient distributed algorithm for calculating connected dominating set in ad hoc wireless networks, where connections of nodes are determined by their geographical distances. In general, nodes in the connected dominating set consume more energy to handle various bypass traffic than nodes outside the set. To prolong the life span of each node, and hence, the network by balancing the energy consumption in the network, nodes should be alternated in being chosen to form a connected dominating set. In this paper, we propose a method of calculating power-aware connected dominating set. Our simulation results show that the proposed approach outperforms several existing approaches in terms of life span of the network.

Keywords: Ad hoc wireless networks, dominating sets, energy level, mobile computing, routing, simulation
1 Introduction

An ad hoc wireless network is a special type of wireless networks in which a collection of mobile hosts with wireless network interfaces may form a temporary network, without the aid of any established infrastructure or centralized administration. If only two hosts, located closely together within wireless transmission range of each other, are involved in the ad hoc wireless network, no real routing protocol or decision is necessary. However, if two hosts that want to communicate are outside their wireless transmission ranges, they could communicate only if other hosts between them in the ad hoc wireless network are willing to forward packets for them.

We can use a simple graph $G = (V, E)$ to represent an ad hoc wireless network, where $V$ represents a set of wireless mobile hosts and $E$ represents a set of edges. An edge between host pairs $\{v, u\}$ indicates that both hosts $v$ and $u$ are within their wireless transmission ranges. To simplify our discussion, we assume all mobile hosts are homogeneous, i.e., their wireless transmission ranges are the same. In other word, if there is an edge $e = \{v, u\}$ in $E$, it indicates $u$ is within $v$’s range and $v$ is within $u$’s range. Thus the corresponding graph will be an undirected graph.

Routing in ad hoc wireless networks poses special challenges. In general, the main characteristics of mobile computing are low bandwidth, mobility, and low power. Wireless networks deliver lower bandwidth than wired networks, and hence, the information collection (during the formation of a routing table) is expensive. Mobility of hosts, which causes topological changes of the underlying network, also increases the volatility of network information. In addition, the limitation of power leads users disconnect mobile unit frequently in order to save power consumption. This feature may also introduce mobile networks more failures (also called switching on/off), which can be considered as a special form of mobility.

Traditional routing protocols in wired networks, that generally use either link state [7, 9] or distance vector [5, 8], are no longer suitable for ad hoc wireless networks. In an environment with mobile hosts as routers, convergence to new, stable routes after dynamic changes in network topology may be slow and this process could be expensive due to low bandwidth. Routing information has to be localized to adapt quickly to changes such as host movements.

*Dominating-set-based routing* [11] is based on the concept of *dominating set* in graph theory [4]. A subset of the vertices of a graph is a dominating set if every vertex not in the subset is adjacent to at least one vertex in the subset. The main idea of this approach is to reduce routing and searching to a subgraph induced from the dominating set. Moreover, the dominating set should be connected for the ease of the routing process within the induced graph consisting of dominating nodes only. Vertices in a dominating set are called *gateway* hosts while vertices that are outside a dominating set are called *non-gateway* hosts. The main advantage of connected dominating-set-based routing is that it simplifies the routing process to the one in a smaller subnetwork generated from the
connected dominating set. This means that only gateway hosts need to keep routing information. As long as changes in network topology do not affect this subnetwork there is no need to recalculate routing tables. In Figure 1, \( v \) and \( w \) are gateway hosts which are connected, \( u, x \) and \( y \) are non-gateway hosts. Each cycle in the figure corresponds to the wireless transmission range of a host. *Backbone-based routing* [1] and *spine-based routing* [2] use a similar approach, where a backbone (spine) consists of hosts similar to gateway hosts. *Cluster-based routing* [6] is another approach based on the notion of cluster. Hosts within vicinity (i.e., they are physically close to each other) form a cluster, or a *clique*, which is a complete subgraph.

Clearly, the efficiency of this approach depends largely on the process of finding a connected dominating set and the size of the corresponding subnetwork. Unfortunately, finding a minimum connected dominating set is NP-complete for most graphs. Wu and Li [11] proposed a simple distributed algorithm that can quickly determine a connected dominating set in a given connected graph, which represents an ad hoc wireless network. It is shown that Wu and Li’s approach outperforms several classical approaches in terms of finding a small dominating set and does so quickly.

In ad hoc wireless networks, the limitation of power of each host poses a unique challenge for power-aware design [10]. There has been an increasing focus on low cost and reduced node power consumption in ad hoc wireless networks. Even in standard networks such as IEEE 802.11, requirements are included to sacrifice performance in favor of reduced power consumption [3]. In order to prolong the life span of each node, and hence, the network, power consumption should be minimized and balanced among nodes. Unfortunately, nodes in the dominating set in general consume more energy in handling various bypass traffic than nodes outside the set. Therefore, a static selection of dominating nodes will result in a shorter life span for certain nodes, which in turn
result in a shorter life span of the whole network. In this paper, we propose a method of calculating
power-aware connected dominating set based on a dynamic selection process. Specifically, in the
selection process of a gateway node, we give preference to a node with a higher energy level. Our
simulation results show that the proposed selection process outperforms several existing ones in
terms of longer life span of the network.

This paper is organized as follows: Section 2 overviews the dominating-set-based routing and Wu
and Li’s decentralized formation of a connected dominating set. Section 3 proposes two extensions
to Wu and Li’s approach: one is based on node degree and the other is based on energy level. An
example is also included to illustrate different extensions methods. Performance evaluation is done
in Section 4. Finally, in Section 5 we conclude the paper.

2 Preliminaries

In this section, we review Wu and Li’s dominating-set-based routing and a marking process that
determines a connected dominating set.

2.1 Dominating-set-based-routing

Assume that a connected dominated set has been determined for a given hoc wireless network.
The routing process in a dominating-set-based routing is usually divided into three steps:

1. If the source is not a gateway host, it forwards the packets to a source gateway, which is one
   of the adjacent gateway hosts.

2. This source gateway acts as a new source to route the packets in the induced graph generated
   from the connected dominating set.

3. Eventually, the packets reach a destination gateway, which is either the destination host itself
   or a gateway of the destination host. In the latter case, the destination gateway forwards the
   packets directly to the destination host.

Each gateway host keeps following information: gateway domain membership list and gateway
routing table. Gateway domain membership list is a list of non-gateway hosts which are adjacent
to gateway hosts. Gateway routing table includes one entry for each gateway host, together with
its domain membership list. For example, given an ad hoc wireless network as shown in Figure 2
(a), the corresponding routing information at host 8 are shown as in Figure 2. Figure 2 (b) shows
that host 8 has three members 3, 10, 11 in its gateway domain membership list. Figure 2 (c) shows
Figure 2: A routing example.

the gateway routing table at host 8, which consists of a set of entries for each gateway together
with its membership list. Other columns of this table, including distance and routing information,
are not shown. The way routing tables are constructed and updated in the subnetwork generated
from the connected dominating set can be different.

2.2 Formation of connected dominating set

Wu and Li [11] proposed a simple decentralized algorithm for the formation of connected domi-
nating set in a given ad hoc wireless network. This algorithm is based on a marking process that
marks every vertex in a given connected and simple graph $G = (V, E)$. $m(v)$ is a marker for vertex
$v \in V$, which is either $T$ (marked) or $F$ (unmarked). We assume that all vertices are unmarked
initially. $N(v) = \{u|\{v, u\} \in E\}$ represents the open neighbor set of vertex $v$.

Marking process:

1. Initially assign marker $F$ to every $v$ in $V$.

2. Every $v$ exchanges its open neighbor set $N(v)$ with all its neighbors.
3. Every $v$ assigns its marker $m(v)$ to $T$ if there exist two unconnected neighbors.

In the example of Figure 1, $N(u) = \{v, y\}$, $N(v) = \{u, w, y\}$, $N(w) = \{v, x\}$, $N(y) = \{u, v\}$, and $N(x) = \{w\}$. After Step 2 of the marking process, vertex $A$ has $N(v)$ and $N(y)$, $v$ has $N(u)$, $N(w)$, and $N(y)$, $w$ has $N(v)$ and $N(x)$, $y$ has $N(u)$ and $N(v)$, and $x$ has $N(w)$. Based on Step 3, only vertices $v$ and $w$ are marked $T$.

Assume that $V'$ is the set of vertices that are marked $T$ in $V$, i.e., $V' = \{v|v \in V, m(v) = T\}$. The induced graph $G'$ is the subgraph of $G$ induced by $V'$, i.e., $G' = G[V']$. The following results show properties of the induced graph.

**Property 1** [11]: Given a graph $G = (V, E)$ that is connected, but not completely connected, the vertex subset $V'$, derived from the marking process, forms a dominating set of $G$.

**Property 2** [11]: The induced graph $G' = G[V']$ is a connected graph.

**Property 3** [11]: The shortest path between any two vertices does not include any non-gateway vertex as an intermediate vertex.

Since the problem of determining a minimum connected dominating set of a given connected graph is NP-complete, the connected dominating set derived from the marking process is normally non-minimum. Wu and Li [11] also proposed two rules based on node ID to reduce the size of a connected dominating set generated from the marking process. First of all, a distinct ID, $id(v)$, is assigned to each vertex $v$ in $G$. $N[v] = N(v) \cup \{v\}$ is the closed neighbor set of $v$, as oppose to the open one $N(v)$.

**Rule 1**: Consider two vertices $v$ and $u$ in $G'$. If $N[v] \subseteq N[u]$ in $G$ and $id(v) < id(u)$, the marker of $v$ is changed to $F$ if vertex $v$ is marked, i.e., $G'$ is changed to $G' - \{v\}$.

The above rule indicates that when the closed neighbor set of $v$ is covered by the one of $u$, vertex $v$ can be removed from $G'$ if the ID of $v$ is smaller than the one of $u$. Note that if $v$ is marked and
its closed neighbor set is covered by the one of \( u \), it implies that vertex \( u \) is also marked. When \( v \) and \( u \) have the same closed neighbor set, the vertex with a smaller ID will be removed. It is easy to prove that \( G' - \{ v \} \) is still a connected dominating set of \( G \). The condition \( N[v] \subseteq N[u] \) implies that \( v \) and \( u \) are connected in \( G' \).

In Figure 3 (a), since \( N[v] \subseteq N[u] \), vertex \( v \) is removed from \( G' \) if \( id(v) < id(u) \) and vertex \( u \) is the only dominating node in the graph. In Figure 3 (b), since \( N[v] = N[u] \), either \( v \) or \( u \) can be removed from \( G' \). To sure one and only one is removed, we pick the one with a smaller ID. We call the above process the selective removal based on node ID.

**Rule 2:** Assume that \( u \) and \( w \) are two marked neighbors of marked vertex \( v \) in \( G' \). If \( N(v) \subseteq N(u) \cup N(w) \) in \( G \) and \( id(v) = \min\{id(v), id(u), id(w)\} \), then the marker of \( v \) is changed to \( F \).

The above rule indicates that when the open neighbor set of \( v \) is covered by the open neighbor sets of two of its marked neighbors, \( u \) and \( w \), if \( v \) has the minimum ID of the three, it can be removed from \( G' \) (see the example in Figure 4). The condition \( N(v) \subseteq N(u) \cup N(w) \) in Rule 2 implies that \( u \) and \( w \) are connected. The subtle difference between Rule 1 and Rule 2 is the use of open and close neighbor sets. Again, it is easy to prove that \( G' - \{ v \} \) is still a connected dominating set. Both \( u \) and \( w \) are marked, because the facts that \( v \) is marked and \( N(v) \subseteq N(u) \cup N(w) \) in \( G \) do not imply that \( u \) and \( w \) are marked. Therefore, if one of \( u \) and \( w \) is not marked, \( v \) cannot be unmarked (change the marker to \( F \)). Therefore, to apply Rule 2, an additional step needs to be added in the marking process.

All the above examples represent just global snapshot of the dynamic topology for a given ad hoc wireless network. Because the topology of the network changes over time, the connected dominating set also needs to be updated. Wu and Li [11] showed the desirable locality feature of the marking process. More specifically, it is shown that only the neighbors of changing hosts need to update their gateway/non-gateway status.

### 3 Extented Rules for Enhancements

In this paper, we consider several extended rules for selective removal. One is based on node degree and the other one is based on energy level associated with each node. The main goals of these two extensions are different: the node-degree-based approach is to reduce the size of the connected dominating set while the energy-level-based approach is to prolong the average life span of each node. In the following, we use term node, host, and vertex interchangeably.
3.1 Node-degree-based rules

In the following, we propose two rules based on node degree (ND) to reduce the size of a connected dominating set generated from the marking process. First of all a distinct ID, $id(v)$, is assigned to each vertex $v$ in $G$. In addition, $nd(u)$ represents the node degree of $u$ in $G$, i.e., the cardinality of $u$’s open neighbor set $|N(u)|$.

Rule 1a: Consider two vertices $v$ and $u$ in $G'$. The marker of $v$ is changed to $F$ if one of the following conditions holds:

1. $N[v] \subseteq N[u]$ in $G$ and $nd(v) < nd(u)$.
2. $N[v] \subseteq N[u]$ in $G$ and $id(v) < id(u)$ when $nd(v) = nd(u)$.

The above rule indicates that when the closed neighbor set of $v$ is covered by the one of $u$, node $v$ can be removed from $G'$ if the ND of $v$ is smaller than the one of $u$. Node ID’s are used to break a tie when the node degrees of two nodes are the same. Note that if $v$ is marked and its closed neighbor set is covered by the one of $u$, it implies node $u$ is also marked. It is easy to prove that $G' - \{v\}$ is still a connected dominating set of $G$. The condition $N[v] \subseteq N[u]$ implies $v$ and $u$ are connected in $G'$.

Rule 2a: Assume that $u$ and $w$ are two marked neighbors of marked vertex $v$ in $G'$. The marker of $v$ is changed to $F$ if one of the following conditions holds:

1. $N(v) \subseteq N(u) \cup N(w)$, but $N(u) \not\subseteq N(v) \cup N(w)$ and $N(w) \not\subseteq N(u) \cup N(v)$ in $G$.
2. $N(v) \subseteq N(u) \cup N(w)$ and $N(u) \subseteq N(v) \cup N(w)$, but $N(w) \not\subseteq N(u) \cup N(v)$ in $G$; and one of the following conditions holds:
   
   (a) $nd(v) < nd(u)$, or
   
   (b) $nd(v) = nd(u)$ and $id(v) < id(u)$.
3. $N(v) \subseteq N(u) \cup N(w)$, $N(u) \subseteq N(v) \cup N(w)$ and $N(w) \subseteq N(u) \cup N(v)$ in $G$; and one of the following conditions holds:

(a) $nd(v) < nd(u)$ and $nd(v) < nd(w)$,
(b) $nd(v) = nd(u) < nd(w)$ and $id(v) < id(u)$, or
(c) $nd(v) = nd(u) = nd(w)$ and $id(v) = \min\{id(v), id(u), id(w)\}$.

The above rule indicates that when the open neighbor set of $v$ is covered by the open neighbor sets of two of its marked neighbors, $u$ and $w$ (or simply $v$ is covered by $u$ and $w$); in case (1), if neither $u$ nor $w$ is covered by the other two among $u$, $v$, and $w$, node $v$ can be removed from $G'$; in case (2), if nodes $v$ and $u$ are covered by the other two among $u$, $v$, and $w$, but $w$ is not covered by $u$ and $v$, node $v$ can be removed from $G'$ if the ND of $v$ is smaller than the one of $u$ or the ID of $v$ is smaller than the one of $u$ when their ND are the same; in case (3), when each of $u$, $v$ and $w$ is covered by the other two among $u$, $v$ and $w$, node $v$ can be removed from $G'$ if one of the following conditions holds: $v$ has the minimum ND among $u$, $v$ and $w$, the ND of $v$ is the same as the ND of $u$ but it is smaller than the one of $w$ and the ID of $v$ is smaller than the one of $u$, or the ND's of $u$, $v$ and $w$ are the same and $v$ has the minimum ID among $u$, $v$ and $w$. The condition $N(v) \subseteq N(u) \cup N(w)$ in Rule 2a implies that $u$ and $w$ are connected. Again, it is easy to prove that $G' - \{v\}$ is still a connected dominating set. Both $u$ and $w$ are marked, because the facts that $v$ is marked and $N(v) \subseteq N(u) \cup N(w)$ in $G$ do not imply that $u$ and $w$ are marked. Therefore, if one of $u$ and $w$ is not marked, $v$ cannot be unmarked (change the marker to $F'$).

3.2 Energy-level-based rules

In the following, we propose two rules based on energy level (EL) to prolong the average life span of a host, and at the same time, to reduce the size of a connected dominating set generated from the marking process. We first assign a distinct ID, $id(v)$, and an initial EL, $el(v)$, to each vertex $v$ in $G'$. In a dynamic system such as an ad hoc wireless network, network topology changes over time. Therefore, the connected dominating set also needs to change. Wu and Li [11] showed that the connected dominating set only needs to be updated in a localized manner; i.e., only neighbors of changing hosts need to update their gateway/non-gateway status. An update interval is the time between two adjacent updates in the network. Assume that $d$ and $d'$ are energy consumption in a given interval for a gateway host and a non-gateway host, respectively. That is, each time after applying both Rule 1b and Rule 2b (discussed below), EL of each gateway host will be decreased by $d$ and EL of each non-gateway host will be decreased by $d'$. When the energy level of $u$, $el(u)$, reaches zero, it is assumed that host $u$ ceases to function. In general, $d$ and $d'$ are variables which depend on the length of update interval and bypass traffic. Given an initial energy level of each host and values for $d$ and $d'$, the energy level associated with each host has multiple discrete levels.
**Rule 1b:** Consider two vertices $v$ and $u$ in $G'$. The marker of $v$ is changed to $F$ if one of the following conditions holds:

1. $N[v] \subseteq N[u]$ in $G$ and $el(v) < el(u)$.
2. $N[v] \subseteq N[u]$ in $G$ and $id(v) < id(u)$ when $el(v) = el(u)$.

The above rule indicates that when the closed neighbor set of $v$ is covered by the one of $u$, vertex $v$ can be removed from $G'$ if the EL of $v$ is smaller than the one of $u$. ID is used to break a tie when $el(v) = el(u)$.

In Figure 3 (a), since $N[v] \subseteq N[u]$, node $v$ is removed from $G'$ if $el(v) < el(u)$ and node $u$ is the only dominating node in the graph. In Figure 3 (b), since $N[v] = N[u]$, either $v$ or $u$ can be removed from $G'$. To ensure that one and only one is removed, we pick the one with a smaller EL.

**Rule 2b:** Assume that $u$ and $w$ are two marked neighbors of marked vertex $v$ in $G'$. The marker of $v$ is changed to $F$ if one of the following conditions holds:

1. $N(v) \subseteq N(u) \cup N(w)$, but $N(u) \not\subseteq N(v) \cup N(w)$ and $N(w) \not\subseteq N(u) \cup N(v)$ in $G$.
2. $N(v) \subseteq N(u) \cup N(w)$ and $N(u) \subseteq N(v) \cup N(w)$, but $N(w) \not\subseteq N(u) \cup N(v)$ in $G$; and one of the following conditions holds:
   
   (a) $el(v) < el(u)$, or
   
   (b) $el(v) = el(u)$ and $id(v) < id(u)$.

3. $N(v) \subseteq N(u) \cup N(w)$, $N(u) \subseteq N(v) \cup N(w)$ and $N(w) \subseteq N(u) \cup N(v)$ in $G$; and one of the following conditions holds:

   (a) $el(v) < el(u)$ and $el(v) < el(w)$,
   
   (b) $el(v) = el(u) < el(w)$ and $id(v) < id(u)$, or
   
   (c) $el(v) = el(u) = el(w)$ and $id(v) = \min\{id(v), id(u), id(w)\}$.

The above rule indicates that when $v$ is covered by $u$ and $w$; in case (1), if neither $u$ nor $w$ is covered by the other two among $u$, $v$, and $w$, node $v$ can be removed from $G'$; in case (2), if nodes $v$ and $u$ are covered by the other two among $u$, $v$, and $w$, but $w$ is not covered by $u$ and $v$, node $v$ can be removed from $G'$ if the EL of $v$ is smaller than the one of $u$ or the ID of $v$ is smaller than the one of $u$ when their ND are the same; in case (3), when each of $u$, $v$ and $w$ is covered by the other two among $u$, $v$ and $w$, node $v$ can be removed from $G'$ if one of the following conditions holds: $v$ has the minimum EL among $u$, $v$ and $w$, the EL of $v$ is the same as the EL of $u$ but it is
smaller than the one of \( w \) and the ID of \( v \) is smaller than the one of \( u \), or the EL's of \( u, v \) and \( w \) are the same and \( v \) has the minimum ID among \( u, v \) and \( w \). The condition \( N(v) \subseteq N(u) \cup N(w) \) in Rule 2b implies that \( u \) and \( w \) are connected. Again, it is easy to prove that \( G' - \{v\} \) is still a connected dominating set. Both \( u \) and \( w \) are marked, because the facts that \( v \) is marked and \( N(v) \subseteq N(u) \cup N(w) \) in \( G \) do not imply that \( u \) and \( w \) are marked. Therefore, if one of \( u \) and \( w \) is not marked, \( v \) cannot be unmarked (change the marker to \( F \)).

In the following, we propose another two rules based on EL to prolong the life span of each node to reduce the size of a connected dominating set generated from the marking process. Unlike Rule 1b and Rule 2b where ID is used when there is a tie in EL, in Rule 1b' and 2b', ND is used when there is a tie in EL and ID is used only when there is a tie in ND.

**Rule 1b':** Consider two vertices \( v \) and \( u \) in \( G' \). The marker of \( v \) is changed to \( F \) if one of the following conditions holds:

1. \( N[v] \subseteq N[u] \) in \( G \) and \( el(v) < el(u) \).
2. \( N[v] \subseteq N[u] \) in \( G \) and \( nd(v) < nd(u) \) when \( el(v) = el(u) \).
3. \( N[v] \subseteq N[u] \) in \( G \) and \( id(v) < id(u) \) when \( el(v) = el(u) \) and \( nd(v) = nd(u) \).

The above rule indicates that when the closed neighbor set of \( v \) is covered by the one of \( u \), node \( v \) can be removed from \( G' \) if the EL of \( v \) is smaller than the one of \( u \). When there is a tie in EL, \( v \) can be removed if the ND of \( v \) is smaller than the one of \( u \), and when there is a tie ND, \( v \) can be removed if the ID of \( v \) is smaller than the one of \( u \). It is easy to prove that \( G' - \{v\} \) is still a connected dominating set of \( G \). The condition \( N[v] \subseteq N[u] \) implies that \( v \) and \( u \) are connected in \( G' \).

**Rule 2b':** Assume that \( u \) and \( w \) are two marked neighbors of marked vertex \( v \) in \( G' \). The marker of \( v \) is changed to \( F \) if one of the following conditions holds:

1. \( N(v) \subseteq N(u) \cup N(w) \), but \( N(u) \not\subseteq N(v) \cup N(w) \) and \( N(w) \not\subseteq N(u) \cup N(v) \) in \( G \).
2. \( N(v) \subseteq N(u) \cup N(w) \) and \( N(u) \subseteq N(v) \cup N(w) \), but \( N(w) \not\subseteq N(u) \cup N(v) \) in \( G \); and one of the following conditions holds:
   (a) \( el(v) < el(u) \), or
   (b) \( el(v) = el(u) \); and \( nd(v) < nd(u) \), or, \( id(v) < id(u) \) when \( nd(v) = nd(u) \).
3. \( N(v) \subseteq N(u) \cup N(w) , N(u) \subseteq N(v) \cup N(w) \) and \( N(w) \subseteq N(u) \cup N(v) \) in \( G \); and one of the following conditions holds:
(a) \( el(v) < el(u) \) and \( el(v) < el(w) \),

(b) \( el(v) = el(u) < el(w) \); and \( nd(v) < nd(u) \), or \( id(v) < id(u) \) when \( nd(v) = nd(u) \), or

(c) \( el(v) = el(u) = el(w) \) and \( v \) satisfies Step 3 of Rule 2a.

The above rule indicates that when \( v \) is covered by \( u \) and \( w \); in case (1), if neither \( u \) nor \( w \) is covered by the other two among \( u \), \( v \), and \( w \), node \( v \) can be removed from \( G' \); in case (2), if nodes \( v \) and \( u \) are covered by the other two among \( u \), \( v \), and \( w \), but \( w \) is not covered by \( u \) and \( v \), node \( v \) can be removed from \( G' \) if the EL of \( v \) is smaller than the one of \( u \), or the EL of \( v \) is the same as the one of \( u \). In the latter case, either the ND of \( v \) is smaller than the one of \( u \) or the ID of \( v \) is smaller than the one of \( u \) when their ND are the same; in case (3), when each of \( u \), \( v \) and \( w \) is covered by the other two among \( u \), \( v \) and \( w \), node \( v \) can be removed from \( G' \) if one of the following conditions holds: the EL of \( v \) has the minimum EL among \( u \), \( v \) and \( w \), the EL of \( v \) is the same as the EL of \( u \) but it is smaller than the one of \( w \) and the ND of \( v \) is smaller than the one of \( u \) or the ID of \( v \) is smaller than the one of \( u \) when the ND of \( v \) is the same as the one of \( u \), or the EL of \( u \), \( v \) and \( w \) are the same when it satisfies Step 3 of Rule 2a. The condition \( N(v) \subseteq N(u) \cup N(w) \) in Rule 2b implies that \( u \) and \( w \) are connected. Again, it is easy to prove that \( G' - \{v\} \) is still a connected dominating set. Both \( u \) and \( w \) are marked, because the facts that \( v \) is marked and \( N(v) \subseteq N(u) \cup N(w) \) in \( G \) do not imply that \( u \) and \( w \) are marked. Therefore, if one of \( u \) and \( w \) is not marked, \( v \) cannot be unmarked (change the marker to \( F \)).

3.3 An example

Figures 6, 7, 8, 9, 10 show an example of using the proposed marking process and its extensions to identify a set of connected dominating nodes. Each node keeps a list of its neighbors and sends this list to all its neighbors. By doing so each node has distance-2 neighborhood information, i.e., information about its neighbors and the neighbors of all its neighbors.

In Figure 6, node 1 will not mark itself as a gateway node because its only neighbors 2 and 4 are connected. Node 4 will mark itself as a gateway node because there is no connection between neighbors 3 and 9 (3 and 11). After node 4 marks itself, it sends its status to its neighbors 1, 2, 3, 9, 10 and 11. This gateway status will be used to apply Rules 1 and 2 to unmark several gateway nodes to non-gateway nodes. Figure 5 (b) shows the gateway nodes (nodes with cycles) derived by the marking process without applying two rules.

After applying Rule 1, node 21 will be unmarked to the non-gateway status as shown in Figure 6 (c). The closed neighbor set of node 21 is \( N[21] = \{21, 22, 23, 24\} \), and the closed neighbor set of node 21 is \( N[22] = \{20, 21, 22, 23, 24, 25, 26, 27\} \). Apparently, \( N[21] \subseteq N[22] \). Also the ID of node 21 is less than the ID of node 22, thus node 21 can unmark itself by applying Rule 1.
After applying Rule 2, node 2 will be unmarked to the non-gateway status as shown in Figure 6 (d). Node 2 knows that its two neighbors 4 and 9 are all marked. This invokes node 2 to apply Rule 2 to check if condition $N(2) \subseteq N(4) \cup N(9)$ holds or not. The neighbor set of node 2 is $N(2) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the neighbor set of node 4 is $N(4) = \{1, 2, 3, 4, 9, 10, 11\}$, the neighbor set of node 9 is $N(9) = \{2, 4, 5, 6, 7, 8, 9, 10\}$, and therefore, $N(4) \cup N(9) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$. Apparently, $N(2) \subseteq N(4) \cup N(9)$. Node 2 has the min ID among nodes 2, 4, and 9. Thus node 2 can unmark itself by applying Rule 2.

After applying Rule 1a, both nodes 21 and 27 will be unmarked to the non-gateway status as shown in Figure 7 (e). The closed neighbor set of node 21 is $N[21] = \{21, 22, 23, 24\}$, and the closed neighbor set of node 22 is $N[22] = \{20, 21, 22, 23, 24, 25, 26, 27\}$. The closed neighbor set of node 27 is $N[27] = \{22, 25, 26, 27\}$. Apparently, $N[21] \subseteq N[22]$ and $N[27] \subseteq N[24]$. Also node 21 has the min ND among nodes 21, 22 and 27, thus both nodes 21 and 27 can unmark themselves by applying Rule 1a.

After applying Rule 2a, nodes 9, 13 and 18 will be unmarked to the non-gateway status as shown in Figure 7 (f). Node 9 knows that its two neighbors 2 and 4 are all marked. This invokes node 9 to apply Rule 2a to check if condition $N(9) \subseteq N(2) \cup N(4)$ holds or not. The neighbor set of node 2 is $N(2) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the neighbor set of node 4 is $N(4) = \{1, 2, 3, 4, 9, 10, 11\}$, the neighbor set of node 9 is $N(9) = \{2, 4, 5, 6, 7, 8, 9, 10\}$, and therefore, $N(2) \cup N(4) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$. Apparently, $N(9) \subseteq N(2) \cup N(4)$, $N(2) \subseteq N(4) \cup N(9)$, but $N(4) \not\subseteq N(2) \cup N(9)$; $N(13) \subseteq N(11) \cup N(15)$, $N(15) \subseteq N(11) \cup N(13)$, but $N(11) \not\subseteq N(13) \cup N(15)$; $N(18) \subseteq N(11) \cup N(20)$, $N(11) \not\subseteq N(18) \cup N(20)$, and $N(20) \not\subseteq N(11) \cup N(18)$. Thus nodes 9, 13 and 18 can unmark themselves by applying Rule 2a.

After applying Rule 1b, node 21 will be unmarked to the non-gateway status as shown in Figure 8 (g), where the number inside each node corresponds to the energy level of that node. The closed neighbor set of node 21 is $N[21] = \{21, 22, 23, 24\}$, and the closed neighbor set of node 22 is $N[22] = \{20, 21, 22, 23, 24, 25, 26, 27\}$. Apparently, $N[21] \subseteq N[22]$, Also the EL of node 21 is less than the EL of node 22, thus node 21 can unmark itself by applying Rule 1b.

After applying Rule 2b, nodes 2, 13 and 18 will be unmarked to the non-gateway status as shown in Figure 8 (h). Node 2 knows that its two neighbors 4 and 9 are all marked. This invokes node 9 to apply Rule 2a to check if condition $N(2) \subseteq N(4) \cup N(9)$ holds or not. The neighbor set of node 2 is $N(2) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the neighbor set of node 4 is $N(4) = \{1, 2, 3, 4, 9, 10, 11\}$, the neighbor set of node 9 is $N(9) = \{2, 4, 5, 6, 7, 8, 9, 10\}$, and therefore, $N(4) \cup N(9) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$. Apparently, $N(2) \subseteq N(4) \cup N(9)$. The EL of node 2 is as same as the EL of node 9 and the ID of node 2 is smaller than the one of node 9. $N(13) \subseteq N(11) \cup N(15)$, $N(15) \subseteq N(11) \cup N(13)$, but $N(11) \not\subseteq N(13) \cup N(15)$ and the EL of node 13 is as same as the one of node 15 and node ID of node 13 is smaller than the one of node 15.
\[ N(18) \subseteq N(11) \cup N(20), \text{ and } N(20) \not\subseteq N(11) \cup N(18), \text{ and node has the min EL among nodes 11, 18 and 20. Thus nodes 2, 13 and 18 can unmark themselves by applying Rule 2b.} \]

After applying Rule 1b', both nodes 21 and 27 will be unmarked to the non-gateway status as shown in Figure 9 (i). The closed neighbor set of node 21 is \( N[21] = \{21, 22, 23, 24\} \), and the closed neighbor set of node 22 is \( N[22] = \{20, 21, 22, 23, 24, 25, 26, 27\} \), and the closed neighbor set of node 22 is \( N[27] = \{22, 25, 26, 27\} \). Apparently, \( N[21] \subseteq N[22] \) and \( N[27] \subseteq N[22] \). Also the EL of node 21 is less than the EL of node 22; the EL of node 22 is as same as the one of node 27 and the ND of node 27 is smaller than the one of node 22; thus both nodes 21 and 27 can unmark themselves by applying Rule 1b'.

After applying Rule 2b', nodes 9, 13 and 18 will be unmarked to the non-gateway status as shown in Figure 9 (j). Node 9 knows that its two neighbors 4 and 9 are all marked. This invokes node 9 to apply Rule 2b' to check if condition \( N(9) \subseteq N(2) \cup N(4) \) holds or not. The neighbor set of node 2 is \( N(2) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \), the neighbor set of node 4 is \( N(4) = \{1, 2, 3, 4, 9, 10, 11\} \), the neighbor set of node 9 is \( N(9) = \{2, 4, 5, 6, 7, 8, 9, 10\} \), and therefore, \( N(9) \cup N(9) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\} \). Apparently, \( N(9) \subseteq N(2) \cup N(4) \). The EL of node 2 is as same as the one of node 9 and the ID of node 2 is smaller than the one of node 9. \( N(13) \subseteq N(11) \cup N(15) \), \( N(15) \subseteq N(11) \cup N(13) \), but \( N(11) \not\subseteq N(13) \cup N(15) \) and the EL of node 13 is as same as the one of node 15 and node ID of node 13 is smaller than the one of node 15; \( N(18) \subseteq N(11) \cup N(20) \), and \( N(20) \not\subseteq N(11) \cup N(18) \); node 18 has the min EL among nodes 11, 18 and 20. Thus nodes 9, 13 and 18 can unmark themselves by applying Rule 2b.
Figure 6: An example of Rule 1 and Rule 2.

Figure 7: An example of Rule 1a and Rule 2a.
(g) Marked gateways by applying Rule 1b
(h) Marked gateways by applying Rule 2b

Figure 8: An example of Rule 1b and Rule 2b.

(i) Marked gateways by applying Rule 1b’
(j) Marked gateways by applying Rule 2b’

Figure 9: An example of Rule 1b’ and Rule 2b’.
4 Performance Evaluation

In this section, we compare different approaches for determining a connected dominating set in an ad hoc wireless network with and without applying two rules and their variations. Specifically, we measure the size of the connected dominating set generated from the marking process and compare it with the size of the connected dominating set after applying different rules, which include the rules based on ID, the rules based on ND, and the rules based on EL and their variations. In addition, the average life spans of the network under different rules are also simulated. The simulation is conducted in a $100 \times 100$ 2-D free-space by randomly allocating a given number of hosts ranging from 3 to 100. The radius of transmission range is assumed to be 25, and the energy level of each host is initialized to 100. $c$ represents the probability of movement for each host ($c$ is 0.5 in our simulation). For each host in an update interval, $\text{rand}(0, 1)$, a random number in $[0...1]$, is associated with each node. If the number is less than $c$, it represents that the corresponding host remains stable in the corresponding interval. If the number is greater than or equal to $c$, the corresponding host moves $l$ units in direction $\text{dir}$, where $\text{dir} = \text{rand}(1, 8)$ is a random number in $[1...8]$ that represents eight directions: E, S, W, N, SE, NE, SW and NW and $l$ is a random number in $[1...6]$. Since each host has the same transmission radius, the generated graph is an undirected one. The simulation is conducted using the following procedure:

1. An undirected graph is randomly generated with each host assigned a uniform energy level.

2. Start a new update interval by applying the marking process to generate gateway hosts, then applying four sets of rules: rules based on ID, rules rules based on ND (1a and 2a), and rules based on EL (1b, 2b, 1b' and 2b'). Record the number of gateway hosts generated in the current interval.

3. The energy level of each host is reduced by $d$ and $d'$ depending on its status (gateway/non-gateway). If the energy level of one host becomes zero, the simulation stops and records the number of update intervals. Otherwise, each host roams around the given 2-D space based on the given probability model and a new graph is generated, and then, goto step 2.

To simplify our simulation, we assume that update intervals are homogeneous, i.e, $d$ and $d'$ are the same for all intervals. $d'$ is assumed be a constant (unit value) which is irrelevant to the size of the network. However, three models are used for $d$ which is a function of bypass traffic at a gateway host.

1. $d$ is a constant (assume $d = 2/|G'|$ in simulation, a normalized constant).

2. $d$ is proportional to the total number of hosts ($N$) (assume $d = N/|G'|$ in simulation).
3. \( d \) is proportional to the total number of different pairs of hosts \((N(N - 1)/2)\) (assume \( d = (N(N - 1)/2)/(10 \* |G'|) \) in simulation).

Models 2 and 3 are more realistic since the pass traffic depends on the total number of nodes \((N)\) in the network.

Two sets of simulation studies have been conducted. In the first one, we record the average number of gateway hosts. In the second one, we record the average number of update intervals when the first host runs out of battery. Figure 10 shows results of the first simulation. Figures 11, 12, and 13 show three results of the second simulation based on different selections of \( d \). In these figures, NR, ID, ND, EL1, and EL2 represent marking process without applying rules (no rule), Rule 1 and Rule 2 (based on ID), Rule 1a and Rule 2a (based on ND), Rule 1b and Rule 2b (based on EL), and Rule 1b' and Rule 2b' (based on EL), respectively. It is clear that ND and EL2 are the best in terms of the smaller number of gateway hosts. In the second simulation, ND, EL1 and EL2 stay very close with ID being clearly the worst in terms of average life span when \( d \) is a constant. When \( d \) is dependent of \( N \) space (the size of the network), EL1 is clearly the winner although it does not generate the smallest set of connected dominating set. Trade offs are possible by increasing the size of the connected dominating set for a longer life span of the network.

5 Conclusions

In this paper, we have extended Wu and Li’s distributed algorithm for calculating a connected dominating set in a given ad hoc wireless network. The connected dominating set is selected based on the node degree and the energy level of each host. The objective is device a selection scheme so that the overall energy consumption is balanced in network, and at the same time, a relatively
Figure 11: Comparison of different selective removal rules when $d = 2/|G'|$.

Figure 12: Comparison of different selective removal rules when $d = N/|G'|$.

Figure 13: Comparison of different selective removal rules when $d = (N(N - 1)/2)/(10 \times |G'|)$. 
small connected dominating set is generated. A simulation study has been conducted to compare
the life span of the network under different selection policies. The results have shown that the
proposed approach based on energy level is clearly the best in terms of the longer life span of the
network. Our future work will perform more in-depth simulation under different settings.

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