Third Order Intermodulation Distortion in Capacitive-Gap Transduced Micromechanical Filters

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Abstract—The third-order intermodulation distortion in properly terminated high-order bridged clamped-clamped beam (CC-beam) channel-select micromechanical filters has been measured for the first time and found to be appreciably higher than seen on unterminated stand-alone CC-beams. In particular, a three-resonator bridged 8-MHz filter with 140nm gaps posts a measured IIP$_3$ of 25dBm and 36dBm for two-tone offsets of 200kHz and 400kHz, respectively, which are much larger than the -11.7dBm for a stand-alone 9.2MHz CC-beam with 200kHz two-tone offset. For the case where the required minimum signal-to-noise ratio is 10dB, the IIP$_3$ of 36dBm translates to an impressive spurious-free dynamic range of 102.3dB. The result matches well the prediction of a new model for non-linearity that incorporates not only parallel-plate capacitor nonlinearity, but also the influence of the filter structure, where Q-loading by the termination reduces the degree of out-of-channel tone suppression, but also reduces the amplitude of resonator motion, which then improves the IIP$_3$. The full model incorporates these phenomena, plus dependencies on dc-bias, gap spacing, and electrode area.

Keywords—Third-order intermodulation, micromechanical resonators, micromechanical filters, channel selection, capacitive-gap transducer.

I. INTRODUCTION

Channel-select filters like those of this work, capable of rejecting all interferer signals relax the dynamic range requirements on subsequent stages, e.g., the LNA and the mixer, thereby greatly reduce power consumption. However, the degree of interferer suppression depends strongly on the linearity of the filter, which if not sufficiently linear, can also generate intermodulation spurs even after rejecting interferers.

A recent 223-MHz capacitive-gap transduced disk micromechanical filter [1] proved that with electrode-to-resonator gaps on the order of 39nm, its transduction was sufficient to achieve a channel-select response with a small percent bandwidth of 0.09% and in-band insertion loss of 2.7dB, as well as a 50dB of out-of-channel rejection, all while maintaining a termination resistance around 500Ω which is easily L-network-matched to conventional antennas. However, use of such devices in RF communication circuits has so far been deferred due to skepticism on the linearity of these filters, which must satisfy the very strict specifications of present-day wireless standards. For example, the European GSM standard for mobile communications [2] requires a minimum total IIP$_3$ of -18dBm in the receiver path to insure adequate suppression of interferer signals from adjacent channels.

Previous works [3, 4, 5] on this issue were limited to two-tone measurements on single resonators, as opposed to multiple resonator filters. Pursuant to determining the linearity of high-order capacitive-gap micromechanical filters, this paper represents a complete analytical formulation for the IIP$_3$ of such devices and then verifies the formulation via experimental measurement on 3rd- and 4th-order 8-MHz clamped-clamped beam filters, shown in Fig. 1. With IIP$_3$ on the order of 36dBm for 400-
kHz tone separations, these filters possess sufficient linearity for a large range of RF front-end or IF applications.

II. ANALYTICAL MODEL

Before presenting the model for multi-resonator filter nonlinearity, it is instructive to first review the existing formulation for stand-alone resonators. This section starts with intermodulation distortion fundamentals, then moves to deeper analysis of intermodulation distortion that accounts for more nonlinear terms than previous work [4], and culminates with a comprehensive formulation for multi-resonator filters.

A. Intermodulation Distortion Fundamentals

Any practical system that entails some nonlinearity is most conveniently modeled by a Taylor series:

\[ y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots \]  

(1)

where \( x \) and \( y \) are input and output and \( a_i \)'s describe system behavior. For the case of the third order intermodulation distortion, the input of consequence takes the form

\[ x = S_0 \cos(\omega_0 t) + S_1 \cos(\omega_1 t) + S_2 \cos(\omega_2 t) \]  

(2)

where two interferer signals at frequencies \( \omega_1 \) and \( \omega_2 \) add to the desired signal at frequency \( \omega_0 \). When this signal passes through a nonlinear transfer function of the form in (1), the output includes not only a scaled version of the input, but also spurious signals at frequencies not present in the original input. Specifically, inserting (2) into (1), harmonics as well as intermodulation components arise

\[ y = \cdots + a_1 S_0 \cos(\omega_0 t) + \frac{1}{2} a_2 S_0^2 \cos(2 \omega_0 t) \]

\[ + \frac{3}{4} a_3 S_0^3 \cos(3 \omega_1 - \omega_2) t + \cdots \]  

(3)

The harmonic terms occupy much higher frequencies, so can be filtered out easily. However, intermodulation between two equispaced input tones can produce output terms at frequencies near the desired signal. In particular, third-order nonlinearity can generate an output component exactly at the desired frequency if one tone is twice as far from \( \omega_0 \) as the other one, as shown in Fig. 2 and Eq. (3). This third-order intermodulation component, \( IM_3 \), can directly impact the signal-to-noise ratio of the desired channel and eventually mask the channel completely, if the system nonlinearity is too large. Hence, \( IM_3 \) distortion must be constrained below a minimum acceptable value.

The third-order input intercept point (IIP3) is defined as the input power level at which the extrapolated intermodulation component has the same power as the fundamental output. In general, a larger IIP3 indicates smaller nonlinearity (or better linearity) in a given system and hence smaller intermodulation component generation, which is a design goal of communication systems.

As shown in Fig. 2, while a micromechanical channel-select filter attenuates out-of-channel interferer signals, its nonlinearity can still result in intermodulation components at its output that corrupt the desired signal, which motivates the necessity to design for maximum filter IIP3. Note that interferer signals for a channel-select filter are outside the filter passband, unlike the case for a band-select filter, for which in-band interferers must be considered.

B. Capacitive-Gap Transducer Nonlinearity

Nonlinearity in either resonator stiffness [7] or capacitive-gap transduction [4] is often the most important contributors to filter nonlinearity. The former becomes significant when large displacement induces non-negligible internal strain in the resonator, which manifests as a stiffness nonlinearity. Since interferer signals are out-of-channel for a channel-select filter, the induced displacement is generally very small, so the stiffness nonlinearity is negligible. Therefore, transducer nonlinearity which translates to input force nonlinearity generates the intermodulation term of a channel-select filter.

As shown in Fig. 3, the transducers in the micromechanical filters of Fig. 1 are simplified to a parallel-plate capacitor composed of a gap spacing of \( d_0 \) separating two polysilicon electrodes: one fixed, the other suspended by an effective stiffness \( k_r \). The transducer takes as input a dc voltage \( V_F \) applied to one electrode and an ac excitation voltage \( v_i \) applied to the other. Since no dc current flows through the capacitor, there is no dc power consumption. The free electrode moves in response to force \( F_{\text{os}} \) generated by the input voltage combination following a biquad frequency response of Fig. 3, where the electrode effective stiffness \( k_r \) and quality factor \( Q \) determine the maximum displacement according to

\[ \frac{X}{F}(\omega) = \frac{1}{k_r} \Theta(\omega) \]  

(4)
\[ \Theta(\omega) = \frac{1}{1 - (\omega/\omega_0)^2 + j\omega/(Q\omega_0)} \]  

where \( \omega_0 \) is the resonance frequency from the ratio of \( k \) and effective mass \( m \).

The actuation force \( F_{\text{act}} \) that drives the resonator to move a displacement \( x \) derives from the co-energy in the capacitor [4]:

\[
F_{\text{tot}} = \frac{1}{2} (V_p - v_i)^2 \frac{\partial C}{\partial x} = \frac{1}{2} (V_p - v_i)^2 \frac{\partial}{\partial x} \left( C_0 \left( 1 - \frac{x}{d_0} \right)^{-1} \right) 
\]

\[
F_{\text{tot}} = \frac{1}{2} (V_p - v_i)^2 \frac{C_0}{d_0} \left( 1 + \frac{2x}{d_0} + \frac{3x^2}{d_0^2} + \frac{4x^3}{d_0^3} + \cdots \right)
\]

where \( C_0 \) is the parallel-plate capacitance at rest, and \( d_0 \) is the electrode-to-resonator gap spacing, neglecting beam bending under applied dc-bias voltage \( V_p \). For purposes of \( IIP_3 \) determination, the input voltage comprises the sum of two out-of-band signals:

\[
v_i = V_1 \cos(\omega_1 t) + V_2 \cos(\omega_2 t)
\]

The \( F_{\text{act}} \) expression (6) has a second-order dependence on input voltage \( v_i \), which combined with the nonlinear nature of the change in the capacitance per unit displacement \( \partial C/\partial x \), results in higher order nonlinearities. In particular, out-of-band interferer signals at \( \omega_1 \) and \( \omega_2 \), equispaced from \( \omega_0 \) and from each other (\( \omega_1 - \omega_0 = \omega_2 - \omega_0 \)), induce small displacements at \( \omega_1 \) and \( \omega_2 \), respectively. Nonlinear interaction between these displacements and with the input voltages produces a displacement spur at \( \omega_0 \) via third-order intermodulation terms, as shown in Fig. 4. Here, the total resulting resonator displacement takes the form:

\[
x = X_0 \cos(\omega_0 t + \phi_0) + X_1 \cos(\omega_1 t + \phi_1) + X_2 \cos(\omega_2 t + \phi_2)
\]

where the \( \phi_0 \)s are the relative phases between \( x \) and input voltage \( v_i \), and the \( X_0 \)s are displacement amplitudes. The equation of motion governs the relation between \( F_{\text{tot}} \) and \( x \):

\[
m_{\text{eff}} \ddot{x} + b_{\text{eff}} \dot{x} + k_{\text{eff}} x = F_{\text{tot}}
\]

Neglecting nonlinear force terms, approximate expression for the out-of-band \( X_1 \) and \( X_2 \) magnitudes take the form

\[
X_1 = \frac{V_p V_1 C_0}{k_r d_0} \Theta(\omega_1)
\]

\[
X_2 = \frac{V_p V_2 C_0}{k_r d_0} \Theta(\omega_2)
\]

If \( \omega_1 \) and \( \omega_2 \) are sufficiently distant from the center frequency \( \omega_0 \), the approximate phases of the displacement components become

\[
\phi_0 = 90^\circ, \phi_1 = \phi_2 = 0 \text{ or } 180^\circ
\]

Substituting (10)-(12) into (8) and using this \( x \) in (6) yields the input force intermodulation term (13) shown at the bottom of the next page, where \( \varepsilon \) is the permittivity in vacuum, \( A_0 = W_i W_c \) is the electrode-to-resonator overlap area, \( \Theta = \Theta(\omega_1) \) and \( \Theta_2 = \Theta(\omega_2) \).

The third-order input intercept point (\( IIP_3 \)) is defined as the input power for which the third-order modulation \( IM_3 \) term is equal to the fundamental term. Assuming the nonlinearity associated with current generation at the output transducer is negligible compared to those associated with force generation at the input transducer, equating \( F_{\text{Im3}} \) (13) to the fundamental force \( F_{\text{fund}} \) (14) when exciting the resonator at \( \omega_0 \), and then solving for \( V_i \), yields the expression for input voltage \( V_{\text{IIP3}} \), as given in (15). Recognizing that the linear force component is

\[
F_{\text{fund}} = \frac{V_i \varepsilon_0 A_0}{d_0^2} V_1
\]

the corresponding input power, \( IIP_3 \), then takes the form

\[
IIP_3 = \frac{V_{\text{IIP3}}^2}{2R_T}
\]

where \( R_T \) is the total resistance in the system, seen at the input, dominated by the resonator motional resistance.

A careful examination of equation (15) shows that \( V_{\text{IIP3}} \) is dependent on not only material properties, but also resonator and electrode geometry. In particular, \( V_{\text{IIP3}} \) increases with decreases in \( V_p \) and \( A_0 \) and with increases in \( d_0 \) and \( k \). Since these parameters also affect the motional resistance \( R \), there is a trade-off between the \( V_{\text{IIP3}} \) and the motional resistance of a capacitive-gap transducer.

C. Filter Considerations in \( IIP_3 \) Calculation

Although the parallel-plate approximation is still valid for a capacitive-gap micromechanical filter, the \( IIP_3 \) expression requires two major modifications to account for the following:

1) Total Resistance:

Proper termination is essential to minimize in-channel ripple and attain a flat passband. Before termination, the resonator \( Q \)'s are too large and the filter passband consists of distinct peaks, as
shown in Fig. 5(a). The termination resistance needed to flatten the passband of a filter with center frequency \( f_0 \), bandwidth \( B \), and small insertion loss is [8]:

\[
R_q = \frac{Q_{f_{\text{itr}}}}{Q_{f_{\text{itr}}}} R_x = \frac{Q_{f_{\text{itr}}}}{Q_{f_{\text{itr}}}} R_x \tag{17}
\]

where \( Q_{f_{\text{itr}}} = f_0/B \). In contrast to the single-resonator case, these termination resistors \( R_q \) dominate the resistance in the system, so \( R_1 \sim R_2 \) in equation (16) when calculating the \( IIP_3 \) of filters.

2) High-Order Mechanical System

The equation of motion (9) describes only single resonators, whereas the micromechanical filters of Fig. 1 comprise several resonators linked by coupling beams. Solving the complete mechanical system yields transfer functions listed in filter cookbooks, such as [8]. For example, the transfer function of a second-order filter coupled by quarter-wavelength coupling beams takes the form

\[
\frac{X}{F}(\omega) = \frac{1}{k_r} \Theta(\omega) \tag{18}
\]

where \( \Theta(\omega) = \left(1 - \left(\frac{\omega/\omega_0}{1 + f_{\text{bw}}/Q_{f_{\text{itr}}}}\right)^2 \right) \left(1 - \left(\frac{\omega/\omega_0}{1 - f_{\text{bw}}/Q_{f_{\text{itr}}}}\right)^2 \right) \right) \frac{\omega}{\omega_0} \tag{19}\)

where \( f_{\text{bw}} \) is the filter percent bandwidth. Note that terminating the filter with \( R_q \) loads the quality factors of its end resonators, which is why \( Q_{f_{\text{itr}}} \) appears in (19). The loading effect reduces resonator displacement at resonance \( X_0 \propto Q/k_1 \) and reduces out-of-band rejection leading to larger out-of-band displacements \( X_1 \) and \( X_2 \). However, compared with a similarly terminated stand-alone resonator, the high-order transfer function of a multi-resonator filter shown in Fig. 5 provides a larger out-of-band attenuation that reduces out-of-band displacement, decreasing \( IM_3 \) hence increasing \( IIP_3 \).

III. COMPLETE FORMULATION FOR \( IIP_3 \)

While the parallel-plate capacitor approximation provides an analytical solution and design insight for the effect of third-order nonlinearities, it neglects phenomena such as beam bending due to de-bias voltage and location-dependent effective stiffness. This can introduce errors in the \( IIP_3 \) calculation.

\( V_p \)-induced beam bending results in \( C_0 \) and \( d_0 \) of (10) and (11) that are not constant, but rather functions of location on the y-axis given by \( d(y) \) in [5], neglecting variation along the z-axis. On the other hand, the effective resonator stiffness is also location dependent and changes according to the mode shape at the point of interest. For similar reasons, \( X_1 \) and \( X_2 \) vary along the beam length (the y-axis in Fig. 1(b)), approaching zero near the anchors. In general, with knowledge of the peak displacement (at the beam midpoint) governed by the resonator lumped model, displacements at other beam locations follow from the resonator mode shape. To correctly determine the total actuation force \( F_{\text{act}} \), intermodulation force components \( dF_{\text{IM}} \) in infinitesimal regions \( dy \) at locations \( y \) should be integrated over the entire beam length.

Since effective stiffness increases dramatically moving away from the beam center, displacement is a strong function of location. This phenomenon can result in a \( V_{\text{IM}} \) value twice as large as that derived using a simple parallel-plate approximation. Including these modifications is essential to better explain the experimental results.

IV. EXPERIMENTAL RESULTS

Third- and fourth-order clamped-clamped beam bridged micromechanical filters [9] like those of Fig. 1 were designed and fabricated to test the efficacy of the filter \( IIP_3 \) formulations. Besides the quarter-wavelength coupling beams connecting adjacent resonators to form the basic filter transfer functions, quarter- and third-quarter-wavelength bridging beams also couple the first and last non-adjacent resonators [9] to provide parallel paths for mechanical signals from input to output. With proper

\[
F_{\text{IM3}} = \frac{1}{k_r} \left\{ \frac{V_p^5 (\epsilon_0 A_0)^4}{d_0^5} \left[ \frac{9}{4} |\theta_1|^2 |\theta_2| + \frac{9}{4} |\theta_1||\theta_2|^2 + \frac{3}{8} |\theta_2|^3 \right] \right\}
\]

\[
+ \frac{3 V_p^2 (\epsilon_0 A_0)^3}{d_0^5} \left[ |\theta_1| + 2|\theta_2| \right] \right\} + \frac{3 V_p^5 (\epsilon_0 A_0)^4}{2 k_r^3 d_0^5} |\theta_1|^2 |\theta_2| \right\} \tag{13}
\]

\[
V_{\text{IM3}} = \frac{1}{k_r} \left\{ \frac{V_p^2 (\epsilon_0 A_0)^3}{d_0^5} \left[ \frac{9}{4} |\theta_1|^2 |\theta_2| + \frac{9}{4} |\theta_1||\theta_2|^2 + |\theta_1|^3 \right] \right\}
\]

\[
+ \frac{3 V_p^2 (\epsilon_0 A_0)^3}{4 k_r^3 d_0^5} \left[ |\theta_1| + 2|\theta_2| \right] \right\} + \frac{3 V_p^5 (\epsilon_0 A_0)^4}{2 k_r^3 d_0^5} |\theta_1|^2 |\theta_2| \right\} \tag{15}
\]
design, these parallel paths are out of phase with the primary paths so insert a zero in the filter transfer function, effectively adding a notch in the filter frequency response and sharpening the passband-to-stopband roll-off. Each resonator in the filters of Fig. 1 has a tuning electrode to compensate any deviation in the resonator center frequency due to fabrication tolerances, which then enables near perfect (flat) passbands.

A. Fabrication

Filters were fabricated using a previously described vertical gap surface-micromachining process [5], summarized by the process cross-sections in Fig. 6, with some modifications to incorporate a damascene process to enable a thick, low resistance interconnect layer. Fabrication starts with deposition of 2µm-thick silicon dioxide and 400nm-thick silicon nitride on the silicon substrate to electrically isolate different interconnects. Then 1.5µm-thick of oxide is deposited and patterned using a negative interconnect mask to form a mold that defines thick interconnects after deposition of 2µm-thick polysilicon and polishing down to the oxide to yield Fig. 6(a). This damascene process removes all the structure topography, allowing for more precise definition of resonator center frequency. Next, 137nm of sacrificial oxide is deposited and patterned to open anchor vias, as shown in Fig. 6(b), followed by successive depositions of 2µm-thick structural P-doped-polysilicon and 500nm of oxide hard mask material, respectively. Patterning via the filter structure mask and etching then yields Fig. 6(c). A wet dip in hydrofluoric acid then releases devices, leaving free standing structures, such as shown in Fig. 6(d). Fig. 6(e) present the SEM of a released third-order λ/4-bridged micromechanical filter.

B. Measurement Results

Fig. 7 presents the frequency response spectra of the terminated bridged filters, biased at $V_R$ and measured via the circuit shown in Fig. 1. These filters post in-band insertion losses and ripples on the order of ~1dB and 0.2dB, respectively, while also exhibiting respectable 40dB out-of-band rejections. Table I summarizes the filter characteristics.

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Table I. HF Micromechanical Filter Summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>3CC-λ/4</th>
<th>3CC-3λ/4</th>
<th>4CC-3λ/4</th>
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<td>µRes. Beam Length, $L_{µ}$</td>
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<td>µRes. Beam Thickness, $h$</td>
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<td>137nm</td>
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<td>&lt;0.2dB</td>
<td>&lt;0.2dB</td>
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<td>Predicted third-order input intercept point, $IP_3$</td>
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<td>Measured third-order input intercept point, $IP_3$</td>
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<tr>
<td>Sp. Free Dyn. Range, $SFDR$ ($SNR_{min}=10$dB)</td>
<td>102.3dB</td>
<td>89.94dB</td>
<td>91.0dB</td>
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Fig. 8 presents $IP_3$ measurements on (a) a 3CC-3λ/4 bridged filter and (b) a 4CC-3λ/4 bridged filter. Here, a spectrum analyzer measured the output power response to two-tone inputs with frequency spacings like those pictured in Fig. 2. These filters show (a) an $IP_3$ of 17dBm for a tone separation of 125kHz; and in (b), $IP_3$’s of 22.7dBm and 27dBm for tone separations of 80 and 160kHz, respectively.
The expression for out-of-band spurious-free dynamic range takes the form [10]:

\[
SFDR = \frac{2}{3} (IIP_3 + 174\text{dBm} - 10 \log(B)) - SNR_{\text{min}}
\]

(20)

where \(SNR_{\text{min}}\) is the required minimum signal-to-noise ratio. Assuming \(SNR_{\text{min}}=10\text{dB}\), these \(IIP_3\)’s correspond to \(SFDR\)’s of 89.94dB for the 3CC-3λ/4 bridged filter with 125kHz tone separation; and 91dB and 93.9dB for the 4CC-3λ/4 bridged filter with 80 and 160kHz tone separations, respectively.

Fig. 9 presents \(IIP_3\) measurements on a 3CC-3λ/4 bridged micromechanical filter. Specifically, (a) shows two-tone measurements for different tone spacings, while (b) plots measured \(IIP_3\) as a function of percent tone spacing (\(\Delta\omega/\omega_0\)). As expected, as out-of-band interferers move further away from the center frequency, induced displacements \(X_1\) and \(X_2\) decrease and \(IIP_3\) increases. This filter achieves \(IIP_3\) of 11dBm, 22dBm and 36dBm corresponding to \(SFDR\)’s of 85.67dB, 93dB and 102.34dB for tone separations of 80, 160 and 400kHz, respectively. These \(IIP_3\)’s are more than 30dBm higher than previous marks for single resonators [4, 5].

V. CONCLUSION

The analytical expression for multi-resonator capacitive-gap transduced channel-select CC-beam bridged micromechanical filters not only matches measured \(IIP_3\)’s as high as 36dBm for a tone separation of 400kHz, but also provides insight into just how good the linearity of such filters can be. In particular, the formulation shows that capacitive-gap transducer nonlinearity depends on not only material properties, but also on structure and electrode geometry, all of which serve as knobs through which one might design for a specific linearity requirement. Channel-select filters like those of this work, capable of rejecting all interferer signals and passing only the desired signal, and doing so with the high \(IIP_3\)’s demonstrated, stand to greatly reduce dynamic range requirements on subsequent stages, in turn enabling much longer battery lifetimes for ultra-low power wireless front-ends.

ACKNOWLEDGMENT

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REFERENCES


Fig. 8. Third-order input intercept point \(IIP_3\) measurements of the (a) 3CC-3λ/4 and (b) 4CC-3λ/4 bridged micromechanical filters.

Fig. 9. (a) Two-tone measurement with tone separations of 80, 160 and 400kHz and (b) measured \(IIP_3\) versus percent tone separation (\(\Delta\omega/\omega_0\)) for a terminated 3CC-3λ/4 filter.

\[
\begin{align*}
\text{Out. Power [dBm]} & = 89.94 \text{dB} \\
\text{Fund.} & \quad 1.5\% \\
\text{1%} & \quad \text{2%} \\
\text{2%} & \quad \text{5%}
\end{align*}
\]