# A MICROMECHANICAL HIGH- $Q$ ELLIPTIC DISK DISPLACEMENT AMPLIFIER 

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#### Abstract

An $89-\mathrm{MHz}$ micromechanical elliptic-disk displacement amplifying resonant switch ("resoswtich") (cf. Fig. 1(c)) with a displacement gain of 2.04 has achieved a $Q$ of 101,600 - more than $9 \times$ higher than that of previous arraybased (Fig. 1(a) [1]) and slotted-disk (Fig. 1(b) [2] [3]) approaches. Key to this demonstration are: 1) gain-inducing axial stiffness differences derived from ellipse geometry rather than previous $Q$-degrading slots; and 2) an elliptic resonance strain field that better negates energy loss at a center anchor, allowing for $Q$ 's even higher than circular disks. This ability to affect displacement amplification while maintaining $Q>100,000$ should provide more than $10-\mathrm{dB}$ sensitivity improvement for recent resoswitch-based zero-quiescent radio receivers [4].


## INTRODUCTION

The recent demonstration of a wireless receiver employing a micromechanical resoswitch (cf. Fig. 1) to permit continuous listening while consuming no power until a valid input signal arrives [4] has sparked renewed interest in ultra-low power sensor networks that dispense with the complexity of sleep/wake cycles and remain awake and listening at all times. Indeed, such a zero-quiescent power receiver, with block diagram pictured in Fig. 2, obviates the need for real-time clocks to synchronize the wake-up times of networked sensor motes, thereby eliminating their cost and power consumption.

Although the -60 dBm sensitivity demonstrated in [4] is already sufficient for many short range wireless sensor applications, its $20-\mathrm{kHz}$ frequency relegates it to low bit rate long-range applications, e.g., time transfer from an atomic reference (perhaps at NIST) to a portable clock thousands of miles away that receives atomic time via radio transmission in the $60-\mathrm{kHz}$ LF WWVB band [5] [6]. To address more abundant higher bit rate applications, operation at higher frequencies spanning 60 MHz to 5.8 GHz is desirable.

Since (as shown in [4]) the sensitivity of a receiver like that of Fig. 2 is proportional to frequency and inversely proportional to the $Q$ of its resoswitch, the $Q$ of 10,500 achieved in [1] must rise to at least 100,000 if reasonable sensitivity is to be maintained at frequencies in the $80-$ 100 MHz range. In addition, displacement amplification is desirable to not only prevent input impacting while promoting output impacting [7], but also to improve impedance matching and in some cases further improve sensitivity. Unfortunately, previously demonstrated displacement amplifying resoswitches in this frequency range suffer $Q$ degradation, negating any sensitivity advantages. In particular, energy-balancing array-based resoswitches post $Q$ 's of only 10,500 [1], while slotted-disk ones on the order of only 5,000 [2], the latter far lower than the $>100,000$ normally exhibited by their base resonator designs. In effect,


Fig. 1: Descriptions of (a) array; (b) slotted-disk; and (c) elliptic-disk displacement amplifiers, along with a table comparing their performance, where the elliptic-disk displacement amplifier clearly presents the highest $Q$.


Fig. 2: Schematic of a resoswitch-enabled zero-quiescent power receiver architecture.
these devices sacrifice $Q$ to attain displacement gain.
Pursuant to breaking the apparent $Q$ versus displacement gain trade-off, this work presents an $89-\mathrm{MHz}$ micromechanical elliptic-disk displacement amplifying resoswitch that preserves $Q$ by 1) generating gain-inducing axial stiffness differences from ellipse geometry rather than previous $Q$-degrading slots; and 2 ) using an elliptic resonance strain field that better negates energy loss at a center anchor, allowing for $Q$ 's in some cases even higher than circular disks. These strategies together permit a displacement gain of 2.04 with a $Q$ of 101,600 more than $9 \times$ higher than that of previous array-based [1] and slotted-disk [2]


Fig. 3: FEM simulated mode shapes comparing displacements along input and output axes for (a) a conventional wine-glass disk resonator; (b) a slotted-disk displacement amplifier; and (c) an elliptic disk.
[3] approaches. Measurements reveal that the $Q$ 's of elliptic disks vary with ellipse aspect ratio $(A R)$ defined as the major axis length $a$ to the minor axis $b$, shown in Fig. 1(c). Specifically, a $95-\mathrm{MHz}$ ellipse with $A R=1.2(a / b=$ $21.91 \mu \mathrm{~m} / 18.26 \mu \mathrm{~m}$ ) exhibits the lowest $Q$, while one with $A R=1.6$ and beyond restores the $Q$ to equal or sometimes exceed that of a circular disk with $A R=1$.

## ADVANTAGES OVER PREVIOUS RESOSWITCHES

Fig. 1 compares the present elliptic design in (c) with previously-demonstrated displacement amplifying methods based on (a) energy-balanced coupling of asymmetric disk array-composites [1]; and (b) stiffness engineering via strategic geometrical cuts [2] [3].

The disk array resoswitch of Fig. 1(a) generates displacement gain by coupling an input disk array-composite to an output one (using a single disk in Fig. 1(a)) via a quar-ter-wavelength beam that effectively balances the energy on both sides. Since the stiffness of the 4-disk half-wave-length-coupled input array ( $n_{i n}=4$ ) in Fig. 1(a) is four times larger than that of the single output disk ( $n_{\text {out }}=1$ ), the output disk must move more than the input ones to possess equal energy-specifically, $\sqrt{n_{\text {in }} / n_{\text {out }}}=2 \times$ more-thereby providing displacement gain. So far, the only demonstrated such mechanical circuit of [1] utilized radial mode disk resonators, which constrained its $Q$ to only 10,500 . Based on [8], a wine-glass disk based version should do much better, but this has yet to be demonstrated.

On the other hand, the slotted-disk of Fig. 1(b) generates displacement gain by cutting slots along one axis (i.e., the output axis) of a wine glass disk thereby lowering the stiffness in that direction, hence raising its displacement relative to that along the orthogonal input axis. The result: displacement amplification in a single mechanical structure. Unfortunately, the slots induce higher energy losses at the slot-induced stressed corners, which degrade $Q$ to $\sim 10,000$, which is many times smaller than commonly exhibited by non-slotted wine-glass disks.

The elliptic resoswitch of Fig. 1(c) basically engineers stiffness in a similar manner as the device of Fig. 1(b), except without the use of slots. Instead, it effects differences in orthogonal axial stiffness via geometric ratioing of the ellipse aspect ratio $A R(a / b)$, where the smaller stiffness along its longer output axis yields a larger displacement than that along its shorter, hence stiffer, input axis. Unlike


Fig. 4: (a) SEM of a fabricated polysilicon elliptic disk with an AR of 1.6. Measurement set-up for extracting displacement gain is indicated. (b) Measured transmissions obtained via the device and set-up in (a), exhibiting a displacement gain $\sim 2.04$ and a measured $Q$ over 100,000.
the slotted disk, the elliptic disk avoids the energy-consuming stress corners of slots, allowing it to retain high $Q$. Fig. 3 compares the FEM-simulated mode shapes of the new design in (c) with those of (a) a conventional wine-glass disk and (b) the slotted disk, showing no displacement gain in (a), but (by red coloring) ample output displacement gain for both the slotted and the elliptic disks.

## DISPLACEMENT GAIN MODELING

Unlike a slotted-disk, for which displacement gain depends on the size, location, as well as shape of its slots, making modeling very complex; the elliptic disk presents a much simpler structure, for which displacement gain depends only on its geometric aspect ratio $A R$ and the Poisson ratio $v$ of its structural material. A semi-empirical model based on FEA that predicts the displacement gain as a function of these two parameters takes the form

$$
\begin{equation*}
G_{d i s p}=A R^{-4.73 v+2.626} \tag{1}
\end{equation*}
$$

The basic form of (1) derives from knowledge that a purely circular disk $(A R=1)$ has an orthogonal axis displacement gain of one regardless of the value of Poisson ratio.

## EXPERIMENTAL RESULTS

Elliptic displacement amplifiers were fabricated using a polysilicon surface micromachining process similar to that of [8]. Table 1 summarizes the designs, which span $A R$ values from 1 to 2 . Note that all elliptic disk designs have a fixed ellipse area of $400 \pi \mu \mathrm{~m}^{2}$ (i.e., $\pi \mathrm{ab}$ ) and their resultant resonance frequencies span from 96.5 MHz for $A R=1$ to 82.4 MHz for $A R=2$. Fig. 4(a) presents the SEM of one such fabricated elliptic device with $A R=1.6$.

Table 1: Summary of Elliptic Disk Designs

| $a, b\left(\mu \mathrm{~m}^{2}\right)$ | $a b=400$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Thickness $h(\mu \mathrm{~m})$ | 3 |  |  |  |  |  |
| Stem Radius $(\mu \mathrm{m})$ | 1 |  |  |  |  |  |
| Stem Height $(\mu \mathrm{m})$ | 0.7 |  |  |  |  |  |
| Aspect Ratio $A R$ | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
| Frequency $(\mathrm{MHz})$ | 96.49 | 95.36 | 92.68 | 89.31 | 85.79 | 82.40 |



Fig. 5: Comparison of displacement gains as a function of aspect ratio obtained from measurement, FEA, and prediction using (1).


Fig. 6: Measured frequency shifts with tuning voltages applied on input ( $V_{B 1}$ ) versus on output $\left(V_{B 2}\right)$, where the lower mechanical stiffness along the output axis yields a larger tuning range.

Devices like this were measured under $10^{-5}$ Torr vacuum provided by a turbo pump using the indicated circuit, where one input electrode is connected to port 1 , and the other input electrode and the output electrode are connected to port 2 and 3, respectively. Fig. 4(b) presents measured frequency transmissions between two input ports (S21) (blue) and between the input and output ports (S31) (red) of an elliptic disk with $A R=1.6$, where the transmitted power difference between the two peaks indicates a displacement gain of $\sim 2.04$.

Fig. 5 plots measured displacement gain as a function of aspect ratio $A R$ varying from 1 (a purely circular disk) to 2 alongside predictions by FEA and Eq. (1) (with $v=$ 0.226 ), showing good agreement. Here, a single elliptic disk with $A R=2$ achieves a displacement gain up to $\sim 3$.


Fig. 7: Measured input to output frequency response spectra as a function of input drive voltage, showing flattening and bandwidth widening of the response at the onset of impacting.


Fig. 8: Measured $Q$ versus aspect ratio from three dies. Here, $A R=1$ corresponds to a circular disk, while higher AR's indicate ellipses. $A R=1.2$ presents the lowest $Q$ while $A R>1.6$ seems to be able to recover $Q$ back to that of a circular disk.

Fig. 6 plots the frequency shift versus electrical-stiffness tuning voltage applied at the input electrode ( $V_{\mathrm{B} 1}$, blue curve) and the output electrode ( $V_{\mathrm{B} 2}$, red curve), where the larger frequency shift of the latter confirms lower stiffness along the output axis. Fig. 7 presents measured frequency responses taken at the output electrode of an 89-MHz elliptic resoswitch as a function of increasing input voltage, showing the expected peak flattening as the elliptic disk impacts the output electrode.

## ELLIPTIC DISK $Q$

Fig. 8 plots measured $Q$ as a function of $A R$ for several elliptic disks spanning $82.4-96.5 \mathrm{MHz}$ on three different dies, indicating similar $Q$ variations regardless of their $Q$ magnitudes. Interestingly, when $A R=1.2 Q$ drops consistently from that of a conventional circular wine glass disk ( $A R=1$ ), but increases afterwards as the $A R$ rises.

The measured $Q$ of 100,000 for a $96-\mathrm{MHz} A R=1$ circular disk is short of the expected intrinsic $Q$ limit, most likely due to a combination of anchor loss and phonon-phonon interaction loss [9] [10] [11]. Since phonon-phonon interaction energy loss is less influenced by geometry, the reduction in $Q$ as $A R$ changes from 1 to 1.2 likely results from anchor dissipation. In particular, the center stem dissipates


Fig. 9: (a) Schematic of an elliptic disk dissipating energy into the substrate while vibrating. (b) FEA-simulated deformation of the center stem bottom (substrate is not shown for clarity). (c) Illustration of $z$ displacement components versus rotating angle along the circumference at the bottom of the center stem when the elliptic disk vibrates in the wine glass mode shape.


Fig. 10: FEA-simulated $z$-displacement components versus aspect ratio, indicating a large vibration magnitude along the $z$ axis when $A R=1.2$.
energy into the substrate via $z$-directed motion perpendicular to the substrate surface, as illustrated in Fig. 9(a).

Fig. 9(b) and (c) plot the FEA-simulated mode shape around the anchor area and the $z$ displacement amplitude as a function of angle $\theta$ for a point along the circumference of the bottom of the center stem where it attaches to the substrate. Here, the $z$ displacement comprises an average component $Z_{\text {disp, avg }}$ and a varying one with peak-to-peak magnitude $Z_{d i s p, p p}$. An increase in either component raises the amount of energy lost to the substrate, thereby lowering $Q$.

Fig. 10 plots the simulated $z$-displacement components, $Z_{d i s p, \text { avg }}$ and $Z_{d i s p, p p}$, as a function of $\theta$ for elliptic disks with varying $A R$ values. Here, the simulated $1 \mu \mathrm{~m}$ radius, $0.7 \mu \mathrm{~m}$-tall center stem seems to best couple the vibrating energy of the $95-\mathrm{MHz}$ elliptic disk when the $A R=1.2$, which produces the largest $Z_{\text {disp, avg }}$, as shown in Fig. 10. This observation aligns well with the measurement results of Fig. 8 where the elliptic disks with an $A R=1.2$ yield the lowest $Q$ 's.

In addition, aside from the circular disk case $(A R=1)$, $A R=1.6$ yields the smallest $Z_{\text {disp, avg }}$, which might explain why this $A R$ yields the highest measured $Q$ 's for two of the
curves in Fig. 8. For $A R>1.6$, on the other hand, although $Z_{\text {disp, avg }}$ magnitudes rise as $A R$ increases, $Z_{\text {disp, avg }}$ 's decrease, perhaps cancelling the former and allowing $Q$ 's on par with that of $A R=1.6$, as shown in Fig. 8.

## CONCLUSIONS

By engineering stiffnesses via dimensional ratioing rather than slots, the demonstrated elliptic disks provide an alternative method to achieve displacement amplifying resoswitches that retain high resonator $Q$ 's exceeding 100,000 that should improve the sensitivities of resoswitch-based zero-quiescent power radios at VHF. In fact, the almost $10 \times$ improvement in $Q$ over previous disk array and slotted disk approaches should yield sensitivity reductions (i.e., improvements) on the order of 10 dB . Of course, this work presented only polysilicon elliptic resoswitches with high contact resistances that preclude use in many desired applications. To be useful in an actual zero-quiescent power radio, next generation devices should employ metals or other more conductive materials at their contact interfaces. Methods for doing so without sacrificing $Q$ are currently under study.

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## REFERENCES

[1] Y. Lin, et al., "Digitally-specified micromechanical ...," in $15^{\text {th }}$ Int. Conf. on Solid-State Sens., Act., \& Microsyst. (TRANSDUCERS'09), 2009 pp. 781-784.
[2] B. Kim, et al., " $\mu$ mechanical resonant displacement gain stages," in $22^{\text {nd }}$ Int. IEEE MEMS Conf., 2009, pp.19-22.
[3] W.-C. Li, et al., "Metal micromechanical filter-power amplifier utilizing a displacement-amplifying resonant switch," in $17^{\text {th }}$ Int. Conf. on Solid-State Sensors, Actuators, \& Microsystems (Transducers'13), 2013, pp. 1445-1448.
[4] R. Liu, et al., "Zero quiescent power VLF mechanical communication ...," in $18^{\text {th }}$ Int. Conf. on Solid-State Sens., Act., \& Microsyst. (Transducers'15), 2015, pp. 129-132.
[5] J. Johler, "Propagation of the Low-Frequency Radio Signal," Proc. of the IRE, vol. 50, no. 4, pp. 404-427, 1962.
[6] Y. Chen, et al., "Ultra-low power time synchronization using passive radio receivers," in 10th Int. Conf. on Information Processing in Sensor Networks (IPSN), 2011.
[7] Y. Lin, et al., "The micromechanical resonant switch ("resoswitch")," in 2008 Solid-State Sen., Act., \& Microsyst. Workshop, (Hilton Head'08), 2008, pp. 40-43.
[8] Y.-W. Lin, et al., "Series-resonant VHF micromechanical resonator reference oscillators," IEEE J. Solid-State Circuits, vol. 39, no. 12, pp. 2477-2491, 2004.
[9] W.-C. Li, et al., "Quality factor enhancement in micromechanical resonators at cryogenic temperatures," in $15^{\text {th }}$ Int. Conf. on Solid-State Sensors, Actuators, \& Microsystems (TRANSDUCERS'09), 2009, pp. 1445-1448.
[10] R. Tabrizian, et al., "Effect of phonon interactions on limiting the f.Q product of micromechanical resonators," in $15^{\text {th }}$ Int. Conf. on Solid-State Sensors, Actuators, \& Microsystems (Transducers'09), 2009, pp. 2131-2134.
[11] J. E.-Y. Lee, et al., "Study of lateral mode SOI-MEMS resonators for reduced anchor loss," J. Micromech. Microeng., vol. 21, no. 4, p. 045010, 2011.

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