
Precision Resonant Beam Strain Sensor
Employing Gap-Dependent Frequency Shift

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Abstract—A micromechanical structure for on-chip strain sensing maps strain-induced gap changes to resonance frequency shifts while employing differential strategies to null out bias uncertainty, all towards repeatable measurement of sub-nm displacement changes that equate to sub-με strain increments. The key enabler here is the use of gap-dependent electrical stiffness to shift resonance frequencies as structural elements stretch or shrink to relieve stress. An output based on the difference frequency between two close proximity structures with unequal stress arm lengths (cf. Fig. 1) removes uncertainty on the initial gap spacing and permits a 206 Hz/με scale factor. The ability to precisely measure the frequency of the high-Q (~4000) structures, down to at least 1 Hz, puts the resolution of this sensor at least 5 nε (or 790 Pa for polysilicon). An on-chip highly sensitive strain sensing device like this will likely be instrumental to managing stress changes over the lifetime of micromechanical circuits, such as oscillators and filters.

Keywords—MEMS, micromechanical resonator, polysilicon, strain, stress, diagnostic, electrical stiffness

I. INTRODUCTION

Recently demonstrated sub-20-nm capacitive-gap transduced resonators with $C_r/C_o$’s of 71% at 10-MHz [1] and 1% at 200-MHz [2] now introduce compelling combinations of high $Q$ and strong electromechanical coupling at HF and VHF [3] [2] that could change the landscape of applications available to MEMS. However, small electrode-to-resonator gaps also make more difficult the design and realization of MEMS-based mechanical circuits, e.g., filters and oscillator arrays [4], that become more prone to post-fabrication residual stress [5] as circuit size and gap spacing increase and decrease, respectively. Although recent design [5] and fabrication [6] strategies have greatly improved yields, a strain sensor able to precisely measure strain in close proximity to a device would permit real-time correction for strain-induced shifts, e.g., from package stress deviations, as well as optimization of the process recipes used to fabricate small-gap devices.

To this point, existing on-chip stress/strain sensors in the literature either use very specific material systems that limit their applicability, or if widely applicable, are not sufficiently sensitive. For example, the high resolution on-chip strain sensor of [7] relies on piezoelectric material, which is often not compatible with surface-micromachining fabrication processes.

Fig. 1: The on-chip beam resonator strain sensor described herein in a typical operating circuit along with the key dimensions. The inset shows the finite element analysis (FEA) simulated mode shape during strain determination.
used to achieve MEMS devices made in conductive materials, e.g., doped polysilicon or metal. This reduces the range of MEMS devices for which they can serve as on-chip strain sensors.

On the other hand, on-chip strain sensors available via surface-micromachining often lack sufficient resolution [8], [9], [10]. For example, the surface-micromachined Vernier stress gauge depicted in Fig. 2 [11] employs visual determination of indicator beam movement under a microscope—a procedure that clearly lacks precision. This structure also is most sensitive when its indicator beam is long, since this amplifies its movement under a given strain. The need for long length, however, renders the device vulnerable to vertical stress gradients, placing a limit on the minimum measurable stress.

The quest for on-chip strain sensing with high resolution calls for a sensor with a more precise readout method, e.g., frequency, that better decouples strain resolution from structure size. The sensor described herein and summarized in Fig. 1 does precisely this via use of a micromechanical structure that maps strain-induced gap changes to resonance frequency shifts while employing differential strategies to null out bias uncertainty, all towards repeatable measurement of sub-nm displacement changes that equate to sub-με increments. The next section describes in more detail the basic operation principle.

II. ELECTRICAL STIFFNESS-BASED STRAIN SENSOR

The strain sensor of Fig. 1 comprises a conductive polysilicon beam resonator suspended by crab legs to folded-beam anchoring structures designed to relieve stress along the beam’s axis, as indicated by the finite-element simulations of Fig. 3. The stress relief is instrumental to preventing beam buckling that might otherwise short the beam to an overlapping side doped polysilicon electrode spaced only 60 nm away. This small gap spacing determines not only the strength of the capacitive-gap transducer used to drive the beam into resonance vibration and sense said vibration, but also the magnitude of the electrical stiffness between the beam and electrode.

The key enabler in the subject strain sensor is the use of a gap-dependent electrical stiffness to shift the resonance frequency as structural elements stretch, shrink, or otherwise move relative to one another with applied stress. The Fig. 1 strain sensor specifically uses this shift in frequency to measure the nm-level change in gap spacing \(d_s\) between the structure and a capacitive-gap transducing electrode. The gap change then indicates the strain. Perhaps the best vehicle with which to describe sensor operation is during measurement of the post-fabrication residual stress often generated via thermal expansion differences between the substrate and the suspended materials.

A. Post-Fabrication Residual Stress Measurement

Measurement of residual stress using the device of Fig. 1 amounts to measuring the nm-level expansion/contraction of a movable stress arm of length \(L_s\) as a change in capacitive transduction gap \(d_o\). Here, before release, the structure is under compressive stress due to thermal expansion differences between the substrate and structural layer, as depicted in Fig. 4(a). After removing the sacrificial spacer as shown in Fig. 4(b), the stress arm \(L_s\) relieves compressive stress by stretching a few nanometers, thereby shrinking the gap in proportion to its length, i.e., a longer arm leads to a larger reduction in the gap, according to

\[
\Delta d_o = -\epsilon L_s
\]

where \(\Delta d_o\) is the strain-based gap change after release and \(\epsilon\) is the residual strain. If the gap also serves in a capacitive-gap transducer for the indicated beam, the reduction in gap spacing induces an increase in electrical stiffness \(k_e\) according to [12]...
where $\Delta f_i = f_{\text{nom}} - f_i$. An opposite change in gap, i.e., an increase, would indicate tensile stress.

The strong dependence of (6) on gap spacing $d_o$ puts a premium on suppressing phenomena other than the strain along the sense axis that might also influence the gap spacing. One such phenomena is stress along the beam axis, which is orthogonal to the sense direction, and can buckle the resonator and thereby change the gap. This underscores the utility of the previously mentioned folded-beam anchors at the ends of the resonant sensing beam that suppress axial stress, as shown in Fig. 3.

B. General Stress Measurement

Knowledge of fabrication residual strain via (6) then permits isolation of strain due other causes, i.e., the external strain, e.g., package stress, by merely subtracting out the gap change due to residual strain from the total gap change. Here, the remaining gap amount corresponds to the external strain.

III. MEASUREMENT RESULTS

Fig. 5 presents an SEM of the residual strain sensor fabricated alongside tiny-gap mechanical filters, i.e., in the same surface-micromachining process [5]. The fabrication run included several designs like this with 41.3-μm-long, 2-μm-wide, 3-μm-thick resonant beams and various stress arm lengths.

Fig. 6 presents frequency spectra measured under 50-μTorr vacuum over a dc-bias $V_p$ range from 1 V to 5 V for two structures fabricated side by side that are identical in all respects except for different support arm lengths ($L_s$) of 10 μm and 20 μm. The different lengths lead to final gaps extracted by curve-fitting the measurement data [12] of 52.4 nm and 45.7 nm, respectively. Assuming the starting gap for each is 60 μm, these correspond to -760 and -715 μstrain, respectively, using (1).

Note that the accuracy of resulting strain depends on knowledge of the initial gap $d_0$. Unfortunately, fabrication
tolerances and statistical deviations make it difficult to know \( d_o \) accurately. (Thus, the need to assume a \( d_o \) in the previous strain determinations.) To address this problem, a differential approach is possible if two close-proximity sensors with different stress arm lengths are available. Specifically, a strain value independent of the initial gap results from taking the difference between extracted gaps for the 10-\( \mu \)m and 20-\( \mu \)m stress arm length cases and dividing this by the difference in stress arm lengths:

\[
\varepsilon = \frac{d_{o2} - d_{o1}}{L_{s2} - L_{s1}} = \frac{45.7 - 52.4}{20 - 10} \times \frac{10^{-9}}{10^{-6}} = -670 \mu \epsilon
\]  

(7)

where \( d_{o1} \) and \( d_{o2} \) are the gaps for the 10-\( \mu \)m and 20-\( \mu \)m stress arm sensors, respectively. This now is a more accurate value of stress.

A. Faster Methods

Curve-fitting is perhaps the most accurate method to extract the final gap and hence the strain. It does, however, require collection of numerous data points to attain an accurate answer. While the procedure for doing this can be automated in a way that provides outputs fast enough for stress compensation, e.g., for an oscillator, (6) provides a simpler, faster method. However, (6) also requires knowledge of the initial gap spacing \( d_o \) and the unbiased nominal resonance frequency \( f_{\text{nom}} \). While the latter might be obtained by extrapolating a resonance frequency \( f_o \) vs. dc-bias \( V_p \) curve to zero dc-bias, this still requires collection of numerous points, after which the extracted \( f_{\text{nom}} \) can be used in all future calculations using (6). But this need only be done once.

Using (6) with an (assumed) initial gap \( d_o \) of 60 nm, \( \alpha \) of 0.3965, \( \rho \) of 2300 kg/m\(^3\), \( E \) of 158 GPa, \( W \) of 2 \( \mu \)m, \( L \) of 41.3 \( \mu \)m, \( L_o \) of 24 \( \mu \)m, \( V_p \) of 5 V, \( \Delta f_{o1} \) of 228.1 kHz, and \( \Delta f_{o2} \) of 334.5 kHz, the strains for the 10-\( \mu \)m and 20-\( \mu \)m stress arm cases are -895 and -190 \( \mu \)strain, respectively. Of the two of these, the former for the 10-\( \mu \)m-long stress arm is more correct, since it derives from a smaller percent gap change. Indeed, the amount of gap change for the 20-\( \mu \)m case is a significant fraction of the initial gap, making (2) and hence (6) much less accurate.

If two strain sensors with differing stress beams have the same \( f_{\text{nom}} \), then the need to know \( f_{\text{nom}} \) goes away via use of the difference in strain-derived frequency shifts between the two structures. In this case, the specific expression for strain becomes

\[
\varepsilon = -8.24 \frac{\alpha \pi^2 W^2 \sqrt{E \rho d_o^4}}{3 \varepsilon_o L_o V_p^2 L_o (L_{s2} - L_{s1})} \left( \Delta f_{o2} - \Delta f_{o1} \right)
\]

(8)

where \( L_{s1} \) and \( L_{s2} \) are the support arm lengths and \( \Delta f_{o1} \) and \( \Delta f_{o2} \) are the frequency shifts for the two designs, respectively. This expression still contains the initial gap spacing \( d_o \), so still contains some uncertainty, but its impact on (8) is smaller than on (6). Here, using a single nominal \( d_o \) in the numerator yields a more intuitive closed-form expression.

Since the nominal resonance frequencies of the two sensors herein are not quite the same, use of (8) is not particularly advantageous, here. Nevertheless, using (8) with data from Fig. 6(a) and (b) and the target \( d_o \) of 60 nm yields -515.3 \( \mu \)strain, which interestingly is not far from the -670 \( \mu \)strain of (7).

B. Scale Factor (Sensitivity)

Scale factor or sensitivity of a resonant sensor indicates how much its frequency shifts in response to a change in strain. Employing this definition by taking the partial derivative of (8) with respect to strain yields

\[
\frac{\partial \Delta f}{\partial \varepsilon} = -8.24 \frac{\alpha \pi^2 W^2 \sqrt{E \rho d_o^4}}{3 \varepsilon_o L_o V_p^2 L_o (L_{s2} - L_{s1})} \]

(9)

The inverse 4\(^{th}\) power dependence on the actuation gap \( d_o \) generates extremely sensitive resonant sensing as gaps become smaller, i.e., sub-100nm. The 106.4-kHz difference in frequency excursions between Fig. 6 (a) and (b) over the measured 1V to 5V dc-bias voltage \( V_p \) range corresponds to
TABLE I. COMPARISON OF THIS WORK AND OTHER TECHNOLOGIES

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<th></th>
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<td>340μV</td>
<td>120Hz</td>
<td>206Hz</td>
<td>με⁻¹</td>
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<tr>
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<td>4</td>
<td>0.969</td>
<td>nε</td>
</tr>
<tr>
<td>Range</td>
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<td>N/A</td>
<td>±2.5</td>
<td>±3000</td>
<td>με</td>
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81.4 MPa compressive stress. This puts the average scale factor at 206.5 Hz/με (1.31 Hz/kPa for polysilicon) over this range.

C. Resolution

Resolution, in the context of a resonant strain sensor, is the strain corresponding to the minimum detectable resonance frequency shift, which is usually limited by short-term frequency instability. Rearranging (8) and replacing \(\Delta f_{\text{st}} - \Delta f_{\text{st}}\) with \(\sigma_{\text{score}}\), where \(\sigma_{\text{score}}\) is the Allan deviation yields

\[
\Delta \varepsilon = 8.24 \frac{\alpha t^2 W^2 \sqrt{E d_s \sigma_{\text{score}}}}{3\varepsilon_0 L_s V_r^2 L (L_{s2} - L_{s1})} f_0
\]  

Again, the 4th power dependence on the gap d_s implies a very high-resolution sensor with small gaps. Using a typical Allan deviation value of 2x10⁻⁸ at 1 s integration time measured for a wine-glass disk resonator fabricated in a similar process yields 0.969 nε resolution.

D. Range

Although there is technically no limit with high enough dc-bias voltages for measuring tensile strain, the actuation gap d_o ultimately limits the maximum measurable compressive strain as follows

\[
\varepsilon_{\text{max}} = -\frac{d_o}{L_s 2}
\]  

Here, a nominal actuation gap value of 60nm with a 20um-long support arm limits the compressive strain measurement range to ±3000 μstrain.

Table I summarizes the scale factor, resolution, and range for the strain sensor of this paper along with some other devices found in the literature.

IV. CONCLUSIONS

Given that the impressive sensitivity and resolution of the resonant strain sensors demonstrated herein derive from small electrode-to-resonator gaps, recent technological advances that reduce gaps even further [2] will likely encourage many more sensors based on electrical stiffness changes. This is not to say that only small-gapped versions are of interest. Indeed, although this work targets small-gapped micromechanical circuits, it should be clear that this approach is applicable to larger gap devices, as well, since electrical stiffness is universal. Larger gap versions would likely use higher voltages and alternative geometries but could probably still achieve similarly impressive sensitivity and resolution.

V. ACKNOWLEDGMENT

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VI. REFERENCES


