DIGITALLY-SPECIFIED MICROMECHANICAL DISPLACEMENT AMPLIFIERS

Yang Lin^{1*}, Wei-Chang Li¹, Ilya Gurin¹, Sheng-Shian Li², Yu-Wei Lin³ Zeying Ren¹, Bongsang Kim¹ and Clark T.-C. Nguyen¹
Contact: *Yang Lin, tel: +1-510-643-9825; linyang@eecs.berkeley.edu
¹University of California at Berkeley, Berkeley, CA, USA
²National Tsing Hua University, Hsinchu, Taiwan
³Broadcom Corporation, Irvine, CA, USA

ABSTRACT

A micromechanical displacement amplifier comprising two asymmetric resonator array composites coupled by a quarter-wavelength beam has been demonstrated that permits specification of gain factor by mere (digital) selection of an appropriate ratio of the number of resonators in an input array to that in an output array. Like the method of [1], this displacement gain circuit is a key enabler for resoswitch-based mechanical power amplifiers and power converters, because it can prevent unwanted drive electrode-to-resonator impact in such circuits. This design, however, differs from that of [1] in that 1) it can be applied to radial-contour mode disks that can achieve much higher frequency than the wine-glass disks of [1]; 2) it preserves the frequency and Q of its constituent resonators (whereas the method of [1] changed the frequency and lowered the O; and 3) its digital method for gain specification is much more straightforward, accurate, and repeatable.

KEYWORDS

Micromechanical devices, resonant switch, radialcontour mode, resonator, RF MEMS, displacement amplifier, power amplifier, power converter.

INTRODUCTION

With a figure of merit [2] (FOM) potentially much larger than transistor counterparts, the micromechanical resonant switch (dubbed "resoswitch"), introduced in [3][4] and depicted in Fig. 1(a), offers the possibility of switched-mode power converters and power amplifiers with substantially better efficiency than presently available. The resoswitch even transcends the capabilities of conventional RF MEMS switches by harnessing resonance and nonlinear dynamical properties to greatly increase switching speed and cycle count (even under hot switching), and lower needed actuation voltages, all by substantial factors. However, as also summarized in Fig. 1(a), the resoswitch described in [3] was only a demonstration prototype with several imperfections, one of which was that it impacted (i.e., closed with) not only its output electrodes, but also its input electrodes. This unwanted input closing limits resoswitch performance in certain applications.

To remedy this, the micromechanical resonant displacement gain stage, described in [1] and shown in Figs. 1(b) and (c), employs stiffness engineering to effect dis-



Figure 1: Micromechanical resoswitch schematics with dashed mode shape lines showing impact along both (a) switch and (b) drive axes for the unslotted wine-glass disk version of [3]; and impact only along the (c) output switch axis, but not (d) input drive axis, for the slotted version of [1].



Figure 2: Circuit topology of a Class E amplifier built using slotted wine-glass mode resoswitch in place of transistor. (The dashed line indicates how the resoswitch contorts along the softer slotted axis to make contact.).

placement amplification from input axis to output axis. Here, a slotted wine-glass disk is used that allows displacements along the switch axis to be much larger than those along the input axis, thereby preventing impacts along the input axis, but allowing them along the switch axis. In so doing, this device enables the use of resoswitches in applications like that depicted in Fig. 2, where a micromechanical resoswitch replaces the switching transistor in a Class E switched-mode power amplifier to greatly improve its efficiency by virtue of the much



Figure 3: Schematic of a single contour-mode micromechanical resonator and its FEA-simulated mode shape.



Figure 4: Schematic of the 152.3MHz contour-mode micromechanical displacement amplifier realized in this work.

higher *FOM* of the resoswitch versus a semiconductor transistor.

Unfortunately, as shown in [1], the introduction of slots into the disk structure ends up decreasing the Q of structure and shifting its frequency, both of which are undesirable. In addition, there is so far no simple closed form analytical expression that governs the displacement gain factor afforded by this method; rather, finite element simulation is used to determine the gain. Furthermore, this method is applicable mainly to geometries where expansion and contraction occur along orthogonal axes, like wine-glass disks. It is less directly applicable to radial-contour mode disks that expand and contract equally along all radial axes (c.f., Fig. 3), so a better solution is desired if micromechanical power amplifiers are to harness the much higher frequency of radial-contour mode disks.

To overcome the above issues, this work takes a mechanical circuit design approach, as opposed to the devicecentric approach of [1], that differs from that of [1] in that 1) it can be more readily applied to radial-contour mode disks capable of much higher frequency than the wineglass disks of [1]; 2) it preserves the frequency and Q of its constituent resonators (whereas the method of [1] changed the frequency and lowered the Q); and 3) its digital method for gain specification is much more straightforward, accurate, and repeatable.

DISPLACEMENT AMPLIFIER DESIGN

Figure 4 presents the schematic of the micromechanical displacement amplifier demonstrated in this work. Here, an array composite of disks on an input side (the left

 Table 1 Micromechanical Displacement Amplifier Data

I	T	
Parameters	Value	Units
Disk radius <i>R</i>	17	μm
Disk thickness h	3	μm
Electrode-to-resonator gap d_0	100	nm
Number of disks in input array N_I	4	N/A
Number of disks in output array N_O	1	N/A
Acoustic wave velocity of poly-silicon	8075.7	m/s
Resonance frequency f_0	153.2	MHz
Acoustic wave length λ	52.71	μm

side) feeds via a quarter-wavelength coupler into a single output disk (or another array) to effect an amplification of output disk displacement over that of the input. More specifically, on the left hand side, 4 disks with identical radii of 17 μ m are coupled by half-wavelength beams to form a composite array input resonant tank; while on the right hand side only one disk (again, with a 17 μ m radius) is used as the output tank. Electrodes for lateral capacitive transduction surround the output disk and three of the input disks, each spaced 100 nm from the disk sidewalls, and each fully surrounding its corresponding disk to favor the radial-contour resonant mode, depicted in Fig. 3. Table 1 summarizes the design parameters of the device.

To operate the circuit, a dc-bias voltage V_P is applied to the conductive resonant structure (via the electrically connected ground plane underlying the disks) and an ac voltage v_i at the resonance frequency applied to the input port. These voltages together generate an input force that drives the structure into resonance vibration. The velocity of the vibrating single output disk is then sensed as an output motional current generated across the V_P -biased timevarying disk-to-electrode capacitive gap.

Analytical Design

The use of a coupling beam dimensioned to correspond to a quarter-wavelength [5] at the frequency of resonance is key to maximizing the displacement gain provided by this design. The extensional mode coupling beam used here is similar in geometry and design to those used in previous vibrating RF MEMS devices, such as the micromechanical resonator array in the GSM-compliant oscillator of [6], and the LSI RF channel-select bandpass filter circuit of [7]. As in these works, the function of a given coupling beam is governed by its dimensions, and specifically on what fraction of a wavelength they correspond. For example, when an array of resonators is designed to form a composite resonator in order to lower motional impedance and raise power handling, as is done in both [6] and [7][8], half-wavelength coupling beams are needed, since they simulate ideally infinite stiffnesses that accentuate one desired mode in which all resonators vibrate with the same amplitude. They further push all other modes to far away frequencies.

As shown in Fig. 4, the present displacement amplifier design utilizes half-wavelength couplers in its input array to realize a composite resonator array that behaves like one

resonator. As in [8], this "one" composite resonator has an effective stiffness at any given location equal to N_i times that at the same location on a single one of its constituent disk resonators, where N_i is the number of resonators used in the array. In effect, arraying can be used to increase the effective stiffness of a composite resonator, which as will be seen, is instrumental to attaining a digitally specifiable displacement gain using the circuit of Fig. 4.

To effect the most efficient displacement gain, a quarter-wavelength coupler is utilized to connect the input and output networks, much like those used to set the bandwidths of filters like that of [5]. In the micromechanical filters so far demonstrated, quarter-wavelength elements generally couple purely symmetric networks that present the same impedance to the coupling element at each of its attaching ends, i.e., the resonators or composites attached to both ends of the coupler are identical. The displacement amplifier of this work differs from filters in that it quarterwavelength couples dissimilar resonator structures, specifically, a 4 disk array composite on one (input) side, to a single disk on the other (output) side.

Displacement amplification occurs because quarterwavelength coupling constrains the mechanical circuit network so that the energy or power on the left and right sides are equal. Since the stiffnesses of the input and output arrays now differ, different amount of displacements ensue to maintain the same energy on both sides. Quantitatively, the gain factor is governed by:

$$E_{I} = E_{O} \to \frac{1}{2} k_{I} \left(X_{I} \right)^{2} = \frac{1}{2} k_{O} \left(X_{O} \right)^{2} \to \frac{X_{O}}{X_{I}} = \sqrt{\frac{k_{I}}{k_{O}}} = \sqrt{\frac{N_{I}}{N_{O}}}$$
(1)

where subscripts I and O denote parameters for the input and the output ports (or "tanks"), respectively; E is the total vibration energy stored in a given resonant tank at resonance; k is the equivalent stiffness of a given tank at the coupling location; X is the displacement amplitude of a given tank; and N is the number of resonators in the composite array (i.e., in the tank) at each port.

Equation (1) indicates that the displacement gain of this mechanical circuit is governed simply by the square root of the ratio of the number of resonators in the input array composite to that in the output composite.

Simulation verification

Figure 5 presents a finite-element modal simulation of the Fig. 4 structure, showing a displacement amplification of exactly 2, which matches and verifies the value predicted by (1).

Beyond finite element simulation, given that it represents a circuit approach to attaining displacement gain, it should be no surprise that the Fig. 4 circuit is quite amenable to simulation via circuit simulators, such as SPICE. In particular, although space constraints do not allow it in this paper, an electrical equivalent circuit for the structure of Fig. 4 similar to those used for mechanical filters in [5] and [7] can easily be drawn up, input into SPICE, and then simulated. Doing so again yields exactly the same value of



Figure 5: FEA modal simulation of the 152.3MHz contour-mode micromechanical displacement amplifier.

gain as (1).

The value of such amenability to circuit representation, of course, is the resultant amenability to simulation and computer aided design programs that can conveniently analyze and perhaps automatically generate more complex and more capable circuits based on the displacement gain principles of the Fig. 4 circuit. Indeed, the analogy is not unlike that for transistor integrated circuits.

EXPERIMENTAL RESULTS

Figure 6(a) and (b) present the SEM and measured frequency characteristics for a fabricated version of the displacement amplifier circuit of Fig. 4, showing a center frequency of 153.2 MHz and a Q of 10,500, which are essentially the same as that of a single resonator, verifying that this approach does not degrade Q. This is consistent with [9], which showed that an array of identical resonators retains the same Q as any one of its constituents.

While the frequency characteristic of Fig. 6(b) was measured using a direct network analyzer set-up [10], such as depicted in Fig. 4, determination of displacement gain requires a more involved set-up. In particular, a set-up that allows simultaneous measurement of input and output disk displacement is needed. To do this, the "RF/LO mixing overtone test setup" introduced in [1] and depicted in Fig. 7 is used. In this set-up, a network analyzer is still used as a signal source, but a spectrum analyzer is used (rather than the network analyzer) to sense the simultaneous output currents of both the input and output tanks. The local oscillator applied to the resonant structure (atop the dcbias) mixes the output current of the input tank to higher frequencies, separating it in the frequency domain from the input drive current of the source, and thereby allowing detection of this current without interference from the drive current.

The detected output power can then be converted to the associated disk displacement amplitude using (2)



Figure 6: (a) SEM photo of the contour-mode 153.2 MHz micromechanical displacement amplifier, (b) measured frequency response and (c) input and output disk displacements using the measurement setup introduced in [1], clearly showing an amplification ratio of 2.17X.

$$X = \sqrt{\frac{P}{R_L}} \frac{d_0^2}{V_P \cdot (2\pi\varepsilon_0 Rh) \cdot \omega_0}$$
(2)

where R_L is the load resistance, d_0 is the disk-to-resonator gap, R and h are the radius and thickness of the disk, respectively, and ω_0 is its resonance frequency. Using (2) and plotting versus frequency yields the curves of Fig. 6(c), where the ratio of the output disk displacement to that of each input disk in the input array composite is 2.17×, which is close to the prediction of (1).



Figure 7: Schematic of the RF/LO mixing overtone test setup used to simultaneously measure the output currents (= displacements) of the input and output resonator networks.

CONCLUSIONS

The successful use of circuit design methodologies to demonstrate a mechanical circuit capable of amplifying displacements by a factor dependent upon the ratio of the number of input and output resonators used represents a significant leap forward in mechanical circuit design capability. The ease with which this approach allows accurate specification of displacement gains for any type of resonator without any need to alter the fabrication process technology used and without any degradation in frequency or Q, encourages similar circuit-centric approaches for even larger mechanical circuits. The displacement gain function achieved here is especially useful for resonant switches, as it further idealizes the operation of such switches, which in turn should greatly improve the performance of switchedmode power amplifiers and converters that utilize them.

Acknowledgment: This work was supported by DARPA.

REFERENCES

- [1] B. Kim, Y. Lin, et al., "Micromechanical resonant ...," *Technical Digest*, 2009 IEEE MEMS Conf., pp. 19-22.
- [2] Z. J. Yao, et al., "Micromachined low-loss ...," *IEEE/ASME JMEMS.*, vol. 8, no. 2, pp. 129-134, June 1999.
- [3] Y. Lin, et al., "The micromechanical resonant switch ("resoswitch")," Tech. Dig., 2008 Hilton Head, pp. 40-43.
- [4] Y. Lin, et al., "A resonance dynamical approach ...," *Proceedings*, 2008 IEEE Freq. Ctrl. Symp., pp. 640-645.
- [5] F. D. Bannon III, et al., "High frequency ...," *IEEE J. Solid-State Circuits*, vol. 35, no. 4, pp. 512-526, April 2000.
- [6] Y.-W. Lin, et al., "Low phase noise array-composite ...," *Technical Digest*, IEEE IEDM, pp. 287-290.
- [7] S.-S. Li, et al., "An MSI micromechanical ...," *Dig. of Tech. Papers,* Transducers'07, pp. 307-311.
- [8] M. U. Demirci, et al., "Mechanically corner-coupled square microresonator array ...," *IEEE/ASME J. Microelectromechanical Systems*, vol. 15, no. 6, pp. 1419-1436, Dec. 2006.
- [9] Y.-W. Lin, et al., "Quality factor boosting ...," *Dig. of Tech. Papers*, Transducers'07, pp. 2453-2456.
- [10] J. R. Clark, et al., "High-Q UHF ... disk ...," IEEE/ASME JMEMS, vol. 14, no. 6, pp. 1298-1310, Dec. 2005.