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QUALITY FACTOR BOOSTING VIA MECHANICALLY-COUPLED ARRAYING

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Abstract: A mechanical circuit-based approach for boosting the *Q* of a vibrating micromechanical resonator has been demonstrated whereby a low *Q* resonator is embedded into a mechanically-coupled array of much higher *Q* resonators to raise its functional *Q* by a factor approximately equal to the number of resonators in the array. Using this method, the low *Q* of 7,506 exhibited by a support-loss-limited 60-MHz wine-glass disk resonator by itself was effectively raised by about 9X to 63,207 when emplaced into a mechanically-coupled array of eight very high-*Q* wine-glass disks that then form a composite resonator. The availability of such a circuit-based *Q*-enhancement technique has far reaching implications, especially considering the possibility of raising the functional *Q* of a piezoelectric resonator by merely mechanically coupling it to an array of much higher *Q* capacitively-transduced ones to simultaneously obtain the most attractive characteristics of both technologies: low impedance from the piezo-device and high-*Q* from the capacitive ones. Furthermore, the methods of this work stand to enhance the manufacturing repeatability of micromechanical resonator-based products, since they present a convenient method for ensuring *Q*’s greater than a specified threshold value, even when some resonator’s *Q*’s are lower than it.

Keywords: micromechanical circuit, capacitive transducer, resonator, quality factor, fabrication yield.

1. INTRODUCTION

Recently, capacitively-driven vibrating micromechanical resonators have been demonstrated with resonance frequencies in the VHF range with *Q*’s larger than 160,000 [1] and in the GHz range with *Q*’s still larger than 11,000 [2], making them very attractive as on-chip frequency selecting elements for oscillators and filters in wireless communications. To date, oscillators comprised of several mechanically-coupled resonators [3] combined with sustaining transistor circuits have been demonstrated with phase noise performance commensurate with GSM cellular phone specifications for reference oscillators [1]. These oscillators owe their performance largely to the sheer *Q* of their constituent resonators. However, there are other applications, such as filters, where both high *Q* and low impedance are desirable.

Unfortunately, these two qualities have so far not been readily available simultaneously in any single CAD-definable micromechanical resonator design. So far, only capacitively-transduced resonators have achieved *Q*’s over 50,000 at UHF frequencies [2], but with high impedance. On the other hand, piezoelectric resonators with CAD-defined frequencies have achieved impedances below 100 Ω, but only with *Q*’s in the single-digit thousands [4]. A method for combining the most attractive individual characteristics of these devices to simultaneously obtain low impedance from the piezo-device and high-*Q* from the capacitive ones is highly desirable.

This paper presents a possible way to achieve the above combination via a mechanical circuit-based approach that boosts the *Q* of a vibrating micromechanical resonator by embedding it into a mechanically-coupled array [3] of much higher *Q* resonators to raise its functional *Q* by a factor approximately equal to the number of resonators in the array. Using this method, the low *Q* of 7,506 exhibited by a support-loss-limited 60-MHz wine-glass disk resonator by itself was effectively raised by about 9X to 63,207 when emplaced into a mechanically-coupled array of eight very high-*Q* wine-glass disks that then form a composite resonator. In addition to boosting *Q*, the methods of this work also stand to enhance the manufacturing repeatability of micromechanical resonator-based products, since they present a convenient method

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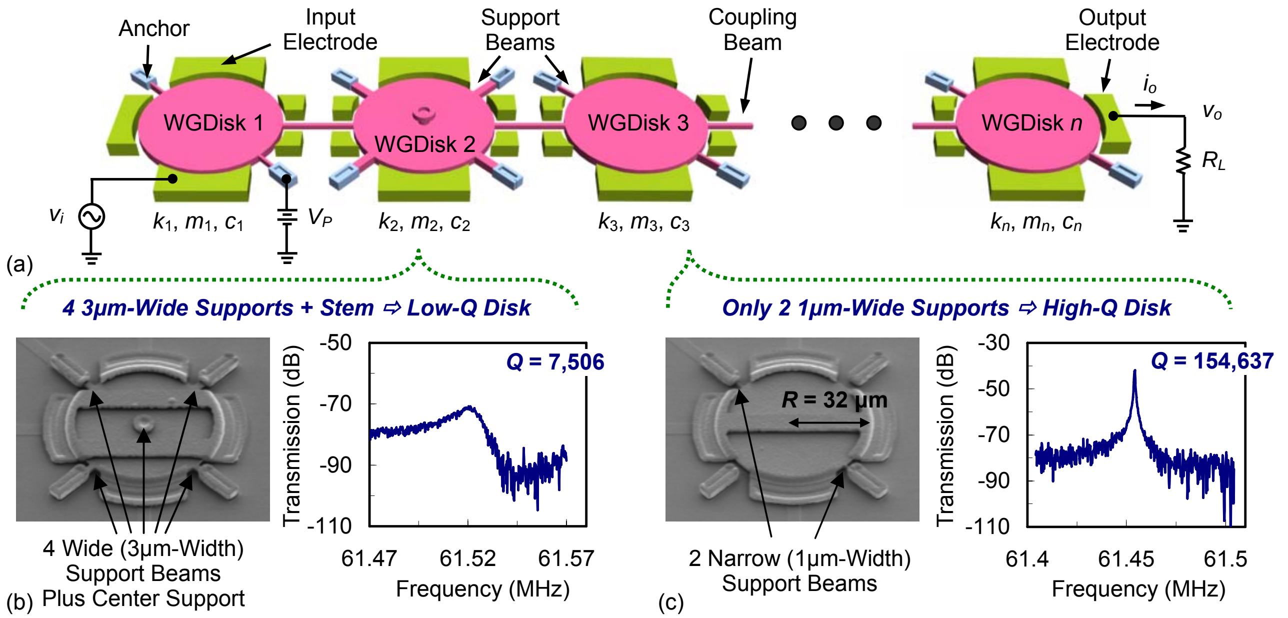

Fig. 1: Perspective-view schematic of an n-resonator version of the mechanically-coupled wine-glass disk resonator array structure, where k_i, m_i, and c_i (i = 1, 2, ..., n) are the stiffness, mass, and damping of the resonator, respectively. The SEM's and measured frequency characteristics of the (b) low-Q and (c) high-Q resonators are shown in the lower portion.

for ensuring Q's greater than a specified threshold value, even when some resonator's Q's are lower than it.

2. WINE-GLASS DISK ARRAY

Fig. 1(a) presents the schematic of an n-resonator version of the mechanically-coupled array structure used to boost Q in this work, together with a typical two-port bias and excitation scheme. Here, n disks, each designed to resonate at 60 MHz in the compound-(2,1) (or "wine-glass") mode shape of Fig. 2, are coupled mechanically [1] by 1µm-wide, half-wavelength coupling beams connecting each adjacent resonator to one another at high-velocity locations.

Each resonator in the array consists of a 32µm-radius, 3µm-thick, electrically conductive disk suspended above the substrate by beams that attach to the disk at quasi-nodal points [5], where displacements are negligible compared to other parts of the disk structure when the disk vibrates in the wine-glass mode shape of Fig. 2, where the disk expands along one axis and contracts in the orthogonal axis. Electrodes surround the disk with a lateral electrode-to-disk gap spacing of only 80 nm. To operate this device, a dc-bias V_P is applied to the disk structure, and an ac voltage v_i is applied to the input electrodes. (Note that there is no dc current flowing once the conductive structure is charged to V_P, so there is no dc power consumption.) This $V_P v_i$ voltage combination generates a time-varying force that drives the disk into the wine-glass mode shape when the frequency of v_i matches the wine-glass resonance frequency f_o, which is inversely proportional to the disk radius. ([5] provides a complete formulation for f_o.)

As shown in [5], the Q of a wine-glass disk resonator is strongly dependent on its supports, where the highest Q's are attained when using the fewest and thinnest supports, such as in Fig. 1(c), and where low Q's can be obtained by increasing the support count and size, such as in Fig. 1(b). Table 1 emphasizes this point with measured Q values for various support number and size combinations. As shown, the Q drops from 154,637 to 121,345 as the number of support beams increases from 2 to 4. In addition, Q drops from 121,345 to 8,989 as support beam width increases from 1 µm to 3 µm. Thus, disk 2 in the Fig. 1(a) array, with 4 wide support beams and a stem, if operated all by itself, would exhibit a considerably lower Q than the adjacent resonators that have only two thin supports each, as verified by the measurements of Fig. 1(b) and Fig. 1(c).

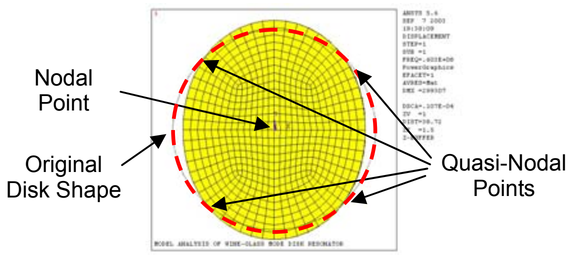

Fig. 2: Wine-glass mode shape simulated via finite element analysis (using ANSYS).

However, when emplaced in the array of Fig. 1(a), the damping factor c_{array} of the total array becomes equal to the sum of the damping factors c_n's of all the resonators. Assuming that the stiffnesses (k_n) and masses (m_n) of all disks are the same and equal to k_r and m_r, respectively, and recognizing that Q is related to damping factor by

$$c_n = \frac{\sqrt{k_n \cdot m_n}}{Q_n} \qquad (1)$$

then

$$\begin{aligned} c_{array} &= c_1 + c_2 + \cdots + c_n \\ k_{array} &= k_1 + k_2 + \cdots + k_n = n \cdot k_r \qquad (2) \\ m_{array} &= m_1 + m_2 + \cdots + m_n = n \cdot m_r \end{aligned}$$

can be rearranged to yield an expression for the Q of the whole array:

$$Q_{array} = n \cdot \left(\frac{1}{Q_1} + \frac{1}{Q_2} + \cdots + \frac{1}{Q_n} \right)^{-1} \qquad (3)$$

For the case where the Q of one of the resonators, say Q_2, is much less than the Q's of all other resonators, (3) reduces to

$$Q_{array} \cong n \cdot Q_2 \qquad (4)$$

In effect, for this case, the array takes on a functional Q that is about n times larger than Q_2.

3. EXPERIMENTAL RESULTS

Wine glass disk array resonators were fabricated via a three-polysilicon self-aligned stem process used previously to achieve disk resonators [6]. Fig. 1(b) and Fig. 1(c) already presented SEM's of fabricated low-Q and high-Q 60-MHz wine-glass disks, while Fig. 3 presents SEM's of fabricated 60-MHz wine-glass disk arrays, one comprised of 1 low-Q and 2 high-Q resonators (c.f., Fig. 3(a)), and the other of 1 low-Q and 8 high-Q resonators (c.f., Fig. 3(b)). Testing was done under vacuum to preserve the anchor-defined differences in the Q's of the micromechanical resonators. In addition, dc bias voltages of V_P = 5 V and input powers of -30 dBm from the port of the network analyzer were utilized.

Table 1: Measured Quality Factor Performance

Single Wine-Glass Disk Resonator Support Beam Parameters		**Measured Quality Factor**
Support Beam Number	Support Beam Width	
2	1 µm	154,637
3	1 µm	133,130
4	1 µm	121,345
4	1.5 µm	55,750
4	2 µm	21,007
4	3 µm	8,989
4 + Center	3 µm	7,506

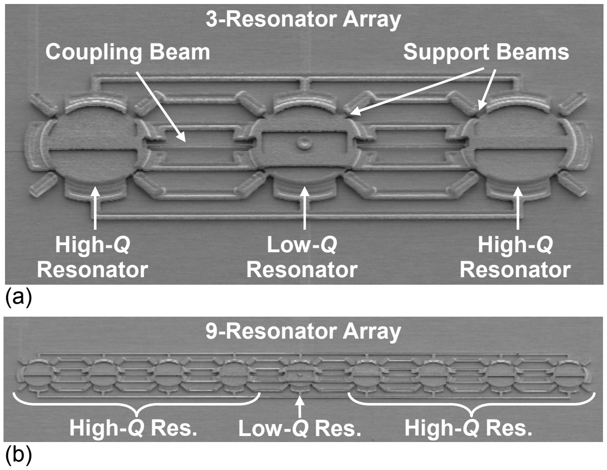

Fig. 3: SEM's of fabricated 60-MHz wine-glass disk resonator arrays with different combinations of high-Q and low-Q resonators.

Fig. 4 combines the measured frequency characteristics for the stand-alone high-Q wine-glass disk of Fig. 1(c) and a three-resonator array of this resonator type. Here, the 135,055 Q of the array is not far from the 154,637 of a single resonator, verifying the prediction of (3).

Fig. 5 combines the measured frequency characteristics of the stand-alone low-Q resonator of Fig. 1(b) with those of the arrays in Fig. 3. Here, whereas the low-Q resonator exhibits a Q of only 7,506 by itself, its functional Q rises to 23,417 when mechanically-coupled to 2 high-Q resonators in the three-resonator array, which is close to the Q = 22,518 predicted by (4). When this low-Q resonator is mechanically-coupled with 8 high-Q

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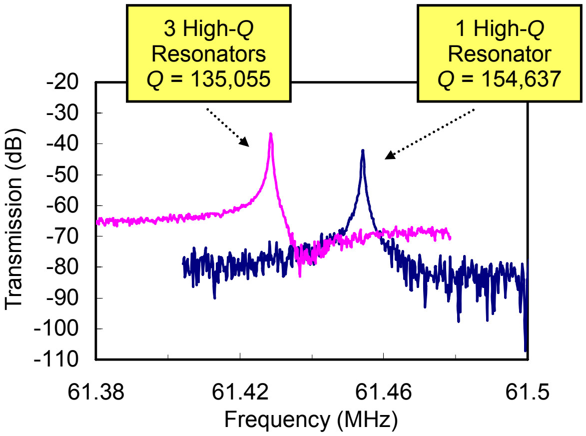

Fig. 4: Measured frequency characteristics for a high-Q wine-glass disk resonator and a mechanically-coupled array of three such resonators.

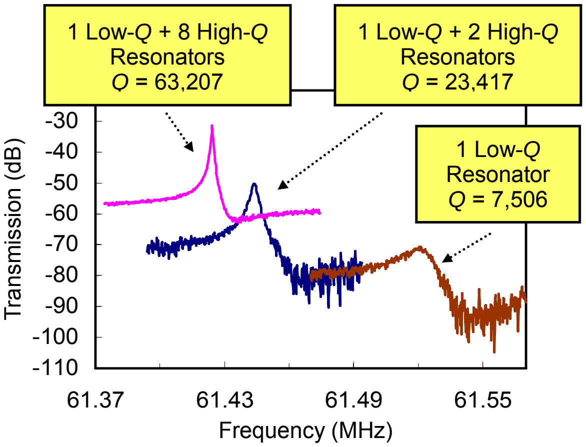

Fig. 5: Measured frequency characteristics for wine-glass disk resonators and arrays with one low-Q resonator and several high-Q resonators, showing that mechanically-coupled arrays can "fix" the Q of a bad resonator.

resonators, the Q of the resulting 9-resonator array is boosted even more dramatically to 63,207, which is about 9X larger than the $Q \sim 7{,}506$ of the embedded low-Q disk, and is again consistent with the prediction of (4).

4. CONCLUSIONS

A micromechanical circuit technique based on mechanically-coupled arraying has been demonstrated to boost the Q's of vibrating micromechanical resonators by factors as high as 9 times. When used to raise the Q's of low-Q resonators, this Q-boosting method can enhance the manufacturing repeatability of micromechanical resonator-based products, since they present a convenient method for ensuring Q's greater than a specified threshold value, even when some of the resonator's Q's are lower than it.

A perhaps more ground-breaking benefit of the described Q boosting is the potential for achieving micromechanical circuits that simultaneously exhibit high-Q and low impedance—a highly desirable combination that so far has not been achieved by stand-alone MEMS resonator designs. If, for example, the ring-shaped aluminum nitride piezoelectric micromechanical resonator of [4] with $Q = 2{,}900$ and motional resistance $\sim 84\,\Omega$ at 472.7 MHz were combined in a mechanically-coupled array with 8 high-Q polysilicon resonators, such as those shown in Fig. 1(c), perhaps with Q's $\sim 50{,}000$ at this frequency, the resulting array-composite resonator would be expected to exhibit a $Q \sim 26{,}100$ (according to (4)), with a motional resistance a bit higher than the original 84 Ω at 472.7 MHz (due to higher mass and stiffness [3]), but still relatively small.

Acknowledgment. This work was supported under DARPA Grant No. F30602-01-1-0573.

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