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A SYSTEMATIC APPROACH TO GENERATING *n*-SCROLL ATTRACTORS

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A new circuitry design based on Chua's circuit for generating n-scroll attractors ($n = 1, 2, 3, \ldots$) is proposed. In this design, the nonlinear resistor in Chua's circuit is constructed via a systematical procedure using basic building blocks. With the proposed construction scheme, the slopes and break points of the v–i characteristic of the circuit can be tuned independently, and chaotic attractors with an even or an odd number of scrolls can be easily generated. Distinct attractors with n-scrolls ($n = 5, 6, 7, 8, 9, 10$) obtained with this simple experimental set-up are demonstrated.

Keywords: n-scroll generation; circuitry implementation; systematic approach.

1. Introduction

One major component for future applications of chaos-based information systems is the hardware implementation of reliable nonlinear circuits for the generation of various chaotic signals. This stimulates the research on generating complex attractors by using electronic devices with lower-order polynomial nonlinearities. A couple of distinct approaches for this purpose have been proposed and can be found in the literature. Particularly, creating attractors with n-scrolls is an interesting topic for research and design.

It was reported that by introducing some additional break points in the piecewise-linear characteristic of the nonlinear resistor in Chua's circuit, the attractor with a maximum of six double-scrolls could be obtained, although by *numerical simulation* [Suykens & Vandewalle, 1993]. From a similar approach [Aziz-Alaoui, 2000], a ten-spiral attractor in Chua's circuit was also generated by *numerical calculation*. Furthermore, the generation of these attractors, with an even or odd number of scrolls, has been considered in [Suykens *et al.*, 1997], where attractors with n-scrolls for $n = 1, 3, 5, 7$, were generated from a generalized Chua's circuit, again via *computer simulation*.

It is well known that it is much more difficult to generate n-scrolls by a physical *electronic circuit*. Although some approaches for constructing the circuit characteristic with additional break points were discussed, only some attractors with a maximum of three-double scrolls that are experimentally observable were reported. In [Arena *et al.*, 1996], cellular neural networks with a piecewise-linear output function were adopted to generate an attractor with three-double scrolls. In [Yalçin *et al.*, 1999], a rescaling break points technique was employed for realizing a six-scroll attractor in a generalized Chua's circuit. Later, in [Wada *et al.*, 1999], the threshold voltage of six diodes was used to set the additional break points of the characteristic, by which a two-double scroll attractor was experimentally observed. Last but perhaps not least, attractors with three- and five-scrolls were experimentally confirmed in a generalized Chua's circuit [Yalçin *et al.*, 2000].

It is also well known that it is especially difficult to realize a nonlinear resistor which has an appropriate characteristic with many segments. The obstacle is two-fold: First, the device must have a very wide dynamic range [Arena *et al.*, 1996; Yalçin *et al.*, 1999]; second, the slopes of those segments and their break points must be adjustable easily and independently. Physical conditions often restrict or even prohibit such circuitry realization.

Considering these difficulties, a new circuitry design is developed here for realizing a multisegment nonlinear resistor. In this design, the nonlinear resistor is constructed via a systematical procedure using negative resistors as building blocks, where the slopes and break points of its v–i characteristic can be independently tuned. The attractors created by this design can have either an even or an odd number of scrolls as desired. For illustration, distinct attractors with n-scrolls for $n = 5$, 6, 7, 8, 9, 10, are shown, which were observed via simple tuning in our experimental setup.

This paper is organized as follows. In Sec. 2, a systematic approach to implementing the multisegment nonlinear resistors is described. In Sec. 3, a local stability analysis is given. Then, the experimental generation of n-scroll attractors for $n = 5$, 6, 7, 8, 9, 10, is reported in Sec. 4. Finally, some concluding remarks are given in Sec. 5.

2. Implemention of a Multisegment Nonlinear Resistor

2.1. *Chua's circuit*

Chua's circuit is a simple autonomous system that can exhibit complex dynamics such as bifurcation and chaos [Zhong & Ayrom, 1985; Kennedy, 1992; Madan, 1993]. A typical Chua's circuit is shown in Fig. 1(a) and its state equations are expressed as follows:

$$
\begin{aligned}
\frac{dv_{C_1}}{dt} &= \frac{1}{RC_1}(v_{C_2} - v_{C_1}) - \frac{1}{C_1} f(v_{C_1}) \\
\frac{dv_{C_2}}{dt} &= \frac{1}{RC_2}(v_{C_1} - v_{C_2}) + \frac{1}{C_2} i_L \\
\frac{di_L}{dt} &= -\frac{1}{L} v_{C_2}
\end{aligned}
\tag{1}
$$

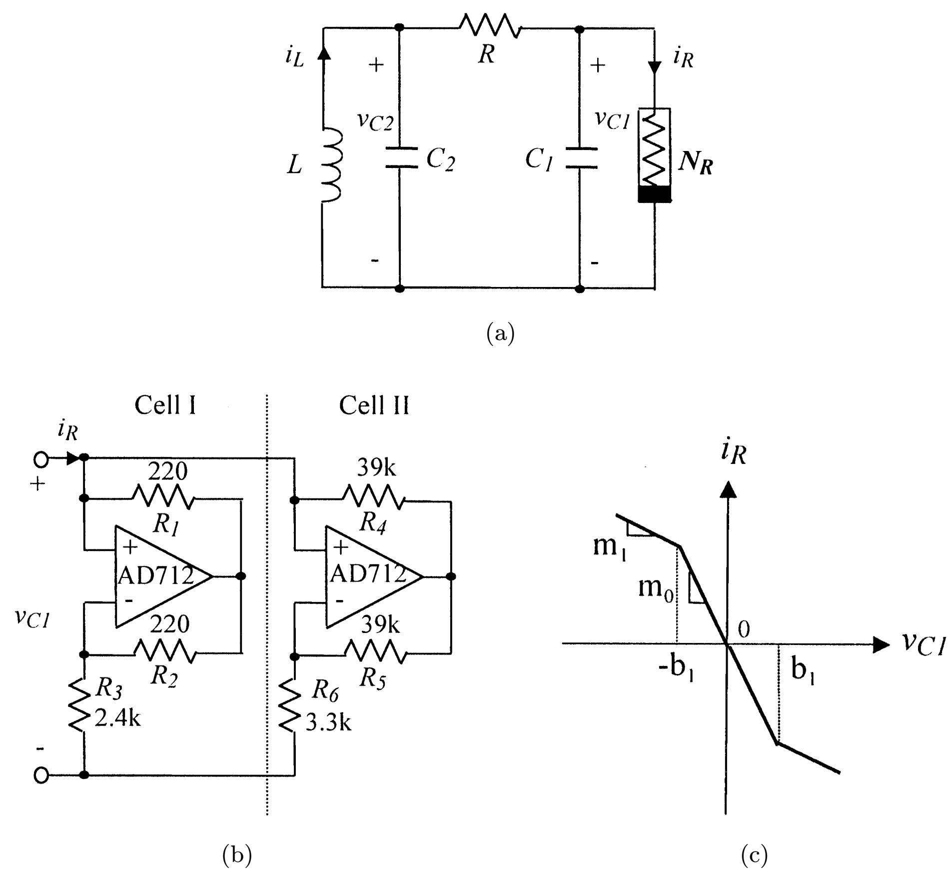


Fig. 1. (a) Chua's circuit; (b) Circuitry for three-segment piecewise nonlinear resistor (Chua's diode); (c) Piecewise-linear v–i characteristic of the nonlinear resistor.

where

$$f(v_{C_1}) = m_1 v_{C_1} + \frac{1}{2}(m_0 - m_1)(|v_{C_1} + b_1| - |v_{C_1} - b_1|) \quad (2)$$

is the v–i characteristic of the nonlinear resistor, the so-called Chua's diode as shown in Fig. 1(b); m_0 and m_1 are the slopes of the inner and outer segments, respectively, and b_1 is the break point voltage of the three-segment piecewise-linear curve, as shown in Fig. 1(c). Chua's circuit can generate the familiar double-scroll attractor and a rich variety of chaotic behaviors.

2.2. *The $v-i$ characteristic of Chua's circuit with a multisegment resistor*

By introducing additional break points into the piecewise-linear function (2), the v–i characteristic of the nonlinear resistor is modified as

$$f(v_{C_1}) = m_{2n-1} v_{C_1} + \frac{1}{2} \sum_{i=1}^{2n-1} (m_{i-1} - m_i) \times [|v_{C_1} + b_i| - |v_{C_1} - b_i|] \quad (3)$$

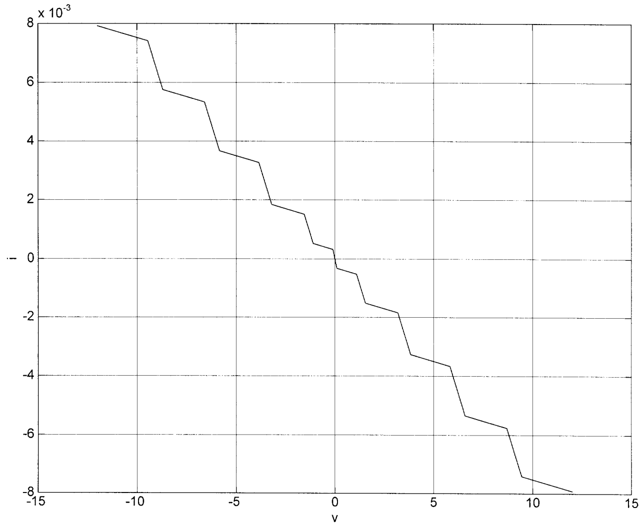
where n is a positive integer, m_i and b_i are the slopes of the ith-segment and the ith-break point, respectively.

Attractors with an even number $2n$ ($n = 1, 2, 3, \ldots$) of scrolls can be generated by this nonlinearity embedded in Chua's circuit. Figures 2(a) and 2(b) illustrate the v–i characteristic calculated by selecting $n = 5$ in (3), and the corresponding attractor with ten-scrolls, simulated by means of the Runge–Kutta method (*ode45* in Matlab), respectively. The chosen parameters are: $C_1 = 10$ nF, $C_2 = 100$ nF, $L = 18.68$ mH, $R = 1380\ \Omega$, $m_0 = -3.2$ mS, $m_1 = m_3 = m_5 = m_7 = m_9 = -0.2$ mS, $m_2 = m_4 = m_6 = m_8 = -2.2$ mS, $b_1 = 0.10$ V, $b_2 = 1.10$ V, $b_3 = 1.55$ V, $b_4 = 3.20$ V, $b_5 = 3.85$ V, $b_6 = 5.84$ V, $b_7 = 6.60$ V, $b_8 = 8.70$ V, $b_9 = 9.45$ V, and the initial conditions are $[0.01, 0.01, 0.001]^T$.

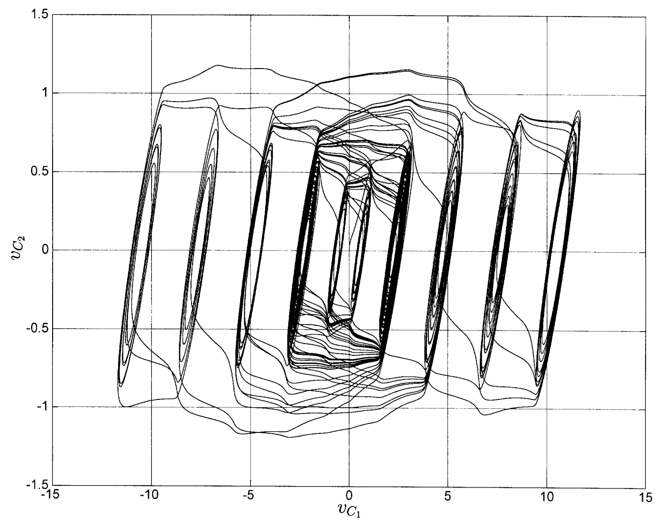
To generate attractors with an odd number $2n - 1$ ($n = 1, 2, 3, \ldots$) of scrolls, b_1 is set to zero and, hence, (3) is recasted in the following form:

$$f(v_{C_1}) = m_{2n-1} v_{C_1} + \frac{1}{2} \sum_{i=2}^{2n-1} (m_{i-1} - m_i) \times [|v_{C_1} + b_i| - |v_{C_1} - b_i|] \quad (4)$$

where notation is as in (3).



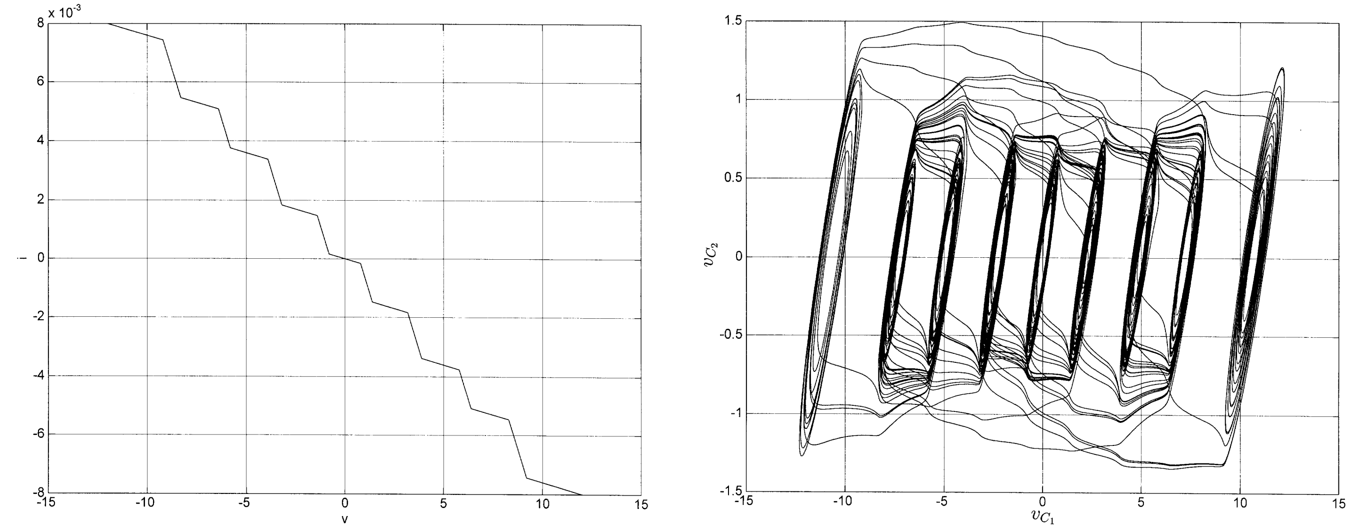
(a)



(b)

Fig. 2. (a) The v–i characteristic with 19 segments of the nonlinear resistor; (b) Phase portrait of the attractor with ten-scrolls in the v_{C_1}–v_{C_2} plane.

Let $n = 5$ and use the following parameters: $C_1 = 10$ nF, $C_2 = 100$ nF, $L = 18.68$ mH, $R = 1380\ \Omega$, $m_1 = m_3 = m_5 = m_7 = m_9 = -0.2$ mS, $m_2 = m_4 = m_6 = m_8 = -2.2$ mS, $b_2 = 0.80$ V, $b_3 = 1.40$ V, $b_4 = 3.20$ V, $b_5 = 3.90$ V, $b_6 = 5.80$ V, $b_7 = 6.40$ V, $b_8 = 8.30$ V, $b_9 = 9.20$ V and initial conditions: $[0.01, 0.01, 0.001]^T$. Then the v–i characteristic calculated and the corresponding attractor with nine-scrolls simulated by means of the Runge–Kutta method (*ode45* in Matlab) are obtained as shown in Figs. 3(a) and 3(b), respectively.



(a) (b)

Fig. 3. (a) The v–i characteristic with 17 segments of the nonlinear resistor; (b) Phase portrait of the attractor with nine-scrolls in the v_{C_1}–v_{C_2} plane.

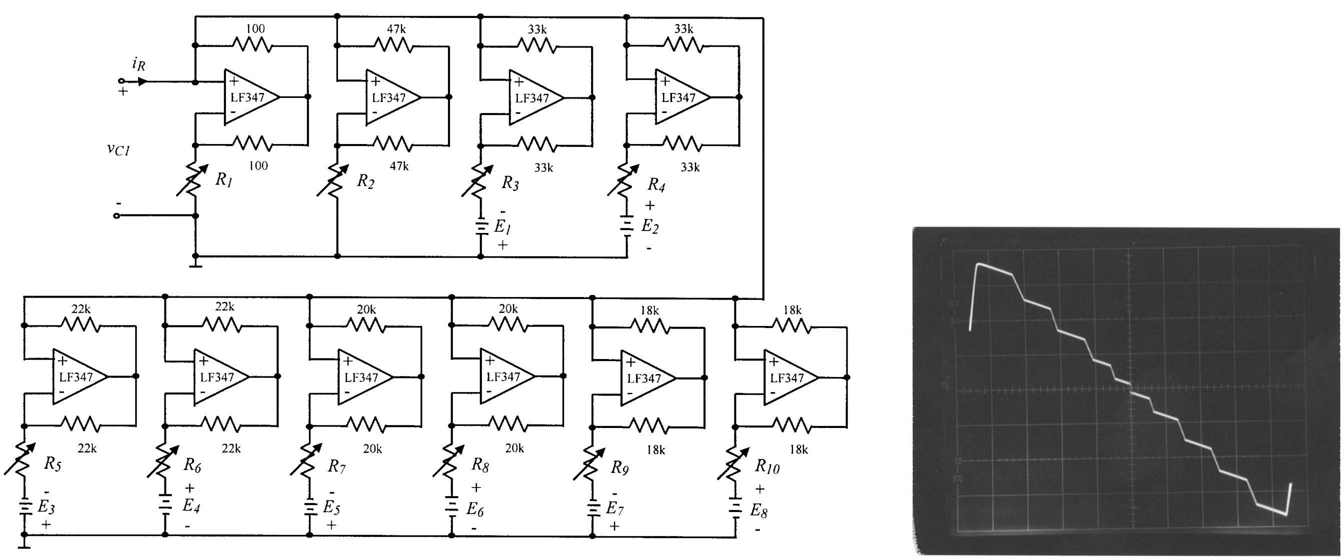
2.3. *Circuitry implementation of nonlinear resistors*

Observing the Chua's diode shown in Fig. 1(b), it can be seen that there are two basic circuit cells: the Op Amp in cell I (left) operates in its linear region and the Op Amp in cell II (right) operates in its whole dynamic range including linear and saturation regions. The saturating point determines the break point of the piecewise-linear function.

Considering these two cells as the building blocks of the circuit, the nonlinear resistors with multisegments are found to be implementable, as described below:

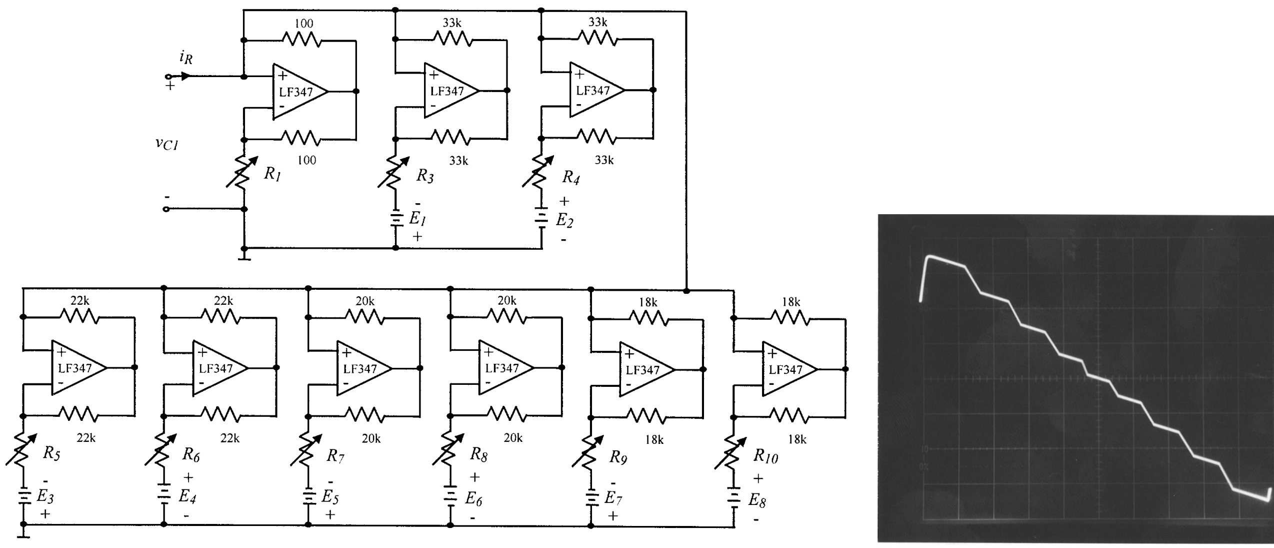
2.3.1. *Implementation of a nonlinear resistor with function (3)*

To implement the nonlinear resistor with v–i characteristic (3) for creating attractors with an even



(a) (b)

Fig. 4. (a) Circuitry realizing the nonlinear resistor with 19 segments; (b) Measured v–i characteristic of the nonlinear resistor.



(a) (b)

Fig. 5. (a) Circuitry realizing the nonlinear resistor with 17 segments; (b) Measured v–i characteristic of the nonlinear resistor.

number $2n$ ($n = 1, 2, 3, \ldots$) of scrolls, additional $n-1$ ($n = 1, 2, 3, \ldots$) pairs of cell II's are connected in parallel with the original Chua's diode shown in Fig. 1(b).

Each cell is offset by an VCVS (voltage-controlled voltage source) in order to tune for the desired break point. Therefore, connecting a pair of cell II's with Chua's diode corresponds to adding two additional break points (with opposite polarities) into the v–i characteristic. For example, a circuit realizing the nonlinear resistor (3) for $n = 5$ is presented in Fig. 4(a). It can be easily observed that four pairs of cells biased by VCVS are connected in parallel with Chua's diode. The v–i characteristic measured in the circuit is shown in Fig. 4(b).

2.3.2. *Implementation of nonlinear resistor with function (4)*

The nonlinear resistor (4) can be realized by simply disconnecting the resistor R_2 in the nonlinear resistor for (3). Then, attractors with an odd number $2n-1$ ($n = 1, 2, 3, \ldots$) of scrolls can be generated. An example of the nonlinear resistor (4) for $n = 5$ is depicted in Fig. 5(a) and the v–i characteristic measured in the circuit is shown in Fig. 5(b).

It is worth noting that the difference between the nonlinear resistors (4) and (3) is only the deletion of the cell II in the original Chua's diode. Taking advantage of such property, it is easy to switch the operational mode of the system from an even number of scrolls to an odd number of scrolls, by simply disconnecting a corresponding cell. This simple yet efficient technique may also benefit potential applications to chaos control and synchronization.

3. Local Stability Analysis

The equilibrium points of system (1) with the v–i characteristics of the nonlinear resistor N_R given by (3) and (4), respectively, will now be determined. Their stabilities will also be discussed.

3.1. *Equilibrium points*

The equilibrium points of system (1) can be obtained by solving the three equations $dv_{C_1}/dt = dv_{C_2}/dt = di_L/dt = 0$. Besides the origin $(0 \;\; 0 \;\; 0)^T$, the other two equilibria are symmetric:

For (3):

$$E_{qn}^{\pm} = \begin{bmatrix} \mp\dfrac{1}{G+m_n}\displaystyle\sum_{i=1}^{n}(m_{i-1}-m_i)b_i \\ 0 \\ \pm\dfrac{G}{G+m_n}\displaystyle\sum_{i=1}^{n}(m_{i-1}-m_i)b_i \end{bmatrix} \tag{5}$$

where $G = 1/R$ and $n = 1, 2, 3, \ldots$.

For (4):

$$E_{qn}^{\pm} = \begin{bmatrix} \mp\dfrac{1}{G+m_n}\displaystyle\sum_{i=1}^{n}(m_{i-1}-m_i)b_i \\ 0 \\ \pm\dfrac{G}{G+m_n}\displaystyle\sum_{i=1}^{n}(m_{i-1}-m_i)b_i \end{bmatrix} \tag{6}$$

where $G = 1/R$, $n = 1, 2, 3, \ldots$ and $b_i = 0$ when $i = 1$.

3.2. *Local stability*

The vector field of system (1) can be decomposed into several distinct affine regions, labeled D_0 and $D_{i+1,i}^{\pm}$, $i = 1, 2, 3, \ldots$, respectively, which are divided by the break points.

Since the autonomous system is linear in each region, the associate Jacobian $\mathbf{J}$ is constant and its eigenvalues are all constant in each region. Hence, the local stability of each equilibrium point can be examined by its corresponding eigenvalues.

The Jacobian matrix for system (1) *with* (3)

In inner region D_0:

$$\mathbf{J} = \begin{pmatrix} -\dfrac{G+m_0}{C_1} & \dfrac{G}{C_1} & 0 \\ \dfrac{G}{C_2} & -\dfrac{G}{C_2} & \dfrac{1}{C_2} \\ 0 & -\dfrac{1}{L} & 0 \end{pmatrix} \tag{7}$$

In outer regions $D_{i+1,i}^{\pm}$:

$$\mathbf{J} = \begin{pmatrix} -\dfrac{G+m_i}{C_1} & \dfrac{G}{C_1} & 0 \\ \dfrac{G}{C_2} & -\dfrac{G}{C_2} & \dfrac{1}{C_2} \\ 0 & -\dfrac{1}{L} & 0 \end{pmatrix} \tag{8}$$

where $i = 1, 2, 3, \ldots$.

The Jacobian matrix for system (1) *with* (4)

In inner region D_0:

$$\mathbf{J} = \begin{pmatrix} -\dfrac{G+m_1}{C_1} & \dfrac{G}{C_1} & 0 \\ \dfrac{G}{C_2} & -\dfrac{G}{C_2} & \dfrac{1}{C_2} \\ 0 & -\dfrac{1}{L} & 0 \end{pmatrix} \tag{9}$$

In outer regions $D_{i+2,i+1}^{\pm}$:

$$\mathbf{J} = \begin{pmatrix} -\dfrac{G+m_{i+1}}{C_1} & \dfrac{G}{C_1} & 0 \\ \dfrac{G}{C_2} & -\dfrac{G}{C_2} & \dfrac{1}{C_2} \\ 0 & -\dfrac{1}{L} & 0 \end{pmatrix} \tag{10}$$

where $i = 1, 2, 3, \ldots$.

Let the parameters for the system with $n = 5$ be selected as in Sec. 2.2. Then, the equilibrium points and the eigenvalues evaluated at the corresponding equilibria of the system (1) with piecewise-linear function (3) are obtained as follows:

$$\begin{array}{ll}
Eq_0 = [0 \quad 0 \quad 0]^T & J(Eq_0): \lambda_1 = 249564;\ \ \lambda_{2,3} = -4637 \pm j2257 \\
Eq_{2,1}^{\pm} = [\pm 0.572\ \text{V} \quad 0 \quad \mp 0.414\ \text{mA}]^T & J(Eq_{2,1}^{\pm}): \lambda_1 = -60876;\ \ \lambda_{2,3} = 583 \pm j21471 \\
Eq_{3,2}^{\pm} = [\pm 1.288\ \text{V} \quad 0 \quad \mp 0.933\ \text{mA}]^T & J(Eq_{3,2}^{\pm}): \lambda_1 = 199754;\ \ \lambda_{2,3} = -29732 \pm j99673 \\
Eq_{4,3}^{\pm} = [\pm 2.287\ \text{V} \quad 0 \quad \mp 1.658\ \text{mA}]^T & J(Eq_{4,3}^{\pm}): \lambda_1 = -60876;\ \ \lambda_{2,3} = 583 \pm j21471 \\
Eq_{5,4}^{\pm} = [\pm 3.525\ \text{V} \quad 0 \quad \mp 2.554\ \text{mA}]^T & J(Eq_{5,4}^{\pm}): \lambda_1 = 199754;\ \ \lambda_{2,3} = -29732 \pm j99673 \\
Eq_{6,5}^{\pm} = [\pm 4.765\ \text{V} \quad 0 \quad \mp 3.453\ \text{mA}]^T & J(Eq_{6,5}^{\pm}): \lambda_1 = -60876;\ \ \lambda_{2,3} = 583 \pm j21471 \\
Eq_{7,6}^{\pm} = [\pm 6.222\ \text{V} \quad 0 \quad \mp 4.509\ \text{mA}]^T & J(Eq_{7,6}^{\pm}): \lambda_1 = 199754;\ \ \lambda_{2,3} = -29732 \pm j99673 \\
Eq_{8,7}^{\pm} = [\pm 7.662\ \text{V} \quad 0 \quad \mp 5.553\ \text{mA}]^T & J(Eq_{8,7}^{\pm}): \lambda_1 = -60876;\ \ \lambda_{2,3} = 583 \pm j21471 \\
Eq_{9,8}^{\pm} = [\pm 9.069\ \text{V} \quad 0 \quad \mp 6.572\ \text{mA}]^T & J(Eq_{9,8}^{\pm}): \lambda_1 = 199754;\ \ \lambda_{2,3} = -29732 \pm j99673 \\
Eq_{10}^{\pm} = [\pm 10.522\ \text{V} \quad 0 \quad \mp 7.624\ \text{mA}]^T & J(Eq_{10}^{\pm}): \lambda_1 = -60876;\ \ \lambda_{2,3} = 583 \pm j21471
\end{array} \tag{11}$$

These parameters yield a ten-scroll attractor as shown in Fig. 2(b). The equilibrium points and the eigenvalues evaluated at the corresponding equilibria of the system (1) with piecewise-linear function (4) are obtained as follows:

$$\begin{array}{ll}
Eq_0 = [0 \quad 0 \quad 0]^T & J(Eq_0) : \lambda_1 = -60876;\ \lambda_{2,3} = 583 \pm j21471 \\
Eq_{3,2}^{\pm} = [\pm 1.085\ \text{V} \quad 0 \quad \mp 0.786\ \text{mA}]^T & J(Eq_{3,2}^{\pm}) : \lambda_1 = 199754;\ \lambda_{2,3} = -29732 \pm j99673 \\
Eq_{4,3}^{\pm} = [\pm 2.287\ \text{V} \quad 0 \quad \mp 1.658\ \text{mA}]^T & J(Eq_{4,3}^{\pm}) : \lambda_1 = -60876;\ \lambda_{2,3} = 583 \pm j21471 \\
Eq_{5,4}^{\pm} = [\pm 3.525\ \text{V} \quad 0 \quad \mp 2.554\ \text{mA}]^T & J(Eq_{5,4}^{\pm}) : \lambda_1 = 199754;\ \lambda_{2,3} = -29732 \pm j99673 \\
Eq_{6,5}^{\pm} = [\pm 4.956\ \text{V} \quad 0 \quad \mp 3.591\ \text{mA}]^T & J(Eq_{6,5}^{\pm}) : \lambda_1 = -60876;\ \lambda_{2,3} = 583 \pm j21471 \\
Eq_{7,6}^{\pm} = [\pm 6.10\ \text{V} \quad 0 \quad \mp 4.42\ \text{mA}]^T & J(Eq_{7,6}^{\pm}) : \lambda_1 = 199754;\ \lambda_{2,3} = -29732 \pm j99673 \\
Eq_{8,7}^{\pm} = [\pm 7.243\ \text{V} \quad 0 \quad \mp 5.249\ \text{mA}]^T & J(Eq_{8,7}^{\pm}) : \lambda_1 = -60876;\ \lambda_{2,3} = 583 \pm j21471 \\
Eq_{9,8}^{\pm} = [\pm 8.676\ \text{V} \quad 0 \quad \mp 6.287\ \text{mA}]^T & J(Eq_{9,8}^{\pm}) : \lambda_1 = 199754;\ \lambda_{2,3} = -29732 \pm j99673 \\
Eq_{10}^{\pm} = [\pm 10.674\ \text{V} \quad 0 \quad \mp 7.735\ \text{mA}]^T & J(Eq_{10}^{\pm}) : \lambda_1 = -60876;\ \lambda_{2,3} = 583 \pm j21471
\end{array} \tag{12}$$

by which a nine-scroll attractor shown in Fig. 3(b) was created.

4. Generation of *n*-Scroll Attractors

To illustrate the proposed systematic design scheme, some attractors with n-scrolls were generated experimentally and reported in this section. Chua's circuit was adopted as a vehicle in our laboratory setup, with the following system parameters: $C_1 = 10$ nF, $C_2 = 100$ nF, and $L = 18.68$ mH. The power supply for all Op Amps was ± 15 V.

Figures 6(a)–6(f) show the attractors with n-scrolls (n = 5, 6, 7, 8, 9, 10) experimentally generated and observed. The selected values of system parameters are listed in Table 1 and shown in the corresponding circuit diagrams otherwise.

Table 1. Selected parameter values for generating the observations shown in Fig. 6.

n	5	6	7	8	9	10
E_1(V)	−2.15	−2.15	−1.30	−1.54	−0.78	−1.43
E_2(V)	2.30	2.30	1.74	1.74	1.27	1.63
E_3(V)	−5.83	−5.83	−3.83	−3.83	−3.17	−3.25
E_4(V)	6.04	6.04	4.31	4.31	3.70	3.99
E_5(V)	–	–	−6.83	−6.83	−5.86	−6.01
E_6(V)	–	–	7.54	7.54	6.53	6.65
E_7(V)	–	–	–	–	−8.96	−8.94
E_8(V)	–	–	–	–	9.37	9.54
$R(\Omega)$	1325	1317	1304	1278	1236	1185
$R_1(\Omega)$	1523	1523	1557	1532	1446	1392
$R_2(\Omega)$	–	10	–	10	–	10
$R_3(\Omega)$	455	455	455	455	454	433
$R_4(\Omega)$	745	745	746	746	745	395
$R_5(\Omega)$	798	798	798	798	797	511
$R_6(\Omega)$	718	718	718	718	718	426
$R_7(\Omega)$	–	–	633	635	633	390
$R_8(\Omega)$	–	–	766	765	765	471
$R_9(\Omega)$	–	–	–	–	741	590
$R_{10}(\Omega)$	–	–	–	–	639	494

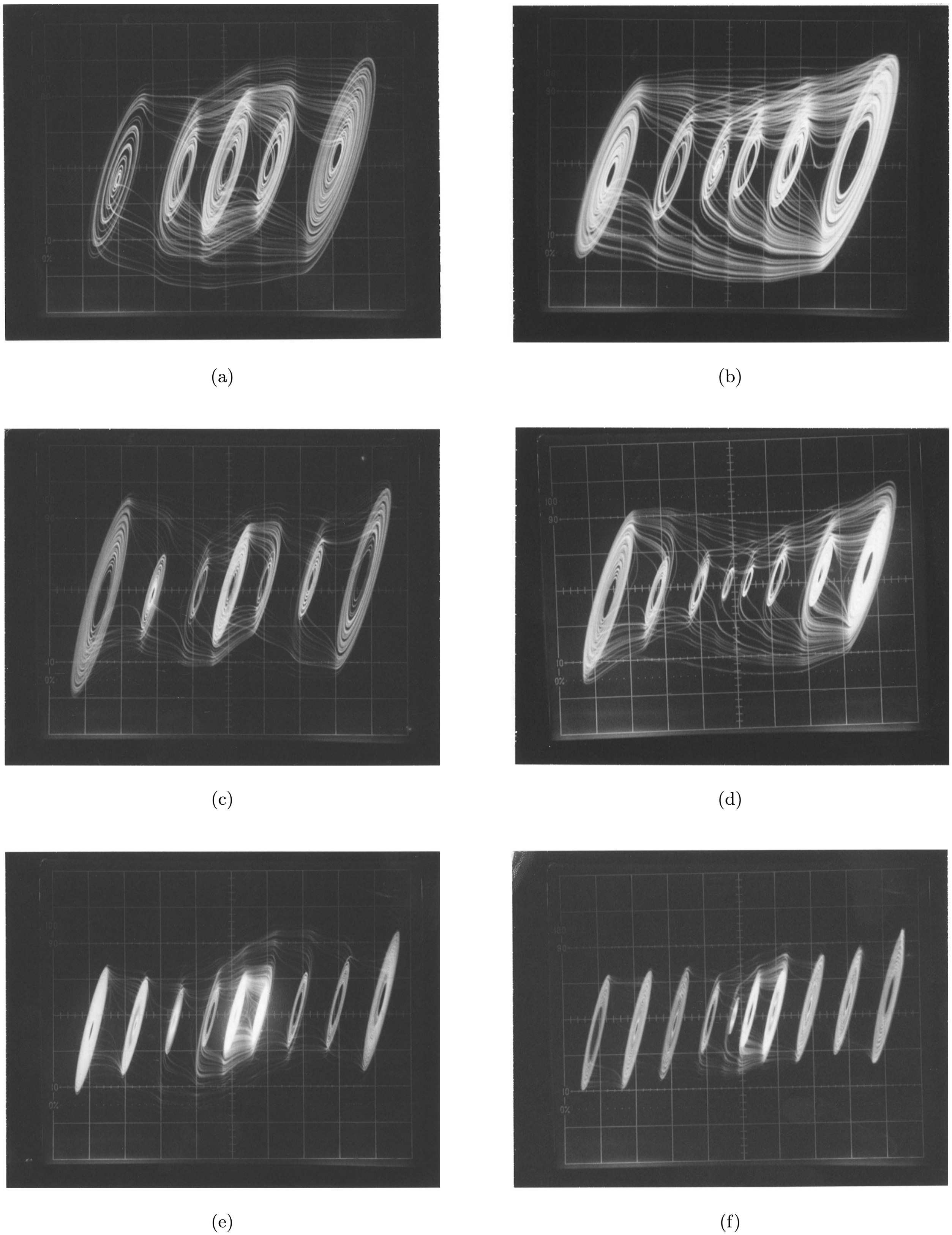


Fig. 6. Phase portrait in v_{C_1}–v_{C_2} plane of n-scroll attractor. Vertical axis: v_{C_2}, 0.5 V/div; Horizontal axis: v_{C_1}, 2.5 V/div. (a) $n = 5$; (b) $n = 6$; (c) $n = 7$; (d) $n = 8$; (e) $n = 9$; (f) $n = 10$.

It can be observed from Table 1 that multiple scrolls are indeed obtained with some minor modifications of system parameters. However, it becomes quite difficult, if not impossible, to generate attractors with a larger number of scrolls, due to the limitation of the dynamic range of the available physical devices.

5. Concluding Remarks

In this paper, a systematic approach to realizing multi-piecewise-linear resistor functions in nonlinear circuits has been proposed and developed. Chaotic attractors with even and odd numbers of scrolls can easily be generated in Chua's circuit experimentally by following the proposed realization scheme and circuitry design. To generate attractors with an even number or an odd number of scrolls, the only difference is found to be simply disconnecting a resistor. Furthermore, the slopes of segments and the break points of the piecewise-linear characteristic are found to be independently tunable, so that the dynamic range of the device can be efficiently exploited. This means that attractors with more scrolls can be obtained for a certain dynamic range of the circuit device. In addition, the circuitry design using building block structure may facilitate monolithic IC realization by using the CMOS technology.

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