An Electronic Real-Time Model of One-Dimensional Discontinuous Conduction of the Cardiac Impulse Realized with Chua's Circuits.


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Abstract - A first approximation of modeling the propagation of excitation through cardiac cells by using nonlinear electronic Chua's circuits is presented here. These circuits are connected by linear resistors and through a continuous passive conductive volume. The functional role of discontinuities is studied within this model showing a good agreement with electrophysiologists results. The effect of some other parameters as the conductivity of the external volume and the distance from the measuring plane to the electronic fiber were also considered.

I. INTRODUCTION AND MOTIVATIONS

A lot of work has been done in order to explain the mechanism of the heartbeat. The propagation of the electrical pulse is followed by the propagation of contraction. Therefore it is important to understand the mechanisms of normal and abnormal conduction of the electrical impulse in the heart and in specific parts of the heart like papillary muscle or ventricular preparations [1].

There are two ways to measure cardiac activation at a specific site with potential sensitive electrodes. One is to impale cells with glass microelectrodes and to measure the intracellular action potential. This procedure is very delicate and cells can be destroyed. The second method is to place small electrodes close to the surface of the preparation surrounded by a conducting salt solution and to measure extracellular potentials. These signals are of lower amplitude but arrays of such electrodes can be fabricated to obtain many simultaneous records at different sites. Such arrays can be scaled down to microscopic size in the order of cell dimensions and even smaller [2].

Electrograms obtained from the epicard are often distorted showing double or multiple deflections of the waveforms [3]. Such irregularities in the electrograms indicate either a spatially complex pathway of the propagating wave and/or effects of a structural and functional discontinuities in the tissue mentioned above.

These experimental phenomena are in contradiction to the theory of syncytial conduction which predicts uniform and elliptic (because of anisotropy of the tissue) wavefronts. Syncytial conduction means that, in the ventricular tissue, the depolarization wavefront can be thought of as a solitary wave propagating through an active, nonlinear and continuous medium.

One possible cause for the irregularities of electrograms might be the discrete microstructure of the tissue if the intercellular resistance increases (electrical uncoupling) or obstacles are embedded between cells (connective tissue) [4]. Because of the complexity of cardiac microstructures, it is very difficult to correlate the multiple deflections to specific microobstacles of the tissue.

On the other hand, systems of discretely coupled cells with reactions and mass, energy or electric charge transfer often serve as standard models for investigating the phenomena occurring in the transformation and transport processes in living cells, tissues, neuron networks, physiological systems and ecosystems, as well as in all forms of chemical and biochemical reactors and combustion systems [5].

A classical example of discrete system used in these modelizations is the nonlinear electronic Chua's circuit [6]. By connecting these cells via resistors, it is possible to build up one, two or three dimensional arrays of circuits that keep the most important features of discrete excitable media. In particular, a linear array can be built that mimics the behaviors observed in some cardiac experiments [7].

In this paper, we present an electronic real-time model of one-dimensional discontinuous conduction of the cardiac impulse made up of Chua's circuits. This model, a row approximation to the cardiac case, exhibits both continuous conduction and the effect of a single discontinuity corresponding to a local obstacle in the tissue and may help to elucidate the functional role of structural discontinuities.

The model consists of a linear array of Chua's circuits connected, first, through resistors in order to form a one-dimensional cable [8] and, second, through some external conductive solution that mimics the conducting path of the local currents via the extracellular space. A local electrical barrier can be simulated by modulating one single coupling resistance and the effect on the propagating wave can be studied at different places in the external medium. The effect of the electrical conductivity of the external conductive volume can also be considered with this setup.
II. EXPERIMENTAL SETUP

The basic unit cell of our array is a Chua's circuit [6], a simple oscillator endowed with an extremely rich gamut of nonlinear phenomena. The circuit contains three linear energy-storage elements (an inductor and two capacitors), a linear resistor and a nonlinear resistor called Chua's diode, described by a three-segment piecewise-linear resistor characteristic.

Two different couplings between cells were considered. First, a short range coupling between neighboring cells, simulating a discrete conduction of signals through the array; i.e., each cell is coupled with its two closest neighbors through a linear resistor, \( R_{\text{int}} \) (see Fig. 1). One or several of these resistances can be increased in order to simulate an electrical barrier to the propagation of excitation. Second, a large range coupling between cells was considered by placing each output of the circuits into a continuous conductive volume (salt solution) that mimics the extracellular volume in the cardiac tissue case.

The scheme of these couplings is depicted in Fig. 1. \( R_{\text{int}} \) are the resistors responsible of the short range coupling and they connect nodes \( N_1 \) of each circuit. In the middle of the array, \( R_{\text{int}} \) is increased to \( R_{\text{gap}} \) in order to simulate an obstacle to the propagation of the signal, which is reflected in a larger delay between the waves measured in both circuits connected through \( R_{\text{gap}} \), than it could be expected under normal conditions. If the value of \( R_{\text{gap}} \) is large enough, the propagation can be blocked (propagation failure) [9]. \( R_{\text{int}} \) are some intermediate resistors between the array and the conductive volume.

Nodes \( N_{\text{ext}} \) in Fig. 1, represent the outputs of the circuits embedded in a rectangular continuous conductive volume (52 x 45 x 1 cm size). The rest of the points plotted in the picture represent other possible positions of measuring. Distances between outputs of the circuits are 1.3 cm; the row of outputs, \( N_{\text{exto}} \), is placed far enough from the borders of the bath in order to avoid deformations in the wave shape.

![Graph](image)

**Fig. 2.** Voltages measured at nodes \( N_1 \) after and before \( R_{\text{gap}} \).

The evolution of the excitation with time was measured at different sites of the setup, namely, at nodes \( N_1 \) of each circuit, at the output of each circuit, nodes \( N_{\text{exto}} \), and at different distances from these outputs, \( N_{\text{ext1}}, \ldots, N_{\text{ext8}} \), separated, respectively, by 1 cm.

Experiments were carried out with a linear array consisting of 40 resistively coupled nonlinear active Chua's circuits connected also through an external passive conductive volume. Initially, all the parameters of the cells were adjusted to have the same initial stable state with the tolerances allowed for the commercially available discrete components used in each unit (10% for inductance, 5% for capacitance and 1% for resistance). In order to study the influence of the external volume on the signal propagation, a train of pulses with low frequency (1 kHz, so the interaction between waves was small) was generated in one end of the array (at circuit number 1). The pulses were triggered by an arbitrary waveform generator (Hewlett-Packard 33120A) with a constant amplitude of 3.4 V, and a pulse width of 20 \( \mu \)s. Circuits were sampled with a digital oscilloscope (Hewlett-Packard 54601) with a maximum sample rate of 20MSA/s, an 8 bits A/D resolution. Data were acquired from the oscilloscope by a PC and, later, filtered in order to diminish the noise.

![Graph](image)

**Fig. 3.** Temporal dependence of the voltages measured at nodes \( N_{\text{ext2}} \) after and before \( R_{\text{gap}} \).

The electronic cell components were fixed throughout this paper as follows: \( C_1 = 1 \, \text{nF} \), \( C_2 = 12 \, \text{nF} \), \( L = 10 \, \text{mH} \), \( r_0 = 10 \, \Omega \), \( G = 3.7 \times 10^{-5} \, \Omega^{-1} \), \( G_0 = 4.56 \times 10^{-5} \, \Omega^{-1} \), \( G_1 = 3.81 \times 10^{-5} \, \Omega^{-1} \), \( G_2 = 3.7 \times 10^{-5} \, \Omega^{-1} \), \( r' = 0 \, \text{\muA} \), \( R_{\text{int}} = 4.7 \, \text{k\Omega} \) and \( R_{\text{gap}} = 0.1 \, \text{k\Omega} \). In order to simulate the gap, or electrical barrier to the propagation of the excitation, the two coupling resistances of circuit number 20 were increased, \( R_{\text{gap}}(19-20) = 3.78 \, \text{k\Omega} \) and \( R_{\text{gap}}(20-21) = 7.57 \, \text{k\Omega} \) and outputs of circuit 19 and 21 were separated by 4 cm in the conductive volume (the output of the circuit number 20 was not embedded into the continuous volume); in this way, the delay introduced by this gap is enhanced and the effect on the propagating signal is optimized. The continuous conductive volume was filled up with different liquids in order to study the influence of conductivity on signal propagation, from distilled water (conductivity 3 \( \mu \)siemens) to ethanol (96% pure and conductivity 0.3 \( \mu \)siemens) different dilutions were used.

III. RESULTS

In Fig. 2, the evolution of the voltage measured at nodes \( N_1 \) is plotted as a function of time. When some critical value of the voltage is reached, the circuit does not
reverses its resting state but describes a full trajectory in the parameter space before reaching its resting state. This excitation propagates through the array with constant velocity \( v(\text{13-16}) = 0.13 \, \text{circuits/\mu s} \). If an obstacle is introduced, at circuit number 20, in the propagating path, the value of some coupling resistor is increased significantly, the velocity of propagation diminishes \( v(\text{19-21}) = 0.06 \, \text{circuits/\mu s} \). This is illustrated in Fig. 2, where the temporal signals of circuits number 16 and 24 measured at nodes \( N_1 \) are plotted for the two cases: with and without gap. Note that when a gap is introduced, the circuit after the gap needs more time to get excited, while the circuit before the gap is excited exactly at the same time as in the case without gap. In this figure, measuring at nodes \( N_1 \), the influence of the gap can only be observed by the delay. If the recording plane is placed far from the outputs of the circuits, which are immersed in a continuous conductive volume filled with water, the situation is quite different. In Fig. 3, the voltage measured at nodes \( N_{\text{ext2}} \) (placed 2 cm far from the array of outputs) is plotted at different positions in this recording plane. Again, these results are compared with the same measurements without gap. Far from the gap (Figs. 3a and d), the waveform is slightly deformed (compare with the case without gap, Figs. 3e and b) but, close to the gap (Figs. 3b and c), the deformation is stronger and a secondary peak is clearly seen. Note that this secondary peak, measured before (after) the gap, corresponds exactly to the time when the main peak occurs measured after (before) the gap (see Fig. 3i, where the signals measured at circuits number 19 and 21 are plotted).

<table>
<thead>
<tr>
<th>Recording Node</th>
<th>Distilled Water Without Gap</th>
<th>Distilled Water With Gap</th>
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Fig. 4. Influence of the distance between the recording and the outputs plane. Only the temporal dependence of the voltages measured at circuits close to \( N_{\text{gap}} \) are plotted, using water (a to i) and ethanol (j to l) in the conductive volume.

In Fig. 4, the influence of the distance between the plane of the circuits outputs and the recording plane as well as the influence of the conductivity of the continuous volume is shown. Each inset shows the temporal evolution of the voltage measured at circuits number 19 and 21 (right before and after the gap respectively). The first two columns (Figs. 4a to j) present the results obtained using distilled water (conductivity 3 microsiemens) as the liquid in the continuous volume, while the last two columns (Figs. 4k to l) were obtained using ethanol (conductivity 0.3 microsiemens). The first and third columns correspond with the case without gap, and the results were recorded at different nodes \( (N_1, N_{\text{ext1}} - 1 \, \text{cm} \) from the plane of the circuits outputs, \( N_{\text{ext3}}, N_{\text{ext5}} \) and \( N_{\text{ext7}} \)). The second and fourth columns were measured considering a gap (as described in section II) in the same points as in the two other columns. Note that as the distance from the recording plane to the outputs is increased, the waveform results attenuated and the amplitude of the second peak (introduced by the gap and only seen in the second and fourth columns) becomes comparable to the first peak. The same is observed in the case with ethanol, just the wave is more attenuated than in the case with distilled water.

The existence of a local discontinuity or gap in the propagation of the excitation through the array results in the appearance of a second peak in the waveform. This second peak is observed when measuring in the conductive volume in front of the circuits placed close to the gap. This effect is shown in Fig. 5. Here, the local excitation time (LET) or time when a local minimum of voltage is recorded are plotted for different circuits (measuring at nodes \( N_{\text{ext2}} \), using distilled water -Fig. 5a- and ethanol -Fig. 5b-). The continuous line represent the LETs when no gap is considered and crosses are the LETs when a local discontinuity is placed in the array. Note that before the circuit number 20 (where the gap is placed), the LETs measured with and without gap coincide. Close to the gap, a second peak appears (and so a second LET is plotted for

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these circuits), far after the gap, the second peak does not appear any longer and propagation continues normally, just with some delay comparing with the case without gap.

The same is observed when the bath is filled up with ethanol (Fig. 5b), just the region where the two peaks coexist is smaller and the delay introduced by the gap is larger. The nonlinear behavior of the LETs far from the gap reflects small differences in the internal parameters of each circuit.

IV. DISCUSSION

In the experiments presented, the excitation of circuits propagates through the array via two different ways; first, through the internal resistors and, second, through an additional external passive conductive volume. The first via a short range coupling, circuits are only affected by the two closest neighbors while the external volume is a sort of large range coupling (all circuits affect each other).

When no continuous conductive volume is considered, the excitation propagates through the array with constant velocity, shape and amplitude. Only the existence of a local discontinuity in the array, gap, introduces a large delay and diminishes drastically the local velocity around the gap. If a large range coupling is considered, the wave form recorded at each point has now two well-distinguished contributions; one coming directly from the neighboring circuits through the internal resistors ($R_{\text{int}}$) and another contribution of all other circuits that now are seen because of the external volume. This second effect reaches simultaneously all circuits, i.e., if a circuit is excited, all others feel it at the same time, only the amplitude and shape of the peak detected diminishes with the distance.

When an obstacle is placed in the middle of the array, the situation becomes more complex. The contribution coming from the other side of the gap and through $R_{\text{int}}$ arrives with a big delay while all contributions coming through the continuous volume arrive instantaneously, just attenuated. As a result, circuits after (before) the gap present a deformation in front of (in the rear part of) the waveform (because of the passive contributions, see Fig. 3). The main peak in every case corresponds with the contributions coming through $R_{\text{int}}$ (the only contribution able to keep constant the amplitude and shape of the wave).

If the distance between the measuring plane and the plane of outputs is increased, all contributions to the signal arrive at the measuring point at the same time. The waveform peaks become broader and the main peak (reflecting the active contribution) becomes of the same order of magnitude as the secondary peak (that reflects the passive contribution), as shown in Fig. 4.

A change in the conductivity of the medium affects the recorded signal in the same way as increasing the distance between the outputs of the circuits and the distance to the measuring plane. The peaks become broader and more attenuated in the ethanol case.

The results presented here constitute a first approach of modeling the propagation of signals in cardiac tissue by using some hardware system -nonlinear electronic Chua’s circuits- that allows a careful control of all the parameters as well as a detailed analysis of the signals (as they can be measured at any point). With this model it was possible to check the role of discontinuities in the propagation of excitation (measurements in the external volume resulted a good tool to detect obstacles in the propagation of signals). Some other parameters, as the distance between the recording plane and the circuits outputs plane, and the conductivity of the external volume and their influence in the system were also studied.

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