Suppressing Chaos in Continuous Systems by Impulse Control

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Abstract

The methods of nonconstant feedback impulse control of chaos are introduced. The approach is based on the similarity of the return maps of dissipative continuous-time systems with one dimensional maps. The methods are illustrated for the Chua's circuit, Rössler oscillator, and phase-locked loop system.

1 Introduction

In this paper, we propose several procedures for controlling chaotic dynamics of continuous-time systems described by ordinary differential equations by ***impulse variation of control parameter*** [1]. These control methods lie, basically, in ***suppressing chaos*** in the return map generated by the trajectories of the continuous system at the Poincaré section.

We consider control in chaotic continuous systems giving one-dimensional maps. We take a chaotic map

$$x_{n+1} = F(x_n)\,. \tag{1}$$

We need to find for the continuous system the impulse control $e(t) = E(x(t), t)$ such that that the resulting map

$$\begin{aligned} x_{n+1} &= F(x_n) + u_n \\ u_{n+1} &= G(x_n, u_n)\,. \end{aligned} \tag{2}$$

where $u_n = u(E)$, should produce a regular trajectory.

Chua's circuit, Rössler oscillator and phase-locked loop (PLL) are used as examples on which chaotic oscillations are suppressed.

2 Chaos suppression by constant impulses

2.1 Map shift

The simplest way to obtain a fixed point for the map (1) producing chaos is to shift this map upwards or downwards [2]) so that the local slope of the map function at the new fixed point x^* should satisfy the stability condition $|\frac{dF(x)}{dx}|_{x^*} < 1$. For the square function F, the transition from chaos to a stable fixed point may be accomplished by means of a constant shift: $x_{n+1} = F(x_n) + u_n$ for $u_n = u^*$. Using this approach we will show how chaos in Chua's circuit may be suppressed by impulse forcing.

2.2 Controlling Chua's circuit

Chua's circuit is an electronic scheme consisting of one nonlinear and four linear elements and is described by three ordinary differential equations [3].

We consider one of the two symmetric spiral chaotic attractors. The return map for this attractor has a square form, which is typical for the return maps depicting the evolution of chaos through a cascade of period-doubling bifurcations. For suppression of chaotic oscillations in Chua's circuit by controlling, the map must be shifted upwards by a certain value so that a stable cycle should appear. This can be attained by adding as a control term external impulse force $e(t)$ to the first equation of the system [4].

The procedure of suppressing chaos in Chua's circuit reduces to feeding external impulse force of constant magnitude E_0 while the map is staying in the ε-neighborhood of the cross-section plane. Note that, since external forcing is fed exclusively in the region of linearity of the system considered, it is obvious that the dependence $u_n(E)$ may be determined analytically. Cycles of periods 4, 2 and 1 are observed for $E_0 = 1.2, 1.4, \text{and} 2.1$, respectively. Original chaotic attractor and controlled period-2 cycle are shown in Fig. 1.

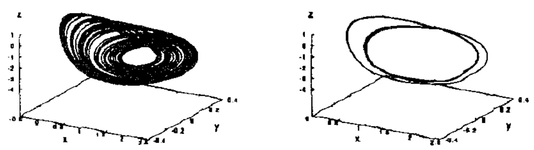

Figure 1: Controlling chaos in Chua's circuit to periodic orbit with impulse feedback.

3 "Local" impulse control

The number of fed impulses and, consequently, the energy consumption on suppressing chaos may be diminished significantly by transforming the map F describing the chaotic behavior of the system not for all the region of variation D of some variable, but only on a certain interval l. We will show below that the magnitude of this interval l may be not greater than 5-10 % of the size of the region D. Hereinafter we will refer to the this method of chaos suppression as to *local* impulse control.

3.1 Local perturbation of the map

We first demonstrate suppression of chaos by local impulse control on an example of a model point map:

$$x_{n+1} = \begin{cases} |1 - 2x_n|, & b \le x_n \le 1 \\ 1 - 2b, & 0 \le x_n \le b. \end{cases} \qquad (3)$$

where the parameter b is responsible for control. In the absence of forcing ($b = 0$), the map (3) produces a chaotic trajectory. This piecewise linear map may be investigated analytically. Periodic orbits of small periods are found relatively easily. The stability of an arbitrary periodic orbit $x_1, x_2, \ldots, x_k$ is determined by $P = |f(x_1)||f(x_2)|\ldots|f(x_k)|$. In our case, the condition $|f(x_j)| = 0$ is fulfilled for an arbitrary point x_j from the set $x_1, x_2, \ldots, x_k$ belonging to the interval $[0, b]$. Consequently, any periodic trajectory produced by the map (3) at $b \neq 0$ and having as one of the coordinates, at least, a point belonging to the interval $[0, b]$ is stable because $P = 0$ for it. We plotted a bifurcation diagram characterizing the evolution of the trajectories realized in the map (3) depending on the magnitude of the parameter b. As b is increased, the chaotic motion existing at $b = 0$ undergoes a cascade of bifurcations which, eventually, lead to emergence of a stable fixed point at $b = 1/3$.

Chaos suppression proceeds as follows. Suppose that we choose a periodic orbit (and the corresponding value of parameter b) which we want to obtain as a result of control. Then, in the course of map iteration, as soon as the current value of the x_n-coordinate gets into the interval $[0, b]$, $u = -2(x_n - b)$ is subtracted from x_{n+1}. After that, we get to a stable orbit of given period only in one iteration. Examples of impulse implementation of this method for continuous systems are given in Sect. 2.2 for fixed u and in Sect. 3.3 for linear controller.

3.2 Controlling Rössler oscillator

The Rössler oscillator is modeled by a system of ordinary differential equations

$$\begin{aligned} \dot{x} &= -y - z, \\ \dot{y} &= x + ay, \\ \dot{z} &= 0.4 + (x - 8.5)z \end{aligned} \qquad (4)$$

As the parameter a is increased in the Rössler oscillator, a chaotic attractor appears through a cascade of period doubling bifurcations of periodic motions. A spiral type chaotic attractor is realized in the system at $a = 0.15$ and $a = 0.18$. The region of permissible changes of the variable y is $D_y = [-13, -6]$. We transform the Poincaré map $y_{n+1} = F(y_n)$ for the Rössler system shifting downwards its portion using impulse force for the values of y_{n+1} near the left boundary of the region D_y. This can be done by including as control the external impulse force $e(t)$ into the second equation of the system.

An example of chaos suppression in the Rössler system by feeding the external impulse force $E_0 = 1.5$ while the map is staying inside the region $|x| < 0.5, y < -12$ is given in Fig. 2(a). A periodic trajectory of period 5 is realized in the system.

Analogous results of chaos suppression we have obtained considering the Poincaré map $x_{n+1} = F(x_n)$ at the section $y = 0$ for $x < 0$. Our numerical experiments verified that for the map $x_{n+1} = F(x_n)$, the decrease in x_{n+1} near the left boundary of the region of permissible changed of the variable D_x leads to the increase of the values of x_{n+1} near the right boundary. Knowing of this fact we performed a series of experiments in which means by external impulse force fed in the region $|y| < 0.5, -5 < x < 0$ we suppressed chaos and realized periodic motions. The results are shown in Fig. 2(b).

4 Automatic impulse control

Numerical experiments on different systems revealed that the magnitude of attractor perturbation (the shift of the one-dimensional map corresponding to it in the simplest case) is proportional to the magnitude of the

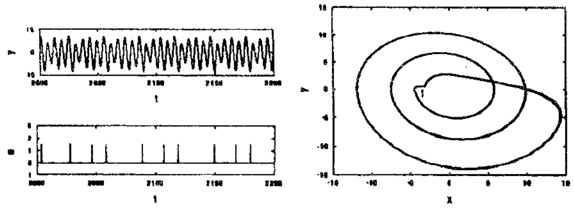

Figure 2: Suppressing chaos in the Rössler oscillator by impulse control applied locally for two different localizations of the impulse control.

fed impulses, and the linear dependence holds in a broad range of impulse magnitudes. This property enables one to control the magnitudes of attractor perturbations, i.e., to control the state of the system in phase space, whereas the linear dependence on impulse amplitude provides a simple implementation of control systems. If the obtained stable periodic solution is a solution of an unperturbed system, then one can say that the periodic orbit *is stabilized* in the chaotic attractor.

4.1 Stabilization of a fixed point of a one-dimensional map by adaptive impulse control

Consider a control system intended for stabilization of a fixed point x^* of the map (1) in the form (2).

For construction of the control law we use the speed-gradient algorithm from the theory of adaptive control [5].

By applying the speed-gradient algorithm to the smooth one-dimensional map (1) and choosing a local target functional in the form

$$Q(x_n, x^*) = (x_n - x^*)^2 ,$$

we obtain a control system of the form (2) with control nonlinearity

$$G(x_n, u_n) = -2\Gamma(F(x_n) - x^*) + (1 - 2\Gamma)u_n , \quad (5)$$

where Γ is the parameter. Note that if x^* is a fixed point of the map (1), then $G(x^*, 0) = 0$ and $(x^*, 0)$ is the fixed point of the control system (2). The presence of a zero second coordinate at the fixed point $(x^*, 0)$ means that, as the trajectory is approaching the control target, the magnitude of the control signal tends to zero, i.e., stabilization of the fixed point occurs by a *small signal.*

The stability conditions of the fixed point $(x^*, 0)$ are defined by

$$\begin{aligned} \beta &> (k-1)\delta , \\ \beta &> (1+k)\delta - 2(1+k) , \\ \beta &< k\delta + 1 - k , \end{aligned} \quad (6)$$

where $k = F'(x^*), \beta = H'_x(0,0) = G'_x(x^*, 0), \delta = H'_u(0,0) = 1 - G'_u(x^*, 0)$ and $G(x_n, u_n) = u_n - H(x_n - x^*, u_n)$. The fixed point $(x^*, 0)$ is rough inside the region (6), consequently, the stabilization effect is structurally stable relative to small parameter variations, which allows us to specify the values of all parameters approximately.

Thus, *weak control* with feedback may be used to stabilize the unstable fixed points of one-dimensional maps of the form (1), including those possessing chaotic dynamics, independent of the form of the function $F(x)$. The only information one needs to have about the map are the coordinate of the fixed point x^* and the value of the derivative $k = F'(x^*)$. The parameters $\beta = H'_x(0,0)$ and $\delta = H'_u(0,0)$ of the control law must satisfy the conditions (6) that guarantee local stability of the fixed point $(x^*, 0)$. The function $H(x_n - x^*, u_n)$ of the form

$$H(x_n - x^*, u_n) = \frac{\beta(x_n - x^*)}{1 + \left(\frac{x_n - x^*}{0.2}\right)^{10}} + \frac{(\delta - 0.2)u_n}{1 + \left(\frac{u_n}{0.2}\right)^{10}} + 0.2u_n , \quad (7)$$

effects local stabilization of the fixed point $(x^*, 0)$ of the map (2) and provides a decrease of the control signal $|u_n|$ at large deviations $|x_n - x^*|$ and $|u_n|$.

4.2 Controlling chaos in a PLL system

Let us employ the control algorithm described above for the problem of impulse stabilization of periodic solution in a continuous model of a nonautonomous phase-locked loop system :

$$\dot{x} = y , \ \ \dot{y} = \gamma - \sin x - (\lambda - r\cos x)y + \mu \sin \omega t , \quad (8)$$

where t is the time, x, y designate the phase variables, and $\gamma, \lambda, r, \mu, \omega$ are the parameters. The system has an asymptotically stable chaotic attractor without rotation along the x-coordinate: $|x(t)| < \pi$. We control the system (8) by periodic rectangular impulses $e(t)$ adding them to the right-hand side of the second equation of the system (8). Such a perturbation leads to the shift of the map (1) by $u(E)$. One can see that the shift of the map is proportional to the magnitude of the impulses: $u(E) \simeq \varepsilon E$. By controlling the magnitude of impulses by means of coordinate feedback according to the law

$$E_{n+1} = E_n - \frac{1}{\varepsilon} G(x_n - x^*, \varepsilon E_n) , \quad (9)$$

we obtain the control scheme (2) for the continuous system (8). In particular, for some fixed parameter values, we have $x^* \simeq 0.23$, $k = F'(x^*) \simeq -1.7$, $\varepsilon \simeq 0.1$. By choosing the control nonlinearity in the form (7) and the parameters δ, β from the region (6): $\delta = -0.1$, $\beta = 2$, we are able to stabilize the saddle periodic solution of the system (8), that corresponds to the unstable fixed point $x^* \simeq 0.23$ (see Fig. 3).

4.3 Linear controller

When the fixed point of the map (1) is stabilized by the procedure described above, the duration of the transi-

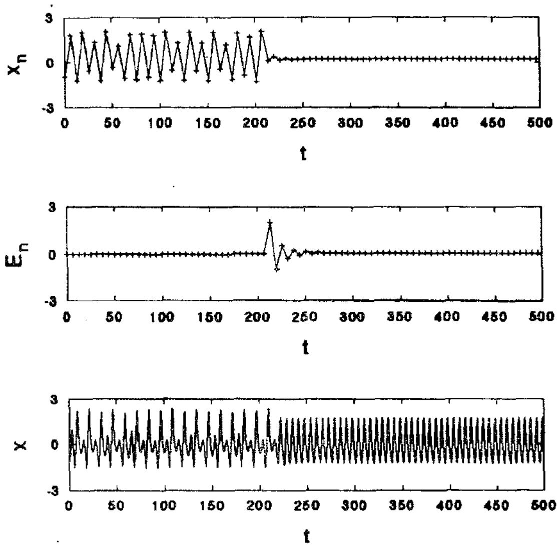

Figure 3: Stabilizing periodic orbit in the model of PLL system using adaptive impulse control.

tion from the initial state on the attractor to the control target (the neighborhood of the fixed point) depends on the eigenvalues of the matrix of linearized map. We consider as an example the impulse implementation of linear controller, within the continuous system (8), driving the orbit of linear map to the fixed point in one iteration. Let us linearize the map (1) in the neighborhood of the fixed point x^*:

$$x_{n+1} = k(x_n - x^*) + x^* , \tag{10}$$

where $k = F'(x^*)$. Let at some iteration $k = M$ the map x_M be in the neighborhood of x^*. Then, with (10) taken into account, there exists the value of the control signal u_M at which x_M is transformed into x^* in one iteration:

$$u_M = -k(x_M - x^*) .$$

For the continuous system (8) with additive impulse control, stabilization of periodic solution is attained in the second equation at the impulse magnitude

$$E_n = \frac{1}{\varepsilon} u_n = -\frac{1}{\varepsilon} k(x_n - x^*) \tag{11}$$

In contrast to the linear case (10), the control target in the continuous nonlinear system (8) is usually not attained in one step, and the control impulses of (11) are fed in each iteration with the number $n \geq M$ until the control target is achieved. Nevertheless, the linear controller (10) may have a rather effective speed in application to the continuous system (8) with impulse control.

5 Conclusion

The methods of controlling chaotic oscillations in continuous dynamic system described above enable one to pass to periodic oscillations. This is attained by a special control in the form of ***additive*** impulse forcing that may be realized by three different methods: control by means of the impulses of constant duration and amplitude that are independent of (i) or dependent on the current state of the system (ii), and control by the impulses whose amplitude, duration and the conditions of feeding obey a definite known law of control (iii).

Similarly, the impulse control of chaos may be realized for variation of the ***system parameters*** [6].

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