SWITCHED CAPACITOR NONLINEAR CIRCUITS DERIVED FROM CHUA'S CIRCUIT

X D Jia and R M M Chen
Department of Electronic Engineering
City University of Hong Kong
83 Tat Chee Avenue, Kowloon, Hong Kong
Fax:(852) 27887791, Email: jia@cpeelgx03.cityu.edu.hk

Abstract: Two switched capacitor (SC) nonlinear circuits derived from Chua's circuits are presented in this paper. These circuits contain SC simulated nonlinear active resistor and SC simulated inductor etc. Computer simulation results are also presented to show the chaotic oscillation generated from these two circuits. Some related issues are also discussed.

I. INTRODUCTION

Switched capacitor networks (SCNs) have many advantages such as easy to be implemented in monolithic IC form, high accuracy, excellent stability, etc. [1-6]. A large number of papers related to SCN have been published since 1980's. Most of the papers deal with about linear SCN, especially switched capacitor filters (SCFs), and only a few of them deal with nonlinear SCN.

More than 200 papers, two special issues and a book on Chua's circuit have been published since its inception a decade ago[7]. The Chua's circuit shown in Fig. 1 is a very famous nonlinear autonomous circuit for research in chaos[7,8]. The circuit shown in Fig. 2. is the simplest sinusoidal driven dissipative second-order circuit made up of three linear circuit elements and a Chua's diode[7,9].

In this paper, two switched capacitor (SC) nonlinear circuits which were derived from Chua's circuits are presented. First, some SC simulated elements used in these two circuits, such as nonlinear active resistor, linear resistor and inductor, are introduced. Then, computer simulation results are presented to show the chaotic oscillation generated from these two circuits. Finally, some related issues are discussed.

II. SC SIMULATED ELEMENTS

To derive SC nonlinear circuits corresponding to these two circuits, we should replace each element in these circuits by an equivalent SC structure. For linear resistors, the linear SC resistor shown in Fig. 3 is used[1]. The equivalent resistance is:

$$R_{eq} = \frac{V^e}{\Delta Q^e / T_c} = \frac{1}{f_c C}$$

where V^e is the even components of the voltage V, $\Delta Q^e = Q^e - Q^o$, Q is the charge, f_C is the clock frequency and $T_C = 1 / f_C$. The input signal frequency f should much less than the clock frequency f_C, that means $f \ll f_C$,

A nonlinear active resistor is a very important element in Chau's circuit. A novel design method for SC nonlinear active resistor (SCNAR) circuits was proposed by the authors[6]. The $\bar{I} \sim V$ characteristic of the SCNAR is a voltage controlled type with piecewise linearity. Its attractive features are insensitive to parasitic capacitance, easy to adjust the breakpoint voltage and the slopes, etc. For the non-autonomous circuit shown in Fig. 2, a SCNAR shown in Fig. 4 is chosen. Since the average current $\bar{I}^e$ for each clock period is equal to $\Delta Q^e / T_c$, the $\bar{I}^e \sim V^e$ characteristic of the SCNAR is given by

$$\bar{I}^e = \begin{cases} (J+1) f_c C_N V^e - J C_N f_c V_s & V > V_s / K_N \\ -(J K_N - J - 1) f_c C_N V^e & V_s / K_N \geq V \geq -V_s / K_N \\ (J+1) f_c C_N V^e + J C_N f_c V_s & V < -V_s / K_N \end{cases}$$

By connecting two such SCNARs with different K_N (e.g. K_{N1} and K_{N2}) in parallel, we obtain a

0-7803-3073-0/96/$5.00 ©1996 IEEE

SCNAR which may be used in the design of SC circuit corresponding to the Chua's circuit as shown in Fig. 1. [6].

For linear inductors, the SC grounded simulated inductor shown in Fig. 5 and the SC floating simulated inductor shown in Fig. 6 are used in the design of SC circuits corresponding to Fig.1 and Fig.2 respectively [4]. The equivalent inductance is:

$$L_{eq} = \frac{C_{L2}}{C_{L1}\,C_{L3}} \cdot \frac{1}{f_C^{\,2}}$$

III. CIRCUIT SIMULATION

These two SC nonlinear circuits were simulated by using a nonlinear SC circuit simulation program named NSCS[5].

For the SC autonomous circuit corresponding to Chua's circuit shown in Fig.1, with $C_1 = 2.51$pF, $C_2 = 10.0$ pF, $C_R = 4.0$ pF, $C_{L3} = 4.3$ pF, $C_{N1} = 1.0$ pF, $C_{N2} = 1.0$ pF, $J_1 = 8.8$, $J_2 = 1$, $K_{N1} = 1.5$, $K_{N2} = 4$, $C_{L1} / C_{L2} = 0.4$ and assuming $V_{C1}(0) = 0.1$ V, $V_{C2}(0)= 0.1$ V, $V_o(0)=0.1$ V, computer simulations were performed and the following results were obtained: the time waveform of $V_o(n)$ is shown in Fig. 7a., the $V_o(n)$ versus $V_{C1}(n)$ Lissajous figures is shown in Fig. 7b, and the power spectrum of time waveform $V_o(n)$ is shown in Fig 7c.

For the SC non-autonomous circuit corresponding to the circuit shown in Fig.2, with $C_1 = 13$ pF, $C_3 = 12.5$ pF, $C_R = 10$ pF, $C_4 = 1.5$ pF, $C_N = 2.5$ pF, $J = 1$, $K_N = 4$, $f(t) = A \cdot \mathrm{Sin}(2 \cdot \pi \cdot i / T_f)$ and assuming $V_1(0) = 0.1$ V, $V(0) = 0.1$ V, $V_o(0) = 0.1$V, computer simulations were performed. The period-doubling, periodic windows, boundary crisis are observed as the forcing parameter A is varied. When $T_f = 33$ and A is chosen as 0.612, the following results were obtained: the $V_o(n)$ versus $V_1(n)$ Lissajous figures is shown in Fig. 8a, and the power spectrum of time waveform $V_o(n)$ is shown in Fig 8b.

For both SC circuits, the power spectra are noise-like and different from that of a periodic or quasi-periodic waveform. These two systems also exhibit sensitive dependence on initial conditions.

IV. CONCLUSION AND DISCUSSION

Two SC nonlinear circuits have been derived from Chua's circuits using SC simulated elements. Computer simulation results show that chaotic oscillation can be generated from these two SC nonlinear circuits.

Since SC circuits have their own features and limitations, we cannot set the parameters in our SC autonomous circuit exactly equivalent to that of Chua's double scroll circuit. However, we can observe some similarities in waveforms, attractors as well as spectrums between the two.

ACKNOWLEDGMENT

The authors are grateful to the University Grants Committee of Hong Kong for the support of this project. Also Prof. L O Chua of UC Berkeley sent many papers about Chua's circuit to us, which are very helpful for our research.

REFERENCES

[1] R. Unbehauen and A. Cichochi. *MOS Switched-Capacitor and Continuous-Time Integrated Circuits and Systems*, Springer-Verlag, 1989.

[2] J. L. Haertas and L. O. Chua, " Nonlinear Switched-Capacitor Network: Basic Principles and Piecewise-linear Design ," *IEEE. Trans. Circuits and Syst.* vol CAS-32 , April 1985, pp.305-318.

[3] A. B. Rodriguezz-Vdzquez, J. L. Huertas and L. O. Chua, " Chaos in a Switched-Capacitor Circuit ," *IEEE Trans. Circuits and Syst.* vol CAS-32, Oct. 1985, pp.1083-1084.

[4] Y. Horio, M. Y. Shinsakamori " Switched-Capacitor General Impedance Converter ", in *Proc. 1985 IEEE International symposium on Circuits and Systems*, May, 1985, pp.789-792

[5] X. D. Jia and G. Y. Qiu , " Novel Switched-Capacitor Oscillators and Their Computer Simulation ," *Journal of China Institute of*

Communication, vol 8, No.6, Nov.1987, pp.51-55.

[6] X. D. Jia and R. M. M. Chen, " Design and Simulation of Nonlinear Switched Capacitor Autonomous Active Resistor ", in *Proc. 1994 IEEE International symposium on Circuits and Systems*, May, 1994, pp.165-168.

[7] L. O. Chua, " Chua's Circuit: An Overview Ten Years Later ", *Journal of Circuits, Systems and Computers,* Vol.4, No.2, 1994, pp117-159.

[8] T. Matsamoto, L. O. Chua and M. Komuro, " The Double Scroll ", *IEEE Trans Circuits and Syst*. vol CAS-32 Aug. 1985, pp.798-1055.

[9] K. Murali and M. Lakshmanan, " Bifurcation and Chaos In The Simplest Dissipative Non-autonomous Circuit ", *International Journal of Bifurcation and Chaos*, vol.4, No.6 1994, pp1151-1524.

[10]T. S. Parker and L. O. Chua, *Practical Numerical Algorithms for Chaotic Systems*, Springer-Verlag, New York, 1989.

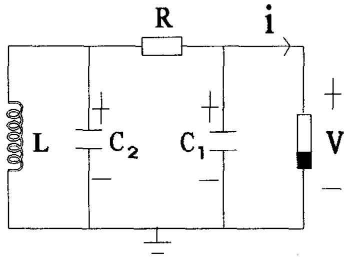
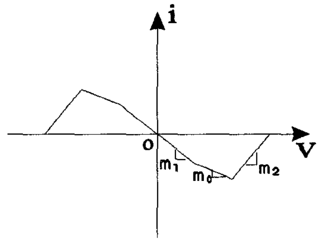


Fig. 1 a) The Chua's circuit.



b). The i-v characteristic of the nonlinear resistor.

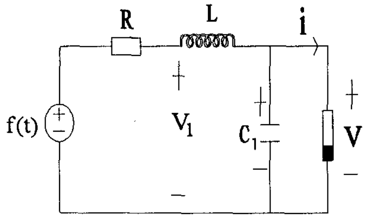


Fig. 2 The non-autonomous circuit.

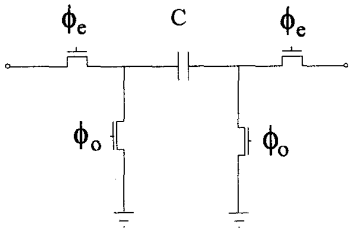


Fig. 3 Switched capacitor resistor.

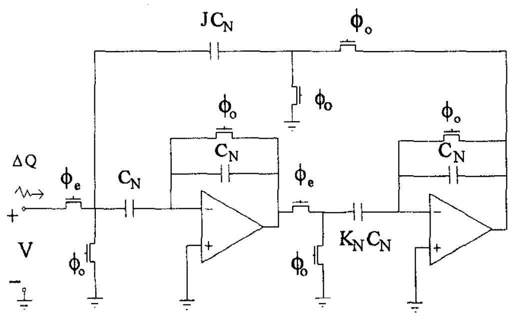


Fig. 4a. The circuit diagram of the SCNAR.

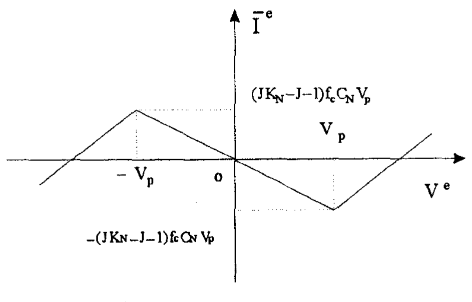


Fig. 4b The $\bar{I}^e \sim V^e$ characteristic of the SCNAR.

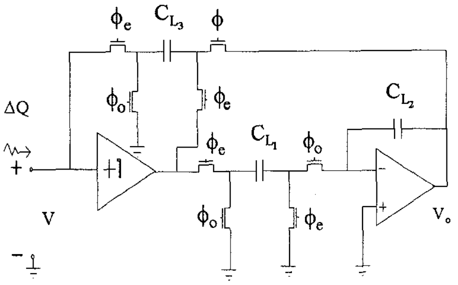


Fig. 5 The circuit diagram of the SC grounded simulated inductor circuit.

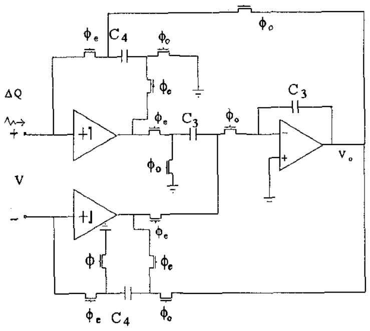
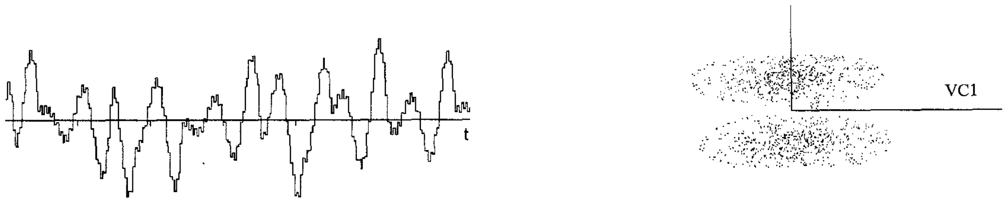
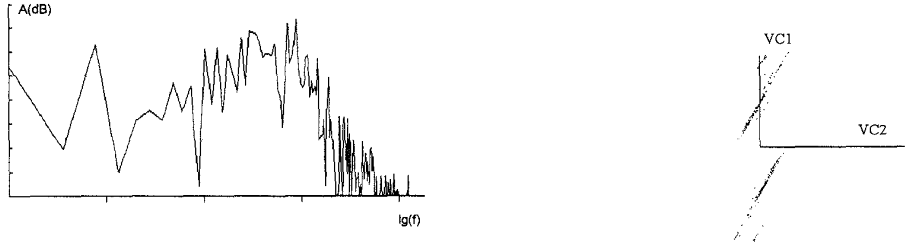


Fig. 6 The circuit diagram of the SC floating simulated inductor circuit



(a) The time waveform of the $V_O(n)$.

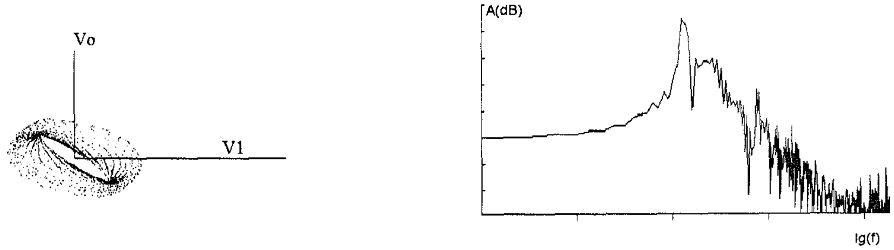
(b) The $V_O(n)$ versus $V_{C1}(n)$ Lissajous figure,



(c) The power spectra A of time waveform $V_O(n)$.

(d) The $V_{C1}(n)$ versus $V_{C2}(n)$ Lissajous figure when Vo=0

Fig. 7 Computer simulation results.



(a) The $V_O(n)$ versus $V_1(n)$ Lissajous figure.

(c) The power spectra A in dB of time waveform $V_O(n)$.

Fig. 8. Computer simulation results.