Design and simulation of nonlinear switched capacitor autonomous circuits containing nonlinear active resistor

X D. Jia and R. M. M. Chen
Department of Electronic Engineering
City Polytechnic of Hong Kong
83 Tat Chee Avenue, Kowloon, Hong Kong
Fax: (052) 788 7791, Email: 00760460@cphkvx.bitnet

ABSTRACT

The design and simulation of nonlinear switched capacitor autonomous circuits containing nonlinear active resistor is described in this paper. First, a new kind of SC nonlinear active resistor circuit is proposed. Then the novel SC nonlinear active resistor is used in the design of nonlinear SC autonomous circuits, in particular, a \( N_R \) L C SCO circuit and an autonomous circuit derived from Chua's circuit. Finally the computer simulation of \( N_R \) L C SCO and chaotic oscillation of the three order SC autonomous circuit are discussed and some results are presented in this paper.

I. INTRODUCTION

Switched capacitor networks (SCNs) have many advantages such as easy to be implemented in monolithic IC form, high accuracy, excellent stability, etc. [1-7]. A large number of papers related to SCN have been published since 1980's. Most of them are about linear SCN, especially switched capacitor filters (SCFs), and only a few of them deal with nonlinear SCN.

A nonlinear active resistor is an important element in nonlinear electronic autonomous circuits such as the Van der Pol oscillator and Chua's double scroll circuit [8-10]. In this paper a novel design method for SC nonlinear active resistor circuit is described and some new SC nonlinear active resistor (SCNAR) circuits are proposed. The \( I-V \) characteristic of the SCNAR is voltage controlled type with piecewise linearity. Its attractive features are stable in structure, insensitive to parasitic capacitance, easy to adjust the breakpoint voltage and the slopes, etc. The novel SCNAR can be used in the design of SC autonomous circuits and the design of two typical autonomous circuits are described in section III. They are a \( N_R \) L C switched capacitor oscillator (SCO) and a nonlinear autonomous circuit derived from Chua's double scroll circuit that may be a chaotic oscillation generator [8-10]. The circuit simulations of these SC circuits are also discussed and some of the results are presented in section IV.

II. NONLINEAR SC ACTIVE RESISTOR CIRCUIT DESIGN

According to the principle of switched capacitor technique [1-7], a linear switched capacitor resistor is shown in Fig. 1. The equivalent resistance is:

\[
R_{eq} = \frac{V^e}{\Delta Q^e / T_c} = \frac{1}{f_c C}
\]

where \( f_c \) is the clock frequency and \( T_c = 1/f_c \). The input signal frequency \( f \) should meet the requirement \( f << f_c \).

\[\text{Fig. 1 Switched capacitor resistor.}\]

By introducing a linear voltage controlled voltage source (VCVS) with \( S[V] = K_i V \) in the circuit in Fig. 1, we obtain an SC linear negative resistor circuit shown in Fig. 2a. Its equivalent resistance is
where $K_N$ is a constant coefficient and it can be proved that $K_N > (J+1)/J$ in order to achieve the effect of negative resistance. When the controlled voltage source is nonlinear and $S[V]$ is a function defined as follows:

$$S[V] = \begin{cases} 
V & V > V_s/K_N \\
K_N V & V_s/K_N \leq V \leq V_s/K_N \\
-V_s & V < -V_s/K_N 
\end{cases}$$

where $V_s$ is the saturation voltage, the circuit in Fig. 2a is an SC nonlinear active resistor (SCNAR) circuit. Since the average current $\overline{I}^e$ for each clock period is equal to $\Delta Q^e/T_c$, the $\overline{I}^e - V^e$ characteristic of the SCNAR is given by

$$\overline{I}^e = \frac{\Delta Q^e}{T_c} = \begin{cases} 
(J+1)f_c C_N V_s^e - J f_c C_N f_s V_s & V > V_s/K_N \\
(J K_N - J - 1)f_c C_N V^e & V_s/K_N \geq V \geq -V_s/K_N \\
(J+1)f_c C_N V^e + J f_c C_N f_s V_s & V < -V_s/K_N 
\end{cases}$$

and the characteristic is shown in Fig. 2b. The breakpoint voltage $V_p = V_s/K_N$ is determined by $K_N$ and the slope is determined by $J$ and $C_N$.

The SC inverting amplifier that is often used in linear SCN may perform the VCVS in Fig. 2a and in this case the $V_s$ is the saturation voltage of op-amps. It can be shown that the detail diagram of SCNAR circuit proposed in this paper is shown in Fig. 3. This SCNAR is stable in structure and insensitive to parasitic capacitance.

**III. SC AUTONOMOUS CIRCUITS DESIGN**

In this section the novel SC nonlinear active resistor is used in the design of the following SC autonomous circuits:

1. N_R L C Switched Capacitor Oscillator Circuit Design

The circuit shown in Fig. 5 is an oscillator and if the $v-i$ characteristic of the nonlinear resistor is voltage controlled type with piecewise linear, then the circuit equation of the oscillator may be Van der Pol equation. Using the SCNAR circuit proposed in this paper and the SC simulated inductor circuit shown in Fig. 6 [6] to replace the nonlinear resistor and inductor of the circuit shown in Fig. 5, we may implement an SCO that will be referred as $N_R L C$ SCO in this paper. The frequency of the SCO can be calculated approximately by:
\[ f_0 \approx \frac{1}{2\pi} \sqrt{\frac{C_{L_1} C_{L_2}}{C_{L_1} C}} f_c \]

The computer simulation of this SCO circuit will be discussed in next section.

![Fig. 5 NLC oscillator.](image)

![Fig. 6 The circuit diagram of the SC simulated inductor circuit.](image)

(2) SC Autonomous Circuit Derived From Chua's Circuit

The Chua's circuit shown in Fig. 7 is a very famous nonlinear autonomous circuit for research in chaos and has been studied extensively in the past [8-10]. Investigation on chaotic oscillation phenomenon is an important research subject in biology, chemistry, mathematics, physics, mechanics, and nonlinear circuit and systems, etc. The phenomenon of chaos in electronic circuits is most easily observed, analyzed, and controlled.

![Fig. 7 a) The Chua's circuit. b) The characteristic of the nonlinear resistor.](image)

Using the new SCNAR with its characteristic shown in Fig. 4, an SC simulated inductor circuit shown in Fig. 6 and a linear SC resistor shown in Fig. 8, we obtained the design of an SC autonomous circuit that corresponds to the Chua's circuit. Computer simulation results demonstrate that this SC autonomous circuit may generate chaotic oscillation. Some simulation results are presented in the next section.

![Fig. 8. SC linear resistor](image)

IV. COMPUTER SIMULATION OF NONLINEAR SC CIRCUIT

Modern SC networks may consist of hundreds or thousands of capacitors, switches and operational amplifiers and normally have two excitations: input signal excitation and clock signal excitation. Circuit simulations using a standard general purpose simulator such as SPICE can be very time consuming. Partitioning algorithm and parallel processing technique can be used to reduce the simulation time for VLSI [11-12]. The speed up is contributed by both decomposition algorithm and multiprocessor system. Using decomposition method and parallel processing technique for practical SC circuit simulation is out of the scope of this paper.

Since ideal nonlinear autonomous SC circuits are not continuous time circuits and have no equivalent z-domain circuit representation, general purpose simulation programs as well as some specialized programs for SCF cannot deal with them. A nonlinear SC circuit simulation program named NSCS was developed [5]. NSCS can be used to compute the oscillation frequency, amplitude, set up time and harmonics, and to draw the output waveform, frequency spectrum, etc.

For the SCO circuit corresponding to Fig. 5, computer simulations were performed for \( C_{L_1} = 2.66 \) pF, \( C_{L_2} = 7.99 \) pF, \( C_{L_3} = 0.8 \) pF, \( C = 10.14 \) pF, \( C_N = 0.8 \) pF, \( V_N = 3.22 \), \( J=1 \). We obtain the following results: the frequency of SCO is \( f_0 = f_c / 40.92 \) and the output amplitude is 2.824 V. Figs. 8a and 8b are the waveform and power spectrum of \( V_c(n) \) respectively.

![Fig. 8. a) The time waveform of \( V_c(n) \). b) The power spectrum of \( V_c(n) \).](image)
For the SC autonomous circuit corresponding to Chua's circuit shown in Fig. 7, with $C_1 = 5.0 \, \text{pF}$, $C_2 = 20.0 \, \text{pF}$, $C_N = 9.0 \, \text{pF}$, $C_{13} = 8.61 \, \text{pF}$, $C_{N1} = 2.0 \, \text{pF}$, $C_{N2} = 2.0 \, \text{pF}$, $J_1 = 10$, $J_2 = 1$, $K_{N1} = 1.5$, $K_{N2} = 4$, $C_1/C_{12} = 0.4$ and assume $V_{C1}(0) = 0.1 \, \text{V}$, $V_{C2}(0) = 0.2 \, \text{V}$, $V_n(0) = 0.3 \, \text{V}$, computer simulations were performed and the following results were obtained: the time waveform of the $V_n(t)$ is shown in Fig. 9a, the $V_n(t)$ versus $V_{C1}(t)$ Lissajous figures is shown in Fig. 9b, and the power spectrum of time waveform $V_n(t)$ is shown in Fig. 9c. The power spectrum is noise-like and different from that of a periodic or quasi-periodic waveform. This system also exhibits sensitive dependence on initial conditions.

**Fig. 9** Computer simulation results.

Since switched capacitor circuits have their own features and limitations, we cannot set the parameters in our SC autonomous circuit exactly equivalent to that of Chua's double scroll circuit. However, we can observe some similarities in waveforms, attractors as well as spectrums between the two.

**V. CONCLUSION**

A novel switched-capacitor nonlinear active resistor (SCNAR) design is proposed in this paper. This SCNAR can be used in the design of switched-capacitor nonlinear autonomous circuits. Two SC autonomous circuits are presented in this paper. The first one is a $N_R \, L \, C$ SCO circuit and the other is an SC autonomous circuit basically derived from the Chua's double scroll circuit. Circuit simulation of SC nonlinear autonomous circuits are also discussed. Simulation results verify the usefulness of the design technique.

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