forward amplifier with resistive output-summing network is given. The inverting amplifier, in particular, is considered in detail, and its performance is compared with that of the amplifier without feedforward error correction. It should be noted that the proposed feedforward technique is also acceptable for wide-band amplifiers with transformer input–output networks.

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Analysis of Chaotic Behavior in Lumped-Distributed Circuits Applied to the Time-Delayed Chua's Circuit

E. A. Hosny and M. I. Sobhy

***Abstract*—A general method for the analysis of prechaotic and chaotic behaviors in lumped-distributed circuits has been developed. The method is applied to the time-delayed Chua's circuit and the analysis predicts the possibility of the existence of multilevel oscillations and chaotic behavior of the circuit. The proposed procedure predicts the presence of multilevel oscillations which may lead to a chaotic behavior of this circuit. A bifurcation diagram, and phase plane are presented which verify the proposed procedure.**

I. Analysis Procedure of Lumped-Distributed Networks

The analysis procedure starts by establishing the state equations describing the lumped-distributed network. Then the equilibrium points are determined. Finally the dynamics of the network can be predicted by studying the local stability at the equilibrium points [1]. The time simulation can obtained by using the approach introduced in [2].

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A. State Space Representation of Lumped-Distributed Networks

The state variables of a general nonlinear lumped-distributed network are represented by capacitor voltages (or charges) and the inductor currents (or fluxes) and the reflected (or incident) voltage waves at the transmission line ports. Either the incident or reflected voltage waves can be chosen as the distributed state variables.

The state and output equations describing a general nonlinear lumped-distributed network [3] are given by

$$\begin{bmatrix} \dot{x}_1(t) \\ x_2(t+T_i) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + B\, u(t) + B_n F_n(x_1, x_2, u) \tag{1}$$

$$y(t) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + D\, u(t) + D_n F_n(x_1, x_2, u) \tag{2}$$

where x_1 is the lumped state vector of order n,
$\dot{x}_1$ is the first derivative of the lumped state vector x_1,
x_2 is the distributed state vector of order m,
F_n is the vector of nonlinear functions,
u is the input vector,
y is the output vector,
T_i is the delay of the ith transmission line, and
$A_1, A_2, A_3, A_4, B, B_n, C_1, C_2, D$, and D_n are real matrices of compatible dimensions.

B. The Equilibrium Points and Their Stability Analysis

The conditions at the equilibrium points (D.C. solutions) are given by

$$\dot{x}_1(t) = 0$$
$$x_2(t+T_i) = x_2(t). \tag{3}$$

Applying conditions (3) into (1), the following matrix equation for the equilibria can be obtained,

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 - I_m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -Bu - B_n F_n(x_1, x_2, u) \tag{4}$$

where I_m is a unit matrix of order m.

The solution of (4) gives the coordinates of the equilibrium points,

$$x_{Qi} = [x_{1Q_i} \quad x_{2Q_i}]^T \quad i = 1, 2, \ldots, l \tag{5}$$

where x_{1Qi}, x_{2Qi} are the coordinates of the lumped and distributed state vectors at the ith equilibrium point, and l is the number of the equilibrium points.

These equilibrium points can be directly determined by finding D.C. solution of the circuit. In this case all the inductors are short-circuited, capacitors are open-circuited, and input and output terminals of each transmission line are connected together. The number of the equilibrium points depends on the parameters of the network and the characteristics of the nonlinear functions $F_n(x_1, x_2, u)$.

For circuits containing a single nonlinear element, the system stability criteria of the linear system and the graph of the nonlinear

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element can be combined in the i-v plane of the nonlinearity. Thus the existence of equilibria, their positions, stability can be obtained. In this case one can identify two types of oscillations $(-\alpha, \beta)$ and multilevel oscillations [2].

The dynamics of the network in the vicinity of the equilibrium points are studied by linearizing the system of state (1) at each equilibrium point. The stability of the equilibrium points can be determined by examining the Jacobian matrix. The Jacobian matrix of (1) at each equilibrium point is given by,

$$J(x_{Qi}) = \begin{bmatrix} \mathbf{A}_{1i} & \mathbf{A}_{2i} \\ \mathbf{A}_{3i} & \mathbf{A}_{4i} \end{bmatrix} \quad i = 1, 2, \ldots, l \tag{6}$$

where $\mathbf{A}_{1i}$, $\mathbf{A}_{2i}$, $\mathbf{A}_{3i}$, and $\mathbf{A}_{4i}$ are, in general functions of state variables x_{1Qi} and x_{2Qi}.

From (6) the characteristic equation is given by,

$$\text{Det} \begin{bmatrix} s\mathbf{I}_n - \mathbf{A}_{1i} & -\mathbf{A}_{2i} \\ -\mathbf{A}_{3i} & \mathbf{e}^{sT_i}\mathbf{I}_m - \mathbf{A}_{4i} \end{bmatrix} = 0 \tag{7}$$

where s is the complex frequency, and
I_n is a unit matrix of order n.

For a lumped network of order n, the characteristic equation is a polynomial of degree n in the complex frequency s, and the number of roots is n in the s-plane. For a distributed network, with commensurate delays, of order m, the characteristic equation is a polynomial of degree m in $z = e^{sT}$, and the number of roots is m in the z-plane. Since the value of z is unchanged for all $s = \sigma + j(\omega + 2\pi k/T), k = 1, 2, \ldots$, the number of roots are infinite in the s-plane and hence the stability analysis is best performed in the z-plane, where the number of roots is finite and the stable region lies inside the unit circle.

In the case of lumped-distributed network with commensurate delays T, the characteristic (7) can be written in the form,

$$P(s, e^{sT}) = \sum_{k=0}^{m} P_{nk}(s) e^{ksT} \tag{8}$$

where $P_{nk}(s)$ is a polynomial of degree n in s.

In the case of a lumped-distributed network with noncommensurate delays, the characteristic equation is a polynomial in the form $P(s, e^{sT1}, e^{sT2}, \ldots, e^{sTm})$. In both cases (commensurate and noncommensurate) the number of roots is independent of the order of the network $(m + n)$ and in general can be infinite.

The study of the stability of the linearized system of (1) is one of the major problems in the sense that the determinant is a bivariate polynomial in the variables s and $e^{sT_i}, i = 1, \ldots, m$. The system stability is governed by Pontrijagin theorem [4], which states that the necessary and sufficient conditions for a bivariate polynomial to be stable is that all zeros of the polynomial have negative real parts in the s-plane. The conditions for stability can be obtained only analytically in very simple cases. Therefore a numerical procedure using Newton-Raphson approach has been incorporated with the analysis program [2] to find the zeros of (7).

II. Time-Delayed Chua's Circuit

The time-delayed Chua's circuit shown in Fig. 2(a) is obtained from Chua's circuit by replacing the parallel LC resonator by a lossless short-circuited transmission line. The dynamics of this circuit is only described for the special case $C_1 = 0$ [5].

Since the output port is short-circuited, $v_0(t) = 0$, the following relations are obtained,

$$\begin{aligned} v_0^+(t) &= -v_0^-(t) \\ v_i(t) &= v_i^-(t) - v_i^-(t+T) \\ i_i(t) &= -\frac{1}{z_0}(v_i^-(t) + v_i^-(t+T)) \end{aligned} \tag{9}$$

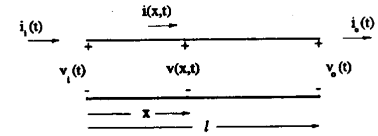

Fig. 1. A lossless transmission line.

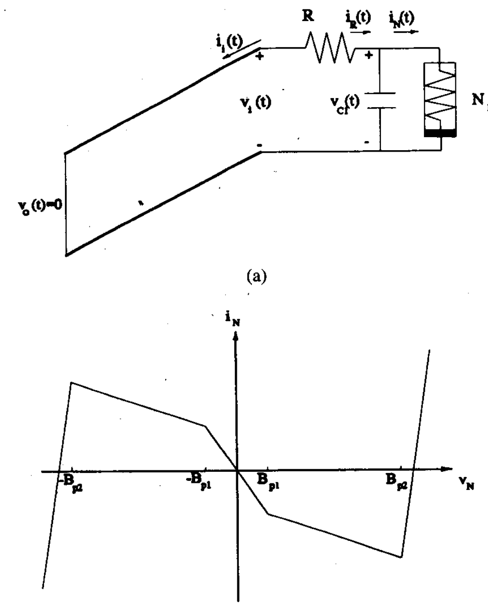

Fig. 2. (a) Time-delayed Chua's circuit. (b) v-i characteristics of the nonlinear resistor N_R.

where $v_i^+(t)$ and $v_i^-(t)$ represent the incident and the reflected voltages at the input port of the transmission line, respectively,
$T = 2l/v$ is twice the delay of the transmission line, and
z_0 is the charecteristic impedance of the transmission line.

From (9) and applying Kirchhoff's voltage and current laws, the state equation can be obtained in the form,

$$\begin{bmatrix} \dot{x}_1(t) \\ x_2(t+T) \end{bmatrix} = \begin{bmatrix} 1/C_1(R+z_0) & 2/C_1(R+z_0) \\ -z_0/(R+z_0) & (z_0 - R)/(z_0 + R) \end{bmatrix} \times \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} -1/C_1 \\ 0 \end{bmatrix} i_n(t) \tag{10}$$

where $x_1(t)$ is the voltage across the capacitor C_1,
$x_2(t)$ is the reflected voltage at input port of the transmission line $v_i^-(t)$, and
i_n is a piecewise-linear function in Fig. 2(b).

The current $i_n(t)$ is given by,

$$\begin{aligned} i_n = m_2 v_n &+ \frac{1}{2}(m_0 - m_1)[|v_n - B_{p1}| + |v_n - B_{p1}|] \\ &+ \frac{1}{2}(m_1 - m_2)[|v_n - B_{p2}| + |v_n - B_{p2}|] \end{aligned} \tag{11}$$

where $v_n = x_1(t)$ is the voltage across the nonlinear resistor.

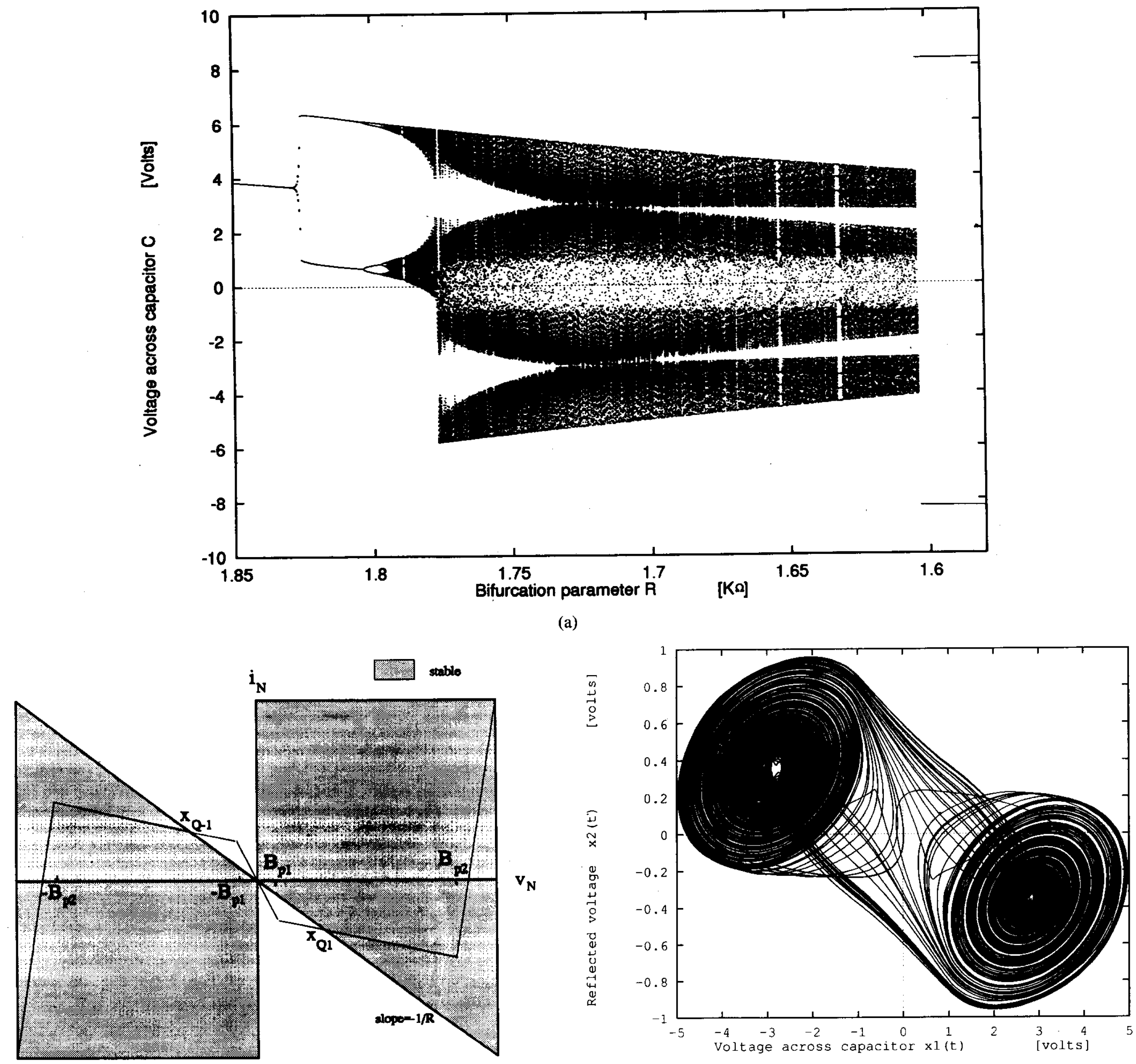

Fig. 3. (a) R-bifurcation diagram of time-delayed Chua's circuit. (b) Absolute stability criteria in the (i-v)plane of the nonlinear characteristics of the Chua's diode (R = 1.85 KΩ). (c) Phase plane of time-delayed Chua's circuit for R = 1.7 KΩ. (Chaotic attractor.)

The system (10) possesses one equilibrium point at the origin if $m_0 > -1/R$, otherwise it will possess three equilibrium points. The coordinates of the equilibrium points (x_{1Q}, x_{2Q}) are:

$$x_{Q0} \equiv (0,0), \quad x_{Q1} \equiv (-kR, k), \text{ and } \quad x_{Q-1} \equiv (kR, -k)$$

where

$$k = B_{p1}(m_0 - m_1)/(1 + m_1 R) \quad \text{for} \quad B_{p1} < v_n < B_{p2}, \quad -B_{p2} < v_n < -B_{p1}$$

or

$$k = (B_{p1}(m_0 - m_1) + B_{p2})/(1 + m_1 R) \quad \text{for} \quad v_n > B_{p2}, \quad v_n < -B_{p2}.$$

The Jacobian matrix of the linearized system at each equilibrium point is given by,

$$J_i = \begin{bmatrix} D_i/C_1(R + z_0) & 2/C_1(R + Z_0) \\ -z_0/(R + z_0) & (z_0 - R)/(z_0 + R) \end{bmatrix} \quad i = 0, 1, 2 \tag{12}$$

where

$$\begin{aligned} D_0 &= 1 + m_0(z_0 + R) && -B_{p1} < v_n < B_{p1} \\ D_1 &= 1 + m_1(z_0 + R) && B_{p1} < v_n < B_{p2}, \\ & && -B_{p2} < v_n < -B_{p1} \\ D_2 &= 1 + m_2(z_0 + R) && v_n < B_{p2}, \\ & && v_n < -B_{p2} \end{aligned}$$

The corresponding characteristic equation is given by

$$\mathrm{Det}\begin{bmatrix} s - D_i/C_1(R+z_0) & -2/C_1(R+z_0) \\ z_0/(R+z_0) & e^{sT} - (z_0-R)/(z_0+R) \end{bmatrix} = 0 \tag{13}$$

The time-delayed Chua's circuit is simulated for the following set of parameters: $m_0 = -11/20$ mS, $m_1 = -9/22$ mS, $m_2 = 10$ mS, $B_{p1} = 1$ V, $B_{p2} = 8$ V, $z_0 = 42/99$ KΩ, and $T = 0.1$ ms. The value of the resistor R is taken as a bifurcation parameter $1.58 < R < 1.85$ KΩ. The R-bifurcation diagram is shown in Fig. 3(a) which shows that the time-delayed Chua's circuit exhibits period doubling route to chaos. This route contains different dynamic modes of behavior of the circuit. In this paper only two modes of operation are investigated. All other modes can be analyzed by the same procedure.

A. Fixed-Point Operation Mode

If the bifurcation parameter $R = 1.85$ KΩ, the system (10) possesses three equilibrium points $x_{Q0} \equiv (0,0)$ and $x_{Q1}(x_{Q-1}) \equiv (\pm 3.86137, 0.44274)$. The absolute stability criteria in the i-v plane of the nonlinear characteristic can be determined by finding zeros of (13) for $m < 0$ (for $m > 0$ the circuit is passive and stable). In this case , as shown in the Fig. 3(b), the only unstable region lies between the two lines :

$$i_n = 0 \quad \text{and} \quad i_n = -(1/R)v_n.$$

The local stability at each equilibrium point is investigated by finding the zeros of (13) at each equilibrium point. The simulation results indicate that each equilibrium point has one real root γ, and an infinite number of complex conjugate poles, $\sigma \pm j\omega$. Although the number of complex poles are infinite, σ remains finite. In this example it can be analytically proven that $\sigma = \ln|(R - z_0)/(R - z_0)|$ as ω tends to infinity. The equilibrium point x_{Qo} is unstable, since it has one positive real pole $\gamma \approx 4.75555$, whereas the other equilibria x_{Q1} and x_{Q-1} are stable (sink points). Therefore the solution is a fixed point at x_{Q1} or x_{Q-1} depending on the initial conditions [2]. It should be noted for lumped-distributed system the initial states should be defined in the interval $-T_{\max} < t < 0$, where $T_{\max}$ is the longest delay of the system. For $R = 1.85$ KΩ, the R-bifurcation diagram of Fig. 3(a) shows that the mode of operation is a fixed-point, as predicted.

B. Chaotic Mode of Operation (Chaotic Attractor)

The previous steps were repeated for the bifurcation parameter $R = 1.7$ KΩ, the system (10) still has three equilibrium points. In this case the three equilibrium points are unstable, hence multilevel oscillatory (or chaotic) response is expected [1]. The phase plane of the reflected voltage at the input port of the transmission line versus the voltage across C_1 is shown in Fig. 3(c) which displays a chaotic attractor.

III. Conclusion

A time domain procedure for the analysis of different modes of behavior of lumped-distributed networks is presented. The analysis procedure and numerical techniques presented in this paper are general with no restriction on the topology of the network. Therefore, the same approach can be applied to study different modes of behavior of any lumped-distributed oscillator.

Time-delayed Chua's circuit is analyzed in the general case ($C_1 \neq 0$) by using the developed procedure. The simulation results show that time-delayed Chua's circuit exhibits period-doubling route to chaos.

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High Input Impedance Insensitive Second-Order Filters Implemented from Current Conveyors

Alain Fabre, Fadi Dayoub, Laurence Duruisseau, and Moez Kamoun

Abstract—**Two high input impedance second-order filters with active and passive sensitivities inferior or equal to the unit are described. Both of them use two second generation current conveyors, with positive current transfer (CCII$^+$) and four passive components. The first circuit achieves a low-pass or high-pass transfer according to the kind of passive components used. The second circuit achieves a band-pass transfer. SPICE simulation results using translinear current conveyors are given and discussed. They confirm the validity of the analysis and they point out the high performances of these filters. Experimental results obtained with the AD844 AN transimpedance operational amplifier show the advantage of these implementations compared to conventional ones.**

I. Introduction

Second-order active filters with infinite input impedance are of great interest because several cells of that kind can be directly connected in cascade to implement higher order filters with no need to interpose active separating stages. As a matter of fact, infinite input impedance cells assure a total uncoupling between the different elementary stages. This will entail an easier determination of the passive component values of each of the elementary cells and an easier perfectionning of the global cicuit. Other than that, theoretical and experimental frequency responses of the filters will generally be closer.

Second generation current conveyor (CCII), [1], whose frequency response remains unchanged up to some hundred of megahertz when they are implemented as application specific integrated circuits (A.S.I.C.'s) using prediffused complementary bipolar arrays [2], are very adapted to the design of that kind of filters. Indeed, they exhibit a high impedance input node: the Y port. Numerous realisations of high

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