Adaptive Control of Uncertain Chua's Circuits

S. S. Ge and C. Wang
Department of Electrical Engineering, National University of Singapore
Singapore 117576
E-mail: elegesz@nus.edu.sg

Abstract

In this paper, the adaptive backstepping with tuning functions method is used for the control of uncertain Chua's circuits with all the key parameters unknown. Firstly, we show that several Chua's circuits of different types including the Chua's oscillator, Chua's circuit with cubic nonlinearity, and Murali-Lakshmanan-Chua circuit, can all be transformed into a class of nonlinear systems in the so-called non-autonomous "strict-feedback" form. Secondly, an adaptive backstepping with tuning functions method is extended to the non-autonomous "strict-feedback" system, and then used to control the output of the Chua's circuit to asymptotically track an arbitrarily given reference signal generated from a known, bounded and smooth nonlinear reference model. Both global stability and asymptotic tracking of the closed-loop system are guaranteed. Simulation results are presented to show the effectiveness of the approach.

1 Introduction

Controlling chaotic systems has recently been in the focus of attention in the nonlinear dynamics literature [6] and the references therein. In particular, many adaptive control schemes have been successfully applied to the control and synchronization of chaotic systems [5][7][14]. All these methods are based on rigorous Lyapunov stability theorem and Lyapunov function methods. But the construction of the Lyapunov functions remains to be a difficult task.

In the past decade, adaptive control of nonlinear systems has undergone rapid developments ([11] and the references therein). By using the backstepping design procedure, Kanellakopoulos et al. [8] have presented a systematic approach of globally stable and asymptotically tracking adaptive controllers for a class of nonlinear systems transformable to a parametric strict-feedback canonical form. The overparametrization problem was soon eliminated by Krstić et al. [10] by elegantly introducing the concept of tuning functions.

In this paper, by noticing that several Chua's circuits [1] of different types, including the Chua's oscillator [2], Chua's circuit with cubic nonlinearity [15], and Murali-Lakshmanan-Chua circuit [13], which have been used as paradigms in the research of bifurcations and chaos, are actually in the form of non-autonomous strict-feedback systems, we extend the adaptive backstepping with tuning functions method to the non-autonomous strict-feedback system in the following form:

\[
\begin{align*}
\dot{x}_i &= b_i g_i(x_i, t) x_{i+1} + \theta_i^T F_i(x_i, t) + f_i(x_i, t), \quad 1 \leq i \leq n-1 \\
\dot{x}_n &= b_n g_n(x_n, t) u + \theta_n^T F_n(x_n, t) + f_n(x_n, t) \\
y &= x_1
\end{align*}
\]

where \( x_i = [x_1, x_2, \ldots, x_i]^T \in \mathbb{R}^i, i = 1, \ldots, n, u \in \mathbb{R}, \) and \( u \in \mathbb{R} \) are the states, input and output, respectively; \( b_i = [b_{i1}, b_{i2}, \ldots, b_{in}]^T \in \mathbb{R}^n \) and \( \theta_i = [\theta_{i1}, \theta_{i2}, \ldots, \theta_{in}] \in \mathbb{R}^n \) are the vectors of unknown constant parameters of interest; \( g_i(\cdot) \neq 0, F_i(\cdot), f_i(\cdot), i = 1, \ldots, n-1 \) are known, smooth nonlinear functions, \( g_n(\cdot) \neq 0, F_n(\cdot), f_n(\cdot) \) are known continuous nonlinear functions. We assume that the signs of parameters \( b_i, i = 1, \ldots, n \) are known.

This design procedure is then applied to control the output of Chua's circuit to asymptotically track any given reference signal generated from a known, bounded and smooth nonlinear reference model. Simulation studies are conducted to show the effectiveness of the proposed method.

2 Chua's Circuits in Strict-Feedback Form

2.1 Chua's Circuit

The famous Chua's circuit [1] is a simple oscillator circuit which exhibits a rich variety of bifurcations and chaos phenomena. It contains three linear energy storage elements (one inductor \( L \) and two capacitors \( C_1 \) and \( C_2 \)), one linear resistor \( R \), and one nonlinear resistor called Chua's diode \( g(u_{C_1}) \). The dynamic equation of Chua's circuit is described by:

\[
\begin{align*}
\frac{dV_{C_1}}{dt} &= \frac{1}{C_1}(V_{C_2} - V_{C_1}) - g(V_{C_1}) \\
\frac{dV_{C_2}}{dt} &= \frac{1}{C_2}(V_{C_1} - V_{C_2}) + i_L \\
\frac{di_L}{dt} &= -V_{C_2}
\end{align*}
\]

where \( C_1, C_2, L \) and \( R \) are all circuit parameters, \( i_L \) is the current through the inductor \( L \), \( V_{C_1} \) and \( V_{C_2} \) are the voltages across \( C_1 \) and \( C_2 \), respectively, and the piecewise linear function \( g(V_{C_1}) \) describes the \( V-I \) characteristics of the Chua's diode \( g \) as follows:

\[
g(V_{C_1}) = G_a V_{C_1} + \frac{1}{2}(G_b - G_a)(|V_{C_1} + 1| - |V_{C_1} - 1|)
\]

with \( G_a < 0 \) and \( G_b < 0 \) being some appropriately chosen constants.

By defining \( b_1 = 1/L > 0, b_2 = 1/R C_2 > 0, \theta_1 = 1/C_1, \theta_2 = 1/R C_2, \theta_3 = 1/R C_1, \theta_4 = 1/R C_1 + G_a/C_1 \) and \( \theta_5 = G_a/C_2 \), and defining the state variables as:

\[
x_1 = i_L, \quad x_2 = V_{C_2}, \quad x_3 = V_{C_1}
\]

then equations (2.1) can be reformulated in the following form:

\[
\begin{align*}
\dot{x}_1 &= b_1 g_1(x_1, t) x_2 + \theta_1^T F_1(x_2, t) + f_1(x_2, t) \\
\dot{x}_2 &= b_2 g_2(x_2, t) u + \theta_2^T F_2(x_2, t) + f_2(x_2, t) \\
y &= x_1
\end{align*}
\]
where the control $u(\cdot)$ is assumed to be introduced into the third equation of (2.4) to form the controlled Chua's circuit.

In comparison with the “strict-feedback” system form (1.1), and in the case when all the system parameters are unknown constants, i.e., $\theta = [\theta_1, \theta_2, \ldots, \theta_k]^T$, $b_1$ and $b_2$ are unknown (except that the signs of $b_1$ and $b_2$ are assumed to be known), we have

$$g_1(x_1) = -1, g_2(x_1, x_2) = 1, g_3(x_1, x_2, x_3) = 1$$

$$f_1(x_1) = 0, f_2(x_1, x_2) = 0, f_3(x_1, x_2, x_3) = 0$$

$$F_1(x_1) = [0 0 0 0]^T, F_2(x_1, x_2) = [x_1 - x_2 0 0 0]^T, F_3(x_1, x_2, x_3) = [0 0 x_3 - x_3 - (x_2 + 1) - |x_3 - 1|]$$

Following the same procedure, it can be verified that several other kinds of Chua's circuits, such as the Chua’s Oscillator and the Chua’s circuit with cubic nonlinearity, can all be transformed into the non-autonomous “strict-feedback” form (1.1).

### 2.2 Murali-Lakshmanan-Chua circuit

The Murali-Lakshmanan-Chua circuit is a simple second order non-autonomous nonlinear circuit, which can exhibit a rich variety of bifurcation and chaos phenomena [13].

The dynamical equation of Murali-Lakshmanan-Chua circuit is described by

$$C_1 \frac{dv_{C_1}}{dt} = i_L - g(v_{C_1})$$

$$L \frac{di_L}{dt} = -v_{C_1} - R_i i_L - R_{iK} + F \sin(\Omega t)$$

where $g(v_{C_1})$ is given by (2.2).

By defining $b_1 = 1/L > 0, b_2 = (R + R_i)/L, b_3 = F$, $\theta = 1/C_1, \theta_1 = G/R_i$, and $\theta_2 = \frac{F_1 - F_2}{2C_1}$, and defining the state variables as

$$x_1 = i_L, x_2 = v_{C_1}$$

then equations (2.5) can be transformed as

$$\dot{x}_1 = -b_2 x_2 - \theta_2 x_1 + \theta_1 \sin(\Omega t)$$

$$\dot{x}_2 = u + \theta_3 x_1 - \theta_3 x_2 - \theta_3 ([x_2 + 1] - |x_2 - 1|)$$

where the control $u(\cdot)$ is assumed to be introduced into the second equation of (2.7).

In comparison with the “strict-feedback” system form (1.1), and in the case when all the system parameters are unknown constants, we have

$$g_1(x_1) = -1, g_2(x_1, x_2) = 1, f_1(x_1) = 0, f_2(x_1, x_2) = 0$$

$$F_1(x_1) = [ -x_1 \sin(\Omega t) 0 0 0]^T, F_2(x_1, x_2) = [0 0 x_2 - x_2 - (x_2 + 1) - |x_2 - 1|]^T$$

In the next section, we will extend the adaptive backstepping with tuning functions method [Krstić, et al., 1992; Krstić, et al., 1995] to the non-autonomous strict-feedback system in form (1.1).

### 3 Adaptive Backstepping with Tuning Functions Method

For the controlled system in form (1.1), consider a known, bounded and smooth reference model as follows

$$\dot{x}_{r,i} = f_{r,i}(x_i, t), 1 \leq i \leq m - 1$$

$$\dot{x}_{r,m} = f_{r,m}(x_i, t)$$

where $x_r = [x_{r,1}, x_{r,2}, \ldots, x_{r,m}]^T \in \mathbb{R}^m (m \geq n)$, $y_r \in \mathbb{R}$ are the states and output respectively; $f_{r,i}(\cdot), i = 1, 2, \ldots, m - 1$ are known smooth nonlinear functions and $f_{r,m}(\cdot)$ is a known continuous nonlinear function.

Our objective is to design an adaptive state-feedback controller for system (1.1) that guarantees global stability and force the output $y = x_{r,1}(t)$ of system (1.1) to asymptotically track the output $y_r = x_{r,1}(t)$ of the reference model, i.e.,

$$|y(t) - y_r(t)| \to 0, \text{ as } t \to \infty.$$
where $z_4 = x_4 - \beta_0 x_{a4} - \alpha_3$, $\alpha_3$ is the virtual control to be defined later, $F_{3a} = F_3 - \beta_0 \alpha_3 F_1 - \beta_0 \alpha_3 F_2$ and $F_{3s} = F_3 - \beta_0 \alpha_3 F_1 - \beta_0 \alpha_3 F_2 + x_3 x_4 - \beta_1 \delta_3 \gamma_1 x_2 - \beta_0 \alpha_3 \gamma_2 x_3 - \beta_0 \alpha_3 \gamma_1 - \delta_3 \gamma_1 \gamma_2 - \sum_{k=1}^n \delta_3 \gamma_1 f_k + \delta_0 \alpha_3 \gamma_1 f_0 + \delta_0 \alpha_3 \gamma_1 f_1 + \delta_0 \alpha_3 \gamma_1 f_2 - \delta_0 f_2 - \delta_3 (x_3 - \beta_0 x_{a4})$. Using $\alpha_3$ as a control to stabilize the $(z_1, z_2, z_3)$-subsystem, we choose the following Lyapunov function candidate

$$V_3 = V_2 + x_2^2 + \frac{1}{2\gamma} (b_0 - b_2)^2 + \frac{|b_0|}{2\gamma} (x_2 - \bar{v}_2)^2$$

The derivative of $V_3$ is

$$V_3' = -c_2 z_2^2 + b_2 g_2 z_2 + b_1 (b_0 - b_2) + \frac{\partial_2}{\partial x_2} (x_2 - \bar{v}_2)$$

Define tuning functions $\gamma_2$, $\pi_2^b$ and $\pi_2^s$ for $\bar{v}_2$, $b_2$ and $b_2$ respectively as $\gamma_2 = x_2 + \alpha_3 x_3 - \beta_1 \delta_3 \gamma_1 x_2 - \beta_0 \alpha_3 \gamma_2 x_3$ and $\pi_2^s = x_2 - \beta_0 \alpha_3 x_2 - \bar{v}_2$. To eliminate the $(b_2 - b_2)$-term from equation (3.15), we choose the parameter update law for $\bar{v}_2$ as $\bar{v}_2 = -\text{sgn}(b_2) \gamma_2 (2 b_2 z_2 + x_2 z_2)$. To make the third term in equation (3.10) be equal to $-c_2 z_2$, we choose

$$\gamma_2 = \frac{\pi_2}{b_2} (\pi_2 - \gamma_2 \pi_2)$$

which yields

$$V_2 = -c_2 z_2^2 - \bar{v}_2 (2 b_2 - b_1 g_2) + \frac{\partial_2}{\partial x_2} (x_2 - \bar{v}_2)$$

Step 3. The derivative of $z_3$ is expressed as

$$z_3 = b_3 g_2 z_4 + b_3 g_3 z_2 + \pi_3^b + \pi_3^s + (b_2 - b_2) \frac{\partial_0}{\partial x_2} (x_2 - \bar{v}_2)$$

where $z_4 = x_4 - \beta_0 x_{a4} - \alpha_3$, $\alpha_3$ is the virtual control to be defined later, $F_{3a} = F_3 - \beta_0 \alpha_3 F_1 - \beta_0 \alpha_3 F_2$ and $F_{3s} = F_3 - \beta_0 \alpha_3 F_1 - \beta_0 \alpha_3 F_2 + x_3 x_4 - \beta_1 \delta_3 \gamma_1 x_2 - \beta_0 \alpha_3 \gamma_2 x_3 - \beta_0 \alpha_3 \gamma_1 - \delta_3 \gamma_1 \gamma_2 - \sum_{k=1}^n \delta_3 \gamma_1 f_k + \delta_0 \alpha_3 \gamma_1 f_0 + \delta_0 \alpha_3 \gamma_1 f_1 + \delta_0 \alpha_3 \gamma_1 f_2 - \delta_0 f_2 - \delta_3 (x_3 - \beta_0 x_{a4})$. Using $\alpha_3$ as a control to stabilize the $(z_1, z_2, z_3)$-subsystem, we choose the following Lyapunov function candidate

$$V_3 = V_2 + \frac{1}{\gamma} (b_2 - b_2)^2 + \frac{|b_2|}{\gamma} (x_2 - \bar{v}_2)^2$$

The derivative of $V_3$ is

$$V_3' = -c_2 z_2^2 + b_2 g_2 z_2 + b_1 (b_0 - b_2) + \frac{\partial_2}{\partial x_2} (x_2 - \bar{v}_2)$$

Define tuning functions $\gamma_2$, $\pi_2^b$ and $\pi_2^s$ for $\bar{v}_2$, $b_2$ and $b_2$ respectively as $\gamma_2 = x_2 + \alpha_3 x_3 - \beta_1 \delta_3 \gamma_1 x_2 - \beta_0 \alpha_3 \gamma_2 x_3$ and $\pi_2^s = x_2 - \beta_0 \alpha_3 x_2 - \bar{v}_2$. To eliminate the $(b_2 - b_2)$-term from equation (3.15), we choose the parameter update law for $\bar{v}_2$ as $\bar{v}_2 = -\text{sgn}(b_2) \gamma_2 (2 b_2 z_2 + x_2 z_2)$. Noting that $\gamma_2 - \bar{v}_2 = \gamma_2 - \bar{v}_2 - \gamma_2 + \bar{v}_2 = \gamma_2 - \bar{v}_2$ in equation (3.15) can be written as

$$V_3 = -c_2 z_2^2 - \bar{v}_2 (x_2 + \alpha_3 x_3 - \beta_1 \delta_3 \gamma_1 x_2 - \beta_0 \alpha_3 \gamma_2 x_3)$$

which yields

$$V_3' = -c_2 z_2^2 - \bar{v}_2 (x_2 + \alpha_3 x_3 - \beta_1 \delta_3 \gamma_1 x_2 - \beta_0 \alpha_3 \gamma_2 x_3)$$

Step 3. The derivative of $z_3$ is expressed as

$$z_3 = b_3 g_2 z_4 + b_3 g_3 z_2 + \pi_3^b + \pi_3^s + (b_2 - b_2) \frac{\partial_0}{\partial x_2} (x_2 - \bar{v}_2)$$

where $z_4 = x_4 - \beta_0 x_{a4} - \alpha_3$, $\alpha_3$ is the virtual control to be defined later, $F_{3a} = F_3 - \beta_0 \alpha_3 F_1 - \beta_0 \alpha_3 F_2$ and $F_{3s} = F_3 - \beta_0 \alpha_3 F_1 - \beta_0 \alpha_3 F_2 + x_3 x_4 - \beta_1 \delta_3 \gamma_1 x_2 - \beta_0 \alpha_3 \gamma_2 x_3 - \beta_0 \alpha_3 \gamma_1 - \delta_3 \gamma_1 \gamma_2 - \sum_{k=1}^n \delta_3 \gamma_1 f_k + \delta_0 \alpha_3 \gamma_1 f_0 + \delta_0 \alpha_3 \gamma_1 f_1 + \delta_0 \alpha_3 \gamma_1 f_2 - \delta_0 f_2 - \delta_3 (x_3 - \beta_0 x_{a4})$.
\[ z_i = b_i g_i z_{i+1} + \frac{\partial_i}{\partial_i} \alpha_i + \theta^T F_{ia} + f_a - (\theta - \hat{\theta})^T F_{ia} + \frac{\partial \alpha_i}{\partial \theta} (\tau_i \dot{\theta}) - \hat{\theta} + \sum_{k=1}^{i-2} \frac{\partial \alpha_i}{\partial \theta_k} (\tau_k \dot{\theta} - \hat{\theta}) + \sum_{k=1}^{i-1} (\hat{\theta}_k - b_k) + \frac{\partial \alpha_i}{\partial \sigma_k} g_k x_{k+1} \]

\[ - (\hat{\theta}_i - b_i) g_i z_{i+1} + (\hat{\theta}_i - 1) (g_i x_{i+1} + \frac{\partial_i}{\partial_i} \alpha_i) \]  

(3.19)

where \( z_{i+1} = x_{i+1} - \hat{\theta} x_{i+1} \) - \( \alpha_i \), \( \alpha_i \) is a fictitious control to be defined later, and

\[ F_{ia} = f_a + g_i x_{i+1} \]

\[ f_{ia} = f_a + g_i x_{i+1} - \sum_{k=1}^{i-1} \frac{\partial \alpha_i}{\partial \theta_k} g_k x_{k+1} - \sum_{k=1}^{i-1} \hat{\theta}_k + \frac{\partial \alpha_i}{\partial \sigma_k} g_k x_{k+1} \]

\[ - (\hat{\theta}_i - 1) - \sum_{k=1}^{i-1} \frac{\partial \alpha_i}{\partial \theta_k} g_k x_{k+1} \]

(3.20)

Using \( \alpha_i \) as a control to stabilize the \( (z_i, \ldots, z_i) \)-subsystem, we choose the following Lyapunov function candidate

\[ V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2} (\hat{\theta}_i - b_i)^2 + \frac{||b_i||}{2} (\hat{\theta}_i - \theta)^2 \]

To make the bracketed term multiplying \( z_i \) in equation (3.22) be equal to \(-\epsilon z_i^2\), we choose

\[ V_i = -\sum_{k=1}^{i-1} c_k z_k^2 + b_i g_i z_{i+1} + \sum_{k=1}^{i-1} (x_{k+1} + \frac{\partial \alpha_i}{\partial \theta_k} (\tau_i - \hat{\theta}) + z_i (\hat{\theta}_i - 1) + \frac{\partial \alpha_i}{\partial \sigma_k} g_k x_{k+1} \]

(3.23)

Step n. Since this is our last step, the derivative of \( z_n \) is expressed as

\[ z_n = \frac{\partial_{n}}{\partial n} u + \theta^T F_{ns} + f_{ns} + \frac{\partial \alpha_{n-1}}{\partial \theta} \]

\[ (\hat{\theta}_n^* - \hat{\theta}_n) + (\hat{\theta}_n - \hat{\theta}_n) \]

(3.24)

129
The derivative of $V_n$ is

$$V_n = - \sum_{k=1}^{n} c_k x_k^2 + \sum_{k=1}^{n} x_{k+1} \frac{\partial x_k}{\partial \dot{x}_k} (\tau_n - \dot{\theta}) + \sum_{k=1}^{n} x_{k+1} \frac{\gamma x_{k+1}}{\delta x_k} (\tau_n - \dot{\theta}) + \sum_{k=1}^{n} x_{k+1} \frac{\gamma x_{k+1}}{\delta x_k} (\tau_n - \dot{\theta}) + \sum_{k=1}^{n} x_{k+1} \frac{\gamma x_{k+1}}{\delta x_k} (\tau_n - \dot{\theta})$$

and the parameter update law (3.28) has a globally uniformly stable equilibrium at $z = [z_1, z_2, \ldots, z_n]^T = 0$. This guarantees the global boundedness of all the signals in the closed-loop system, including the states $z = [x_1, x_2, \ldots, x_n]^T$, the control $u$ and parameter estimates $\dot{\theta}$, $\dot{b}_1, \dot{b}_2, \ldots, \dot{b}_n$, and $\dot{g}_1, \dot{g}_2, \ldots, \dot{g}_n$, and $\lim_{t \to \infty} x(t) = 0$, i.e., subsequently,

$$\lim_{t \to \infty} [y(t) - y_r(t)] = 0$$

(3.31)

Proof: The $(z_1, \ldots, z_n)$-system corresponds to the closed-loop adaptive system, which consists of the linear system (1.1), the reference model (3.1), the controller (3.29) and the parameter update law (3.28). The derivative of the Lyapunov function (3.13) along the $(z_1, \ldots, z_n)$-system is (3.30), which proves that equilibrium $z = 0$ is globally uniformly stable.

Combining (3.26) with (3.30), we conclude that $\dot{\theta}, \dot{b}_1, \ldots, \dot{b}_n = \dot{g}_1, \ldots, \dot{g}_n = \dot{\theta}_n$, are bounded. Since $z_1 = x_1 - x_{r1}$ and $z_{r1}$ is bounded, we see that $z_1$ is also bounded. The boundedness of $z_1$ follows from the boundedness of $\alpha_{n-1}$ and $\dot{b}_1, \ldots, \dot{b}_n$, and the fact that $z_1 = x_1 - x_{r1}$. Using (3.29), we conclude that the control $u$ is also bounded.

From the LaSalle-Yoshizawa theorem [11], it further follows that, all the solutions of the $(z_1, \ldots, z_n)$-system converge to the manifold $z = 0$ as $t \to \infty$. From the definition $z_1 = x_1 - x_{r1}$, we conclude that $|y(t) - y_r(t)| \to 0$ as $t \to \infty$.

4 Example: Tracking Control of Chua's Circuit

We assume that the controlled Chua's circuit is originally $(u = 0)$ in the periodic state, period-1 attractor [9], with parameters $C_1 = 0.11364, C_2 = 1, L = 0.9525, R = 1, G_2 = -1.143$ and $G_3 = -0.714$, i.e., $b_1 = 16, b_2 = 1$ and $\theta = [1.0000, 1.0000, 8.7997, 2.5167, -1.8875]^T$. The objective is to force the output $y = x_1(t)$ of the controlled Chua's circuit (2.4) to asymptotically track the chaotic reference signal $y_r = x_{r1}(t)$ generated from another uncontrolled Chua's circuit (2.4) $(u = 0)$ in chaotic state, double-scroll attractor [9], with parameters $C_1 = 0.10204, C_2 = 1, L = 0.9525, R = 1, G_2 = -1.143$ and $G_3 = -0.714$.

The design parameters of controller (3.29) and parameter update law (3.28) are chosen as $c_1 = 10, c_2 = 20, c_3 = 50, \gamma = 0.1$ and $\Gamma = diag[0.03, 0.1, 0.1, 0.02, 0.07]$. These gains are chosen by trial and error for better performance. The initial conditions are chosen that $x_1(0) = 0.3, x_2(0) = 0.4, x_{r1}(0) = 0.2, x_{r2}(0) = 0.5$ and $x_3(0) = 0.3$.

Numerical simulation results are shown in Figures 1-3. As shown in Figure 1, the output $y = x_1(t)$ of the controlled Chua's circuit (2.4) asymptotically track the chaotic reference signal $y_r = x_{r1}(t)$. It can be shown that at the same time the states $x_2(t)$ and $x_3(t)$ of the controlled Chua's oscillator (2.4), the parameter estimates $\dot{\theta}, \dot{b}_1, \dot{b}_2, \ldots, \dot{b}_n$ and the control $u$ remain bounded. The boundedness of parameter estimates and control signal $u$ is shown in Figures 2 and 3 respectively.

5 Conclusion

In this paper, firstly we showed that several Chua's circuits of different types, including Chua's oscillator, Chua's circuit
with cubic nonlinearity, and the non-autonomous Chua’s circuit, can all be transformed into the class of nonlinear system in the so-called non-autonomous "strict-feedback" form. Then, an adaptive backstepping with tuning functions method has been extended to the non-autonomous "strict-feedback" system, and it is used to control the output of the Chua’s circuit to asymptotically track arbitrarily given reference signal generated from known, bounded and smooth nonlinear reference model.

References


