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Dynamics of Hyperchaotic Coupled Chua Circuits: Generation of new *N*x*M*-Scroll Attractors

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Abstract

In this paper the attention is focused on the generation of new hyperchaotic attractors from coupled Chua circuits. By exploiting sine functions as nonlinearities, the proposed technique enables *n*x*m*-scroll attractors to be obtained. In particular, it is shown that *n*x*m*-scroll dynamics can be easily designed by modifying four parameters, which characterize the circuit nonlinearities. Finally, some examples are reported to illustrate the capability of the proposed design technique.

Keywords: Chaotic circuits, coupled Chua circuits, hyperchaotic dynamics.

1. Introduction

Recently, Chua circuits with modified voltage-current characteristic have been introduced, with the aim of generating more complex chaotic attractors [1]-[2]. By exploiting the idea of adopting sine-type functions as non-linearities, the aim of this paper is to show that the coupling of two Chua circuits can generate very complex dynamics [3]-[5]. In particular, new hyperchaotic attractors are obtained by modifying suitable parameters related to the circuit nonlinearities.

The paper is organized as follows. Based on the stability properties of the equilibria of the single Chua's circuit, in Sec. 2 hyperchaotic *n*x*m*-scroll attractors are generated by combining the chaotic *n*-scroll attractor related to the first Chua's circuit with the chaotic *m*-scroll attractor related to the second Chua's circuit. Finally, in Sec. 3 some examples illustrate in detail the generation of new hyperchaotic *n*x*m*-scroll attractors.

2. Generation of Hyperchaotic *N*x*M*-Scroll Attractors

At first the forming mechanism of 3x4-scroll attractors is illustrated. Successively, by generalizing such result, a design technique for generating hyperchaotic *n*x*m*-scroll attractors is developed.

2.1 Generation of hyperchaotic 3x4-scroll attractors

By coupling two identical Chua circuits with sine functions, the following dynamic equations are obtained:

$$\begin{aligned} \dot{x}_1 &= a\left[x_2 - f(x_1)\right] \\ \dot{x}_2 &= x_1 - x_2 + x_3 + H(x_5 - x_2) \\ \dot{x}_3 &= -b\,x_2 \\ \dot{x}_4 &= a\left[x_5 - g(x_4)\right] \\ \dot{x}_5 &= x_4 - x_5 + x_6 + M(x_2 - x_5) \\ \dot{x}_6 &= -b\,x_5 \end{aligned} \quad (1a)$$

$$f = \begin{cases} \dfrac{bp}{2a}\left(x_1 - 2ac_1\right) & x_1 \ge 2ac_1 \\ -b\sin\left(\dfrac{p\,x_1}{2a} + d_1\right) & -2ac_1 < x_1 < 2ac_1 \\ \dfrac{bp}{2a}\left(x_1 + 2ac_1\right) & x_1 \le -2ac_1 \end{cases} \quad (1b)$$

$$g = \begin{cases} \dfrac{bp}{2a}\left(x_4 - 2ac_2\right) & x_4 \ge 2ac_2 \\ -b\sin\left(\dfrac{p\,x_4}{2a} + d_2\right) & -2ac_2 < x_4 < 2ac_2 \\ \dfrac{bp}{2a}\left(x_4 + 2ac_2\right) & x_4 \le -2ac_2 . \end{cases} \quad (1c)$$

where a = 10.814, b = 14, a = 1.3, b = 0.11, H = M = 0.25, whereas c_1, d_1, c_2 and d_2 are design parameters defined in the following (see eq.(2)). Given system (1), it is easy to show that the equilibrium points x_{eq} of system (1) can be obtained by combining the equilibria of the first Chua's circuit $(x_{1eq}, 0, -x_{1eq}) = (-2ak_1, 0, 2ak_1)$, $k_1 = 0,1,\ldots,c_1$, with the equilibria of the second Chua's circuit $(x_{4eq}, 0, -x_{4eq}) = (-2ak_2, 0, 2ak_2)$, $k_2 = 0,1,\ldots,c_2$. Namely, coupled Chua circuits posses 35 equilibrium points in the form $x_{eq} = (x_{1eq}, 0, -x_{1eq}, x_{4eq}, 0, -x_{4eq}) \in \Re^6$. In [5] it has been proved that the stability properties of all the 35 equilibria of system (1) can be derived from the stability of the equilibria of the single Chua circuits. In particular, the results in [5] highlight that hyperchaotic 3x4-scroll attractors (Fig.1) can be generated by combining chaotic 3-scroll attractors (first circuit) with chaotic 4-scroll attractors (second circuit). By taking into account that *saddle points index r* are characterized by r eigenvalues with positive real parts, some properties about the stability-type of each equilibrium point can be stated [5]. Namely:

Property 1. The combination of *saddle point index 1* (x_{1eq}) with *saddle point index 1* (x_{4eq}) generates in system (1) *saddle point index 2* (x_{eq}), characterized by 2 positive real eigenvalues.

Property 2. The combination of *saddle point index 1* (x_{1eq}) with *saddle point index 2* (x_{4eq}) generates in system (1) *saddle points index 3* (x_{eq}), characterized by 2 complex and 1 real eigenvalues with positive real parts. The same property holds if the stability-type of x_{1eq} and x_{4eq} is exchanged.

Property 3. The combination of *saddle point index 2* (x_{1eq}) with *saddle point index 2* (x_{4eq}) generates in system (1) *saddle point index 4* (x_{eq}), characterized by 4 complex eigenvalues with 4 positive real parts.

Property 4. Similarly to single Chua circuits, where 3-scroll and 4-scroll attractors are generated only around equilibria having all the complex eigenvalues with positive real parts (*saddle points index 2*), in the case of coupled Chua circuits the scrolls of the 3x4-scroll attractor are generated only around the 12 equilibria having all the complex eigenvalues with positive real parts (*saddle points index 4*).

Details about the proofs of Properties 1-4 can be found in [5].

2.2. Generation of hyperchaotic *nxm*-scroll attractors

By generalizing previous results, it is easy to show that hyperchaotic *nxm*-scroll attractors can be generated by combining the chaotic *n*-scroll attractor related to the first Chua's circuit with the chaotic *m*-scroll attractor related to the second Chua's circuit. Herein, the theoretical considerations reported in [5] can be condensed into some simple design guidelines. Namely, by considering that $f(x_1)$ and $g(x_4)$ depend on parameters c_1, d_1, c_2 and d_2, it follows that *nxm*-scroll attractors can be generated by taking (1) with:

$$c_1 = n - 1,\quad c_2 = m - 1,$$
$$d_1 = \begin{cases} 0 & \text{if } n \text{ is even} \\ p & \text{if } n \text{ is odd} \end{cases},\quad d_2 = \begin{cases} 0 & \text{if } m \text{ is even} \\ p & \text{if } m \text{ is odd} \end{cases} \qquad (2)$$

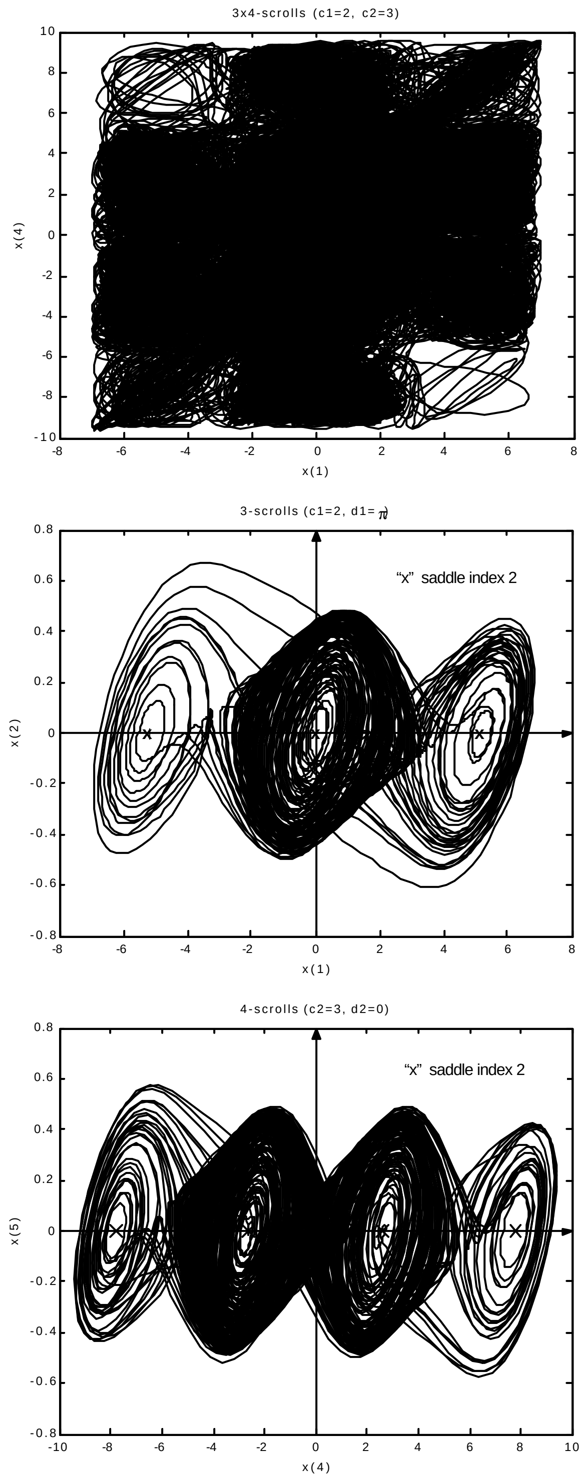


Fig.1. From the top to the bottom: hyperchaotic 3x4-scroll attractor on (x_1, x_4) plane; chaotic 3-scroll attractor on (x_1,x_2) plane; chaotic 4-scroll attractor on (x_4, x_5) plane.

3. Design examples

The hyperchaotic 8x4-scroll attractor reported in Fig.2 is obtained by considering guidelines (2) with $c_1 = 7$, $c_2 = 3$, $d_1 = 0$ and $d_2 = 0$. Such hyperchaotic behavior is achieved since the chaotic 8-scroll attractor related to the first Chua's circuit has been combined with the chaotic 4-scroll attractor related to the second Chua's circuit. Similarly, the hyperchaotic 9x9-scroll attractor reported in Fig.3 is obtained for $c_1 = 8$, $c_2 = 8$, $d_1 = \pi$ and $d_2 = \pi$. In particular, Fig.3(*a*) highlights how the dynamics of the 81 scrolls are generated starting from the origin, whereas Fig.3(*d*) shows the "final" hyperchaotic attractor.

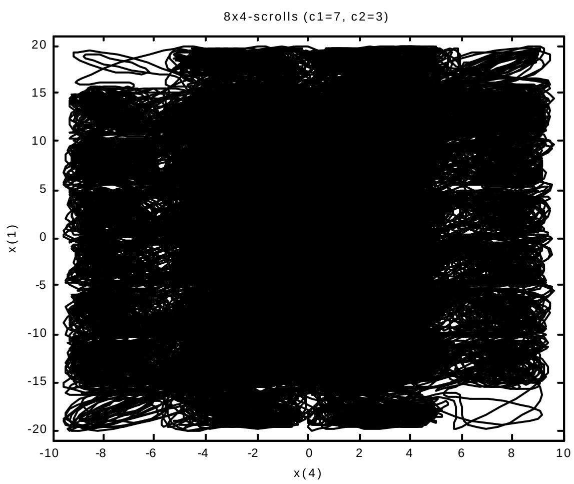
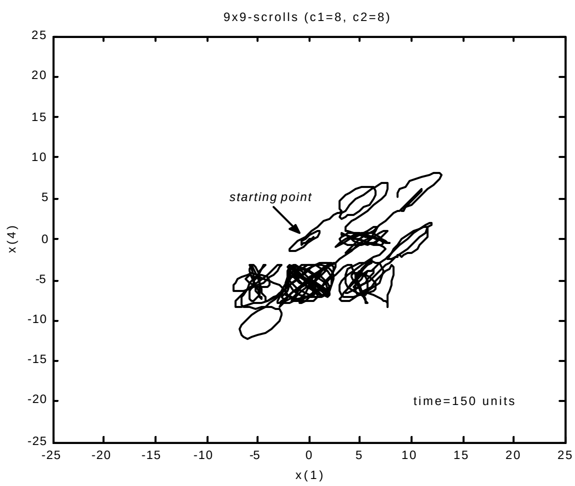
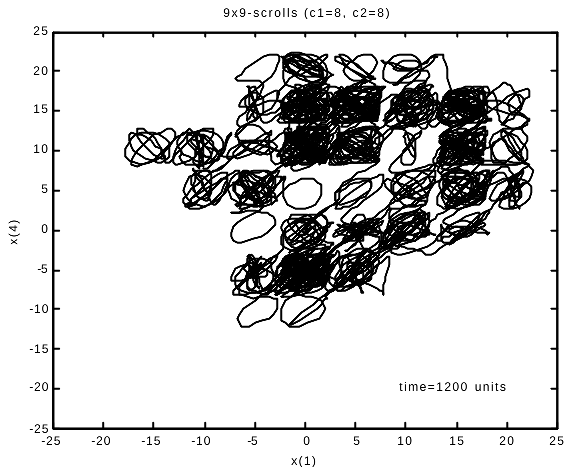


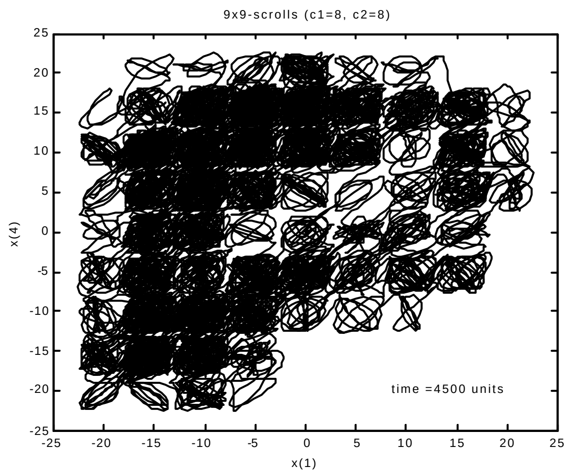
Fig.2. Hyperchaotic 8x4-scroll attractor on (x_4, x_1) plane.



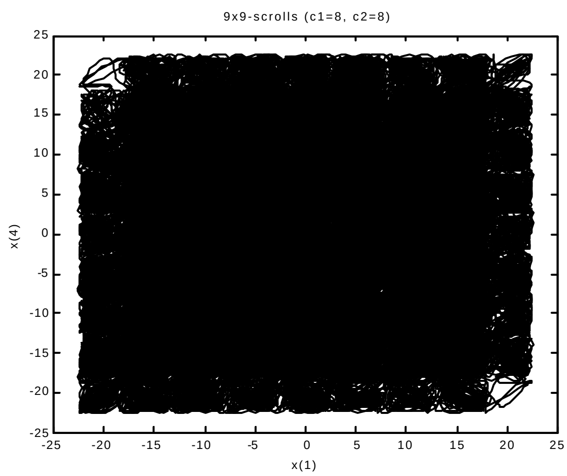
(a)



(b)



(c)



(d)

Fig.3. Hyperchaotic 9x9-scroll attractor on (x_1, x_4) plane. (*a*): time t=150; (*b*): time t=1200; (*c*): time t=4500; (*d*): final attractor.

4. Conclusions

In this paper the attention has been focused on the hyperchaotic dynamics of coupled Chua circuits. By exploiting sine functions as nonlinearities, a novel approach for generating *nxm*-scroll attractors has been developed. Design examples have been reported to illustrate the capability of the approach.

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