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ADAPTIVE OBSERVER-BASED SYNCHRONIZATION FOR COMMUNICATION

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The problem of synchronizing two nonlinear systems (transmitter and receiver) is considered. A simple design of an adaptive observer for estimating the unknown parameters of the transmitter is proposed based on the design of quadratic Lyapunov function for the error system. The results are illustrated by an example of signal transmission based on a pair of synchronizing Chua circuits.

1. Introduction

In recent years a growing interest was observed in the problem of synchronizing chaotic systems [Afraimovich *et al.*, 1987; Pecora & Carroll, 1990; Cuomo *et al.*, 1993; Blekhman *et al.*, 1995; Nijmeijer & Mareels, 1997; Pogromsky & Nijmeijer, 1999]. It was motivated not only by scientific interest in the problem, but also by practical applications in different fields [Blekhman, 1988; Lindsey, 1972], particularly in telecommunications [Kocarev *et al.*, 1992; Cuomo *et al.*, 1993; Dedieu *et al.*, 1993]. However most design methods were suggested and justified under conditions that all the system parameters are known and states are available for measurement. Also, some methods apply only to low-dimensional systems.

Of practical interest is the problem of synchronizing two or more systems when not only initial state but also some parameters are not known to the designer of the receiver. This more complicated problem, which may correspond to the case where parameter modulation is used for message transmission, is referred to as *adaptive synchronization* [Fradkov, 1994, 1995; Wu *et al.*, 1996; Markov & Fradkov, 1997]. Control theory opens new horizons in the synchronization problem and allows to give general framework for its study [Blekhman *et al.*, 1997].

This paper is devoted to design of an adaptive observer oriented to the synchronization for the purpose of communications. A simple design of an adaptive observer for estimating the unknown parameters of the transmitter is proposed based on the design of the Lyapunov function for error system. It provides necessary and sufficient conditions for the existence of a quadratic Lyapunov function for

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the error system. The results are illustrated by an example of information transmission via adaptive synchronization with a pair of Chua circuits.

2. Problem Setting

We consider a transmitter described as

$$\begin{cases} \dot{x}_d = Ax_d + \varphi_0(y_d) + B\sum_{i=1}^{m} \theta_i\varphi_i(y_d)\,, \\ y_d = Cx_d \end{cases} \tag{1}$$

where $x_d \in \mathbb{R}^n$ is the transmitter state vector, $y_d \in \mathbb{R}^l$ is the vector of outputs (transmitted signals), $\theta = \mathrm{col}(\theta_1, \ldots, \theta_m)$ is the vector of transmitter parameters (possibly representing a message, which is either a piecewise constant or slowly time-varying signal). It is assumed that the (piecewise) smooth nonlinearities $\varphi_i(\cdot)$, $i = 0, 1, \ldots, m$, matrices A, C and vector B are known.

The receiver will be designed as another dynamical system which provides estimates $\hat{\theta}_i$, $i = 1, \ldots, m$ of the transmitter parameters based on the observations of the transmitted signal $y_d(t)$. The problem is to design receiver equations

$$\dot{z} = F(z, y_d)\,, \tag{2}$$

$$\hat{\theta} = h(z, y_d) \tag{3}$$

ensuring convergence

$$\lim_{t\to\infty} [\hat{\theta}(t) - \theta] = 0\,. \tag{4}$$

where $\hat{\theta}(t) = \mathrm{col}(\hat{\theta}_1(t), \ldots, \hat{\theta}_m(t))$ is the vector of parameter estimates.

The proposed receiver is a kind of adaptive observer. Its simplest version for the case when A, B, C are known consists of a copy of (1)

$$\begin{aligned} \dot{x} &= Ax + \varphi_0(y_d) \\ &\quad + B\left[\sum_{i=1}^{m} \hat{\theta}_i\varphi_i(y_d) + \hat{\theta}_0 G(y_d - y)\right], \\ y &= Cx\,, \end{aligned} \tag{5}$$

$$\dot{\hat{\theta}}_i = \psi_i(y_d, y)\,, \quad i = 0, 1, \ldots, m\,, \tag{6}$$

where $x \in \mathbb{R}^n$, $y_d \in \mathbb{R}^l$, $\theta_0 \in \mathbb{R}$ and $G \in \mathbb{R}^l$ is the vector of weights and $\psi_i(y_d, y)$, $i = 1, \ldots, m$ are suitably defined functions. The adaptation algorithm (6) will be determined later. Thus the state of receiver is $z = [x, \hat{\theta}_0, \hat{\theta}_1, \ldots, \hat{\theta}_m]$, the right-hand sides of (2) are determined from (5) and (6).

Since the structure of (5) is similar to (1), a natural secondary goal might be

$$\lim_{t\to\infty} e(t) = 0\,, \tag{7}$$

where $e(t) = x(t) - x_d(t)$ is the observation error.

Although (7) is not necessary in order to provide (4), it may give a hint how to choose a Lyapunov function for a proper design of an adaptation algorithm (6).

To solve the problem we write down the error equation:

$$\begin{cases} \dot{e} = Ae + B\left[\sum_{i=1}^{m} \tilde{\theta}_i\varphi_i(y_d) + \hat{\theta}_0 G\tilde{y}\right], \\ \tilde{y} = Ce \end{cases} \tag{8}$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$, $i = 1, \ldots, m$ are the parameter errors. The adaptation algorithm is provided by standard gradient — or speed-gradient — techniques as follows:

$$\dot{\hat{\theta}}_i = -\gamma_i(y - y_d)\varphi_i(y_d)\,, \quad i = 1, \ldots, m\,, \tag{9}$$

$$\dot{\hat{\theta}}_0 = -\gamma_0(y - y_d)^2\,, \tag{10}$$

3. Main Result

In order to formulate the conditions required for a successful applicability of the proposed scheme we need some definitions and auxiliary results.

Definition 1 [Fradkov, 1990]. The system $\dot{x} = \overline{A}x + \overline{B}u$, $y = \overline{C}x$ with transfer matrix $W(\lambda) = \overline{C}(\lambda I - \overline{A})^{-1}\overline{B}$, where u, $y \in \mathbb{R}^l$ and $\lambda \in \mathbb{C}$ is called *hyper-minimum-phase* if it is *minimum-phase* (i.e. the polynomial $\varphi(\lambda) = \det(\lambda I - \overline{A}) \det W(\lambda)$ is Hurwitz), and the matrix $\overline{CB} = \lim_{\lambda\to\infty} \lambda W(\lambda)$ is symmetric and positive definite.

Note that for $l = 1$ the system of order n is hyper-minimum-phase if the numerator of its transfer function is a Hurwitz polynomial of degree $n-1$ with positive coefficients.

Definition 2 [Yuan & Wonham, 1977]. A vector-function $f : [0, \infty) \to \mathbb{R}^m$ is called *persistently exciting* (*PE*) on $[0, \infty)$, if it is measurable and bounded on $[0, \infty)$ and there exist $\alpha > 0$, $T > 0$ such that

$$\int_t^{t+T} f(s)f(s)^T ds \geq \alpha I \tag{11}$$

for all $t \geq 0$.

Lemma 1 [Fradkov, 1976]. *Let matrices $\overline{A}$, $\overline{B}$, $\overline{C}$, G of sizes $n\times n$, $n\times m$, $l\times n$, $m\times l$ be given. Assume rank$(\overline{B}) = m$. Then there exist a positive definite $n\times n$ matrix $P = P^T > 0$ and a $l\times m$ matrix θ_* such that*

$$PA_* + A_*^T P < 0\,,\ P\overline{B} = \overline{C}^T G^T\,,\ A_* = \overline{A} + \overline{B}\theta_* G\overline{C}$$

if and only if the system $\dot{x} = \overline{A}x + \overline{B}u$, $y = G\overline{C}x$ is hyper-minimum-phase.

Lemma 1 establishes conditions of existence of feedback $u = \theta_* y + v$ making the closed loop system with input v and output Gy strictly passive. It is closely related to the Kalman–Yakubovich lemma and can be called a "Feedback Kalman–Yakubovich lemma", see [Andrievsky *et al.*, 1996].

Lemma 2 [Yuan & Wonham, 1977]. *Consider vector-functions f, $\tilde{\theta} : [0, \infty) \to \mathbb{R}^m$. Assume that $\tilde{\theta}(t)$ is continuously differentiable, $\dot{\tilde{\theta}}(t) \to 0$ as $t \to \infty$ and f is PE. Then $\tilde{\theta}(t) \to 0$ as $t \to \infty$ provided that $\tilde{\theta}(t)^T f(t) \to 0$ as $t \to \infty$.*

Theorem 1. *Assume that all the trajectories of the transmitter (1) are bounded and the linear systems with the transfer function $W(\lambda) = GC(\lambda I - A)^{-1}B$ be hyper-minimum-phase. Then all the trajectories of the receiver (5), (9) and (10) are bounded and the relation (7) holds. If, in addition, the vector-function $[\varphi_1(y_d), \ldots, \varphi_m(y_d)]$ satisfies the PE condition, then also (4) holds.*

Proof of Theorem 1. To prove the theorem consider the Lyapunov function candidate

$$V(x,\, \hat{\theta}_0,\, \hat{\theta},\, t) = \frac{1}{2}e^T Pe + \frac{1}{2}\sum_{i=0}^{m} \|\hat{\theta}_i - \theta_i\|^2/\gamma_i + \|\hat{\theta}_0 - \theta_{*0}\|^2/\gamma_0 \qquad (12)$$

where a matrix $P = P^T > 0$ and a number θ_{*0} are to be determined. Calculation of $\dot{V}$ gives that $\dot{V} < 0$ for $e \neq 0$ if and only if the following conditions are valid:

$$\begin{cases} \dot{\hat{\theta}}_0 = -\gamma_0 e^T PBGCe\,, \\ \dot{\hat{\theta}}_i = -\gamma_i e^T PB\varphi_i(y_d)\,, \\ e^T(PA_* + A_*^T P)e < 0\,. \end{cases} \qquad (13)$$

Using Lemma 1 we obtain that $\dot{V} < 0$ for $e \neq 0$ if and only if the adaptation algorithm has the form (9) and (10), and the system $\dot{x} = Ax + Bu$, $y = Cx$ is hyper-minimum-phase. Therefore, under the given conditions the function $V(t) = V(x(t),\, \hat{\theta}_0(t),\, \hat{\theta}(t),\, t)$ is bounded. Since $\varphi_i(y_d(t))$, $i = 1, \ldots, m$ are bounded, the functions $e(t)$, $\hat{\theta}_i(t)$ are bounded too. Equations (13) imply that $\dot{V} = e^T(PA_* + A_*^T P)e \le -\mu\|e(t)\|^2$ for some $\mu > 0$. Integration of the last inequality over the interval $[0, t]$ gives: $V(t) - V(0) \le -\mu\int_0^t \|e(s)\|^2 ds$. Taking into consideration that $V \ge 0$ we obtain: $V(0) \ge \mu\int_0^t \|e(s)\|^2 ds$. This yields the inequality

$$\int_0^\infty \|e(t)\|^2 dt < \infty\,. \qquad (14)$$

Since $\varphi_i(y_d)$, $i = 1, \ldots, m$ are bounded, $\dot{e}(t)$ is also bounded in view of (8). From (14) and Barbalat's lemma we obtain that the goal (7) is achieved.

To prove (4) we first note that $\dot{\tilde{\theta}}(t) \to 0$ as $t \to \infty$ from (9) and (7). Differentiating (8), from boundedness of functions e, $\tilde{\theta}$, φ_d, $\tilde{y}$, $\hat{\theta}_0$ and their time-derivatives we conclude that $\ddot{e}(t)$ is bounded. Barbalat's lemma then implies that $\dot{e}(t) \to 0$ as $t \to \infty$. This and (9) yield $\tilde{\theta}(t)^T\varphi_d(t) \to 0$ as $t \to \infty$. Hence (4) follows from the PE condition and Lemma 2. ■

Remark. Theorem 1 in fact gives necessary and sufficient conditions for the existence of a Lyapunov function of the form (12) with the properties

$$\left\{\begin{aligned} &V(x,\, \hat{\theta}_0,\, \hat{\theta},\, t) > 0 \quad \text{for } e \neq 0\,, \\ &\dot{V}(x,\, \hat{\theta}_0,\, \hat{\theta},\, t) < 0 \quad \text{for } e \neq 0\,. \end{aligned}\right\} \qquad (15)$$

It means that it is not possible to find another adaptation algorithm based on the Lyapunov function (12) with the properties (15).

As an example we consider the problem of synchronizing two Chua circuits with unknown parameters and incomplete measurements.

4. Signal Transmission and Reconstruction

In recent years much attention has been devoted to methods for secure communications utilizing chaos [Kocarev *et al.*, 1992; Cuomo *et al.*, 1993; Dedieu *et al.*, 1993]. Various methods for transmitting signals via chaotic synchronization were proposed like chaotic signal masking [Kocarev *et al.*, 1992; Cuomo *et al.*, 1993], chaotic binary communications [Dedieu *et al.*, 1993; Cuomo *et al.*, 1993], etc.

A possible application of the synchronization scheme proposed in Sec. 2 to chaotic binary communications algorithms goes as follows and is based on the dependence of the synchronization effect on the matching of the corresponding parameters of the systems. The transmitter and receiver have identical structure as in the previous section. The basic idea is to modulate this coefficient with an information-bearing binary waveform and transmit the chaotic signal. At the receiver side the coefficient modulation will produce a synchronization error between the received signal and the corresponding transmitter reconstructed signal: If the coefficients of transmitter and receiver are identical the signals will synchronize, otherwise synchronization fails. Using the synchronization error the modulation can be detected. Security of communications is possibly enhanced by a set of other transmitter parameters.

Consider as an example of information transmission where both transmitter and receiver system are implemented as a Chua circuit, similarly to [Dedieu *et al.*, 1993]. The transmitter model in dimensionless form is given as:

$$\begin{aligned} \dot{x}_{d_1} &= p[x_{d_2} - x_{d_1} + f(x_{d_1}) + sf_1(x_{d_1})] \\ \dot{x}_{d_2} &= x_{d_1} - x_{d_2} + x_{d3} \\ \dot{x}_{d_3} &= -qx_{d_2} \end{aligned} \tag{16}$$

where $f(z) = M_0 z + 0.5(M_1 - M_0)f_1(z)$, $f_1(z) = |z+1| - |z-1|$, M_0, M_1, p, q are the transmitter parameters, $s = s(t)$ is the signal to be reconstructed in the receiver. The parameters are obviously chosen in such a way that the Chua circuit (16) exhibits chaotic dynamics for the signal $s(t)$ being within its range. Assume that the transmitted signal is $y_d(t) = x_{d_1}(t)$, and the values of the parameters p, q are known.

The parameters M_0, M_1 are assumed to be *a priori* unknown which motivates the use of an adaptation for the receiver design. The receiver designed according to the results of Sec. 2 is modeled as

$$\begin{aligned} \dot{x}_1 &= p[x_2 - x_1 + f(y_d) \\ &\qquad + c_1 f_1(y_d) + c_0(x_1 - y_d)]\,, \\ \dot{x}_2 &= x_1 - x_2 + x_3\,, \\ \dot{x}_3 &= -qx_2\,, \end{aligned} \tag{17}$$

where c_0, c_1 are the adjustable parameters. The adaptation algorithm (9) and (10), takes the form

$$\dot{c}_0 = -\gamma_0 (y_d - x_1)^2\,, \tag{18}$$

$$\dot{c}_1 = -\gamma_1 (x_1 - y_d) f_1(y_d)\,,$$

where γ_0, γ_1 are the adaptation gains.

First we examine the ability of the system (17) and (18) to receive and to decode messages. To this end we verify the conditions of Theorem 1 assuming that $s(t) = \text{const}$. Clearly, if $s(t)$ is a time-varying binary signal, we can only expect that the results of Theorem 1 can be used if the parameter estimation is fast enough, at least much faster than the actual parameter modulation. Writing the error equations yields

$$\begin{cases} \dot{e}_1 = p[e_2 - e_1 + (c_1 - s)f_1(y_d) + c_0 e_1] \\ \dot{e}_2 = e_1 - e_2 + e_3 \\ \dot{e}_3 = -qe_2\,, \end{cases} \tag{19}$$

where $e_i = x_i - x_{d_i}$, $i = 1, 2, 3$. The system (19) is, obviously in Lur'e form (8), where

$$A = \begin{bmatrix} -p & p & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad C = [1 \quad 0 \quad 0],$$

$\hat{\theta}_1 = c_1$, $\theta_1 = s$, $\theta_0 = c_0$.

The transfer function of the linear part is

$$W(\lambda) = \frac{\lambda^2 + \lambda + q}{\lambda^3 + (p+1)\lambda^2 + q\lambda + pq} \tag{20}$$

We see that the order of the system is $n = 3$, while the numerator polynomial is Hurwitz and has degree 2 for all $q > 0$ and all real p. Therefore the hyper-minimum-phase condition holds for $q > 0$ and any p, M_0, M_1. Thus, Theorem 1 yields the boundedness of all receiver trajectories $x(t)$ and convergence of the observation error: $e(t) \to 0$. In particular, $y_d(t) - x_1(t) \to 0$. Furthermore, to be able to reconstruct the signal $s(t)$ the receiver should provide convergence $c_1(t) - s \to 0$ for constant s. According to Theorem 1, this will be the case if the PE condition (see Definition 2) holds, which reads as

$$\int_{t_0}^{t_0+T} f_1^2(y_d(t))dt \geq \alpha \tag{21}$$

for some $T > 0$, $\alpha > 0$ and all $t_0 \geq 0$. To verify (21), we note that condition (21) basically means that the trajectory of the transmitter $x_d(t)$ does not converge to the plane $x_{d_1} = 0$ when $t \to \infty$. This is not the case, at least when the system (16) exhibits chaotic behavior. Indeed, in this case the value $x_{d_1}(t)$ leaves the interval $(-1, 1)$ (where $f_1(z)$ is linear) infinitely many times, say at t_k, $k = 1, 2, \ldots$. The time intervals $\Delta t_k = t_{k+1} - t_k$ between t_k can be overbounded by constant, if the trajectory does not converge to the set $x_{d_1} = 0$.

We may also evaluate a lower bound for α in (21)

$$\alpha_0 = \lim_{T\to\infty} \frac{1}{T} \int_0^T f_1^2(x_{d_1}(t))dt\,. \qquad (22)$$

The value of α_0 characterizes the parameter convergence rate. It follows from the standard convergence rate results (see e.g. [Sastry & Bodson, 1989]) that if $\alpha_0 > 0$, then the convergence $c_1(t) - s \to 0$ is exponential, with rate $\gamma_1\alpha_0$, at least for sufficiently small $\gamma_1 > 0$. Ergodicity arguments suggest that

$$\alpha_0 \geq \frac{\overline{x}_{d_1}^2}{\mu}\,, \qquad (23)$$

where $\overline{x}_{d_1}^2$ is the average value of $x_{d_1}^2(t)$ over the attractor Ω, and $\mu = \sup_{x\in\Omega} |x_{d_1}(t)|$.

5. Simulation Results

We carried out simulations for the above scheme. Parameter values were selected as $p = 9$; $q = 14.286$; $M_0 = 5/7$; $M_1 = -6/7$. For these parameter values the system (16) possesses a chaotic attractor (see Fig. 1), resembling that of the system used in [Dedieu *et al.*, 1993] (after some rescaling of space and time variables).

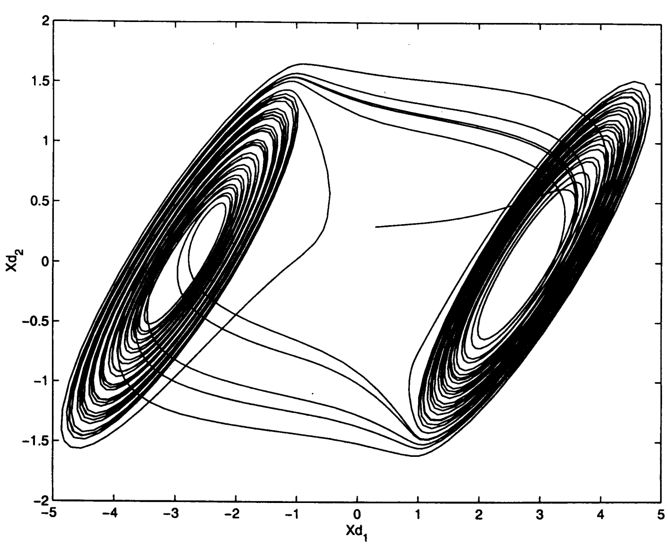

Fig. 1. Attractor of the system (16).

The initial conditions for the transmitter were taken as $x_d(0) = [0.3 \quad 0.3 \quad 0.3]$. For the receiver zero initial conditions were chosen for the state x_0 as well as for the adjustable parameters $c_0(0)$, $c_1(0)$. In order to eliminate the influence of initial conditions no message was transmitted during the first 20 sec ("tuning" or "calibration" of the receiver), i.e. $s(t) \equiv 1$ for $0 \leq t \leq 20$ s. The time history of observation errors (Fig. 2) and parameter estimates (Fig. 3) during tuning show that all observation errors and parameter estimation error $c_1(t) - s$ tend to zero rapidly. The value $c_0(t)$ tends to some constant value.

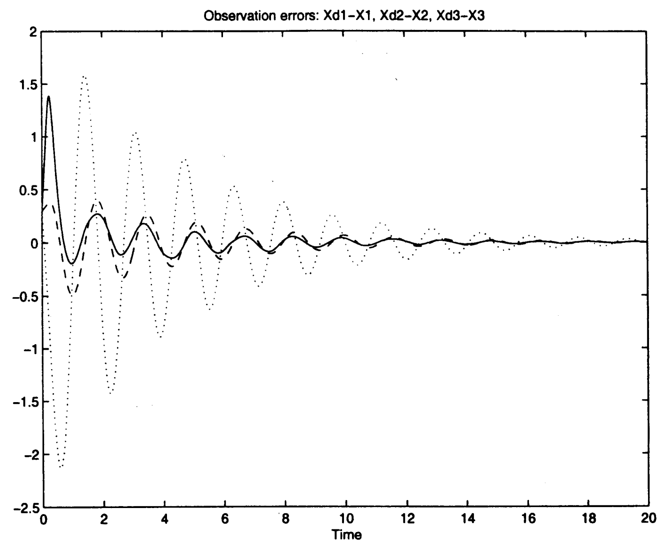

Fig. 2. Time history of observation errors during tuning.

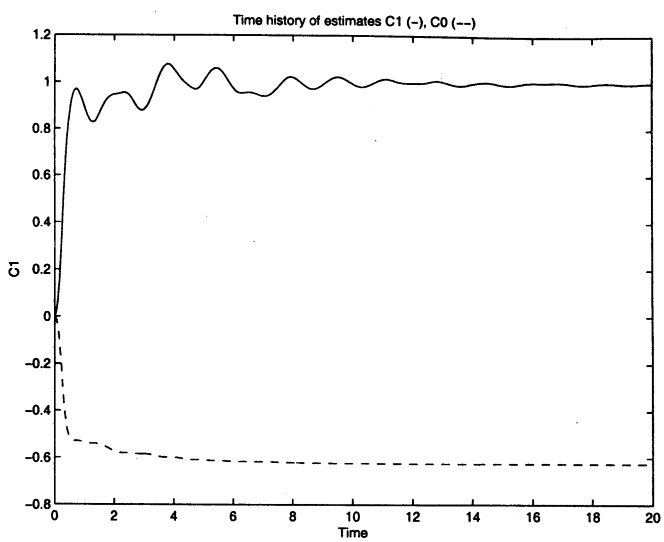

Fig. 3. Time history of parameter estimates during tuning.

After the tuning period the square wave message

$$s(t) = s_0 + s_1 \operatorname{sign} \sin\left(\frac{2\pi t}{T_0}\right), \qquad (24)$$

where $s_0 = 1.005$, $s_2 = 0.005$ was sent. Simulation results for $T_0 = 5.0$ s, $\gamma_1 = 1.0$ are shown in Figs. 4 and 5. It is seen that the reconstructed signal $y(t)$ coincides with the transmitted signal $y_d(t)$ with very good accuracy (the error $y_d(t) - y(t)$ is shown in Fig. 3, solid line). However both observation errors (Fig. 3) and estimation errors (Fig. 4) do not decay completely during the interval when $s(t)$ is constant. Nevertheless, a reliable reconstruction of the signal $s(t)$ is very well possible. The accuracy of estimation can be easily improved by increasing the adaptation gain γ_1, which is confirmed by simulation results for $\gamma_1 = 5.0$ (Figs. 6 and 7). The achievable information transmission rate depends on the highest frequencies in the carrier spectrum.

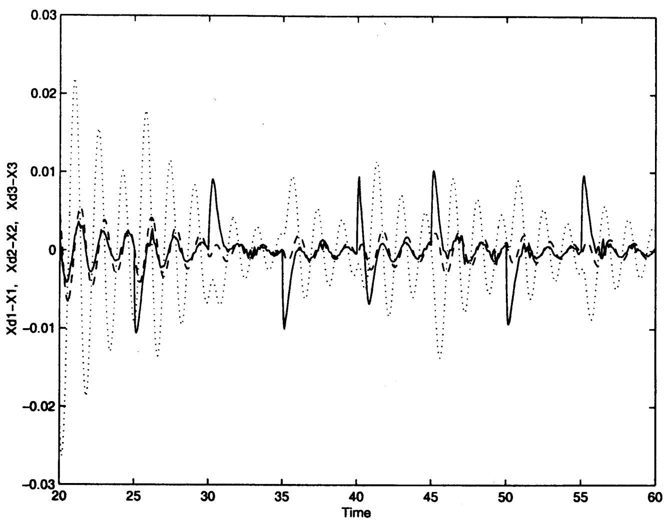

Fig. 4. Time history of observation errors for $\gamma_1 = 1.0$.

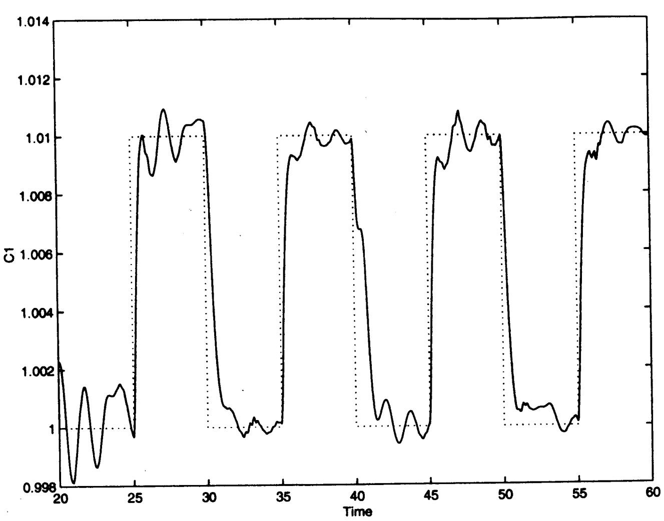

Fig. 5. Time history of parameter estimates for $\gamma_1 = 1.0$.

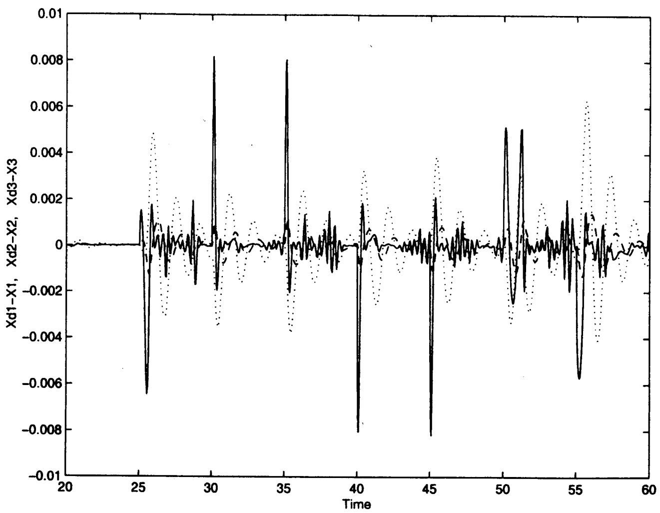

Fig. 6. Time history of observation errors for $\gamma_1 = 5.0$.

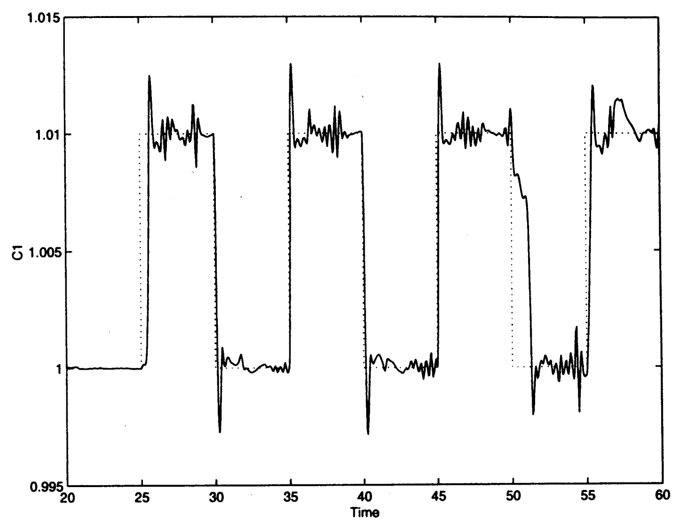

Fig. 7. Time history of parameter estimates for $\gamma_1 = 5.0$.

6. Conclusion

The proposed adaptive observer-based synchronization scheme demonstrates good signal and parameter reconstruction abilities. It allows to achieve high information transmission rate. The results of the paper demonstrate the fruitfulness of modern nonlinear and adaptive control theory application to synchronization problems.

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