CHAOS PRESERVATION TH ROUGH CONTINUOUS
CHAOTIC PULSE POSITION MODULATION

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ABSTRACT
In this paper it is examined an extension of the Chaotic Pulse Position Modulation to the case in which the chaotic sequence is generated by an analog chaotic circuit instead of a discrete chaotic map. The suitability of the approach has been confirmed by carrying out a real case in which a Chua’s circuit has been used, and showing that chaos is preserved after the modulation phase.

1. INTRODUCTION
Chaotic communications conjugate a low probability of detection and intercept with the simplicity of the underlying circuitry, along with low power consumption. The high sensitivity of the chaotic generator to initial conditions and to parameter values allows to establish a great variety of keys in order to generate unpredictable sequences. Since it has been shown that two chaotic systems can be synchronised, despite the broadband nature of their spectra, the noise-like behaviour and sensitivity to initial conditions [1], a tremendous interest has been generated towards the idea of using chaotic signals to transmit information. The transmitted signal can then be retrieved by exploiting the properties of synchronization of chaotic systems.

Apart from the techniques based on synchronization, a wide class of chaotic communications schemes, which do not use synchronization, have been introduced [3], in order to cope with the drawbacks encountered in real systems, i.e. noise, limited band and distortion, which could limit the possibility of performing synchronization-based schemes. The techniques not based on synchronization have evolved from very simple schemes like Chaotic On-Off Keying and the Chaos Shift Keying, up to more complicated ones like the Differential Chaotic Shift Keying and the modern Frequency Modulation Differential Chaotic Shift Keying.

Chaotic communications are very effective in case of multi-path propagation and in multi-user scenarios. In this framework, particular attention has been paid to the Chaotic Pulse Position Modulation (CPPM) [2]. Such a modulation can be realised either by considering a synchronous or an asynchronous approach.

The main idea underlying CPPM is to generate a sequence of pulses distilled by time intervals generated by a chaotic map. The main features of this kind of modulation are low power consumption due to the small duty cycle of the transmitted signal, along with a great robustness to external noise, due to the fact that information is contained in the distance between two consecutive pulses, rather than in the pulse amplitude. CPPM has been applied in wireless communications [4].

The chaotic sequence establishing the distance between two pulses in this modulation scheme as it appears in literature is usually generated by a chaotic discrete map. In this paper it is shown that such a sequence can be generated by an analog chaotic circuit as well. This can be useful to simplify the needed circuitry without involving the use of a digital processor or computational resources. In particular, the real case concerning the use of the well-known Chua’s circuit [5] to perform the modulation has been adopted.

Focusing the attention on the real continuous systems as candidate modulators, it is obviously not guaranteed that the chaotic behaviour is still present in the sequence of pulses after modulation. The reason of this phenomenon is twofold. Firstly, in a real continuous-time case the dynamics of the modulator cannot be neglected. The second reason is the inevitable loss of information introduced when considering only finite samples of a chaotic analog signal.

This paper is structured as follows. After having described the scheme of the CPPM modulator with an analog chaotic circuit as a chaos generator, the chaos preservation is verified by investigating the demodulated signal versus the modulated one by comparison of both time trends of the signals and their frequency spectra. Finally, the embedding
of the demodulated signal has been reported and a comparison with the original one has been performed.

2. CHAOTIC PULSE POSITION MODULATION

As remarked above, in traditional Chaotic Pulse Position Modulation the signal to be transmitted is coded by a sequence of pulses of equal amplitude and duration. Information is actually contained into the distance between a pulse and the following one. The position of a pulse is directly related to the value generated by a chaotic map. This allows us to assume a direct correspondence between the chaotic sequence and the sequence of the time intervals between two pulses as follows:

\[ x_k \rightarrow \tau_k \]  \hspace{1cm} (1)

When, as in our case, a chaotic continuous circuit is used as chaotic generator instead of the discrete map, relationship (1) is no more trivial for two reasons. First, the chaotic sequence is not directly available, but it has to be constructed by starting from a continuous circuit. Moreover, a real circuit is used in this case, involving noise, parameter uncertainties, and so on.

In other words, the continuous CPMM consists essentially in a voltage-to-time conversion. This requires a circuit that is nonlinear and has its own dynamics which cannot be a priori neglected, so (1) has to be rewritten in order to include this issue.

Thus, the dynamics of the modulator necessary to realize (1) can actually modify the chaotic behaviour of the chaotic generator. In other words, in the assumptions we made, a continuous signal has to be mapped at first to a discrete sequence by using an adequate circuitry. Then, the sequence obtained has to be coded as traditional CPMM by a sequence of pulses. This process obviously does not allow to make the assumption expressed in (1). It in fact involves a more complicated mapping, that can be resumed as:

\[ x \rightarrow x_k \rightarrow g(x_k) \rightarrow \tau_k \] \hspace{1cm} (2)

The \( g(.) \), which is obviously not known, takes into account all the effects examined.

Let us consider in more details the continuous modulator, which is depicted in Fig. 1 by a block scheme. The chaotic sequence is generated in the following way. For sake of simplicity, we focus attention on a single state variable. A sample-and-hold circuit performs a S/H operation on the continuous signal. Then, a ramp is generated and is stopped when its value equals that of the sampled signal. When this occurs, a pulse is emitted from the modulator, the ramp is reset and a further S/H operation is performed. It is clear that, as the ramp signal is reset when the pulse is emitted, it reaches the value of the chaotic sample after a time which is proportional to the value itself.

![Fig. 1. Scheme of the continuous pulse position modulator.](image)

The circuit implementing such a modulator is shown in Fig. 2. It is worth remarking that some parameters of the circuit have to be tuned with respect to the considered chaotic attractor. Such a parameter is, for example, the resistance R3, related to the slope of the ramp. This has to be tuned for the specific chaotic attractor. For this reason, it has been implemented with a potentiometer. Another component that has been introduced in order to make possible its fine tuning is the resistance R4, which fixes the duration of the pulse. However, in this case this parameter is not related to the specific attractor, but other design issues have to be taken into account.

In particular, the pulse duration plays a fundamental role in considering asynchronous schemes, because it is directly related to the amplitude of the correlation of the received signal. This consideration allows us to establish the lowest value for the pulse duration. The upper bound to this parameter is instead related to the power of transmission required.

![Fig. 2. The circuit implementation of the continuous chaotic pulse position modulator.](image)
The presented modulator does work only for positive signals, which are easily achievable with discrete chaotic maps. In case of real chaotic circuits, which often assume both positive and negative values (as it happens for the double scroll Chua attractor [5]), it can be necessary to apply an offset to make the signal positive. Obviously, this offset allows to impose the minimum distance between two pulses.

3. EXPERIMENTAL RESULTS

As explained above, to examine a real case, we have to focus our attention on a specific chaotic system. It is worth noticing that only the modulator parameters depend on the specific attractor to be modulated, whereas the general structure of the modulator is not affected by it. The results shown in this section are obtained by considering the well-known double scroll Chua attractor of the Chua’s circuit, realised by using State Controlled Cellular Neural Networks [6], and the modulator shown in Fig. 2.

The equations, in non-dimensional form, of the Chua’s circuit are the following:

\[
\frac{dx}{dt} = \alpha [y - h(x)] \\
\frac{dy}{dt} = x - y + z \\
\frac{dz}{dt} = -\beta \cdot y \\
h(x) = m_1 \cdot x + 0.5 \cdot (m_0 - m_1) \cdot |x + 1| - |x - 1| 
\]

while the following choice of the parameters has to be taken into account to obtain the double scroll Chua attractor:

\[
\alpha = 9 \\
\beta = 14.286 \\
m_0 = \frac{1}{7} \\
m_1 = \frac{2}{7}
\]

Fig. 3 shows an example of the pulse train obtained by using the \(x\) state variable of the double scroll Chua attractor to generate the chaotic sequence.

In particular, the continuous chaotic pulse position modulated signal is shown in Fig. 3(a), whereas the corresponding chaotic signal generating the sequence is shown in Fig. 3(b) along with the sawtooth signal.

![Fig. 3](image)

(a) The Continuous Chaotic Pulse Position Modulated signal. (b) The original chaotic signal (state variable \(x\) of the double scroll Chua attractor) and sawtooth waveform (SPICE simulation).

In order to verify that the continuous CPPM scheme allows us to preserve chaos, in the following we will demodulate the pulse-coded signal and compare it with the original one. It is worth to remark that in the communication scheme the demodulation is not included, and is used in this work only for validation purpose. The demodulation of the pulse-coded signal can be simply performed by tracking the trend of the duty cycle of the signal.

In particular, Fig. 4(a) depicts the trend of the \(x\) state variable generated by the SC-CNN-based Chua’s circuit, while the demodulated signal is reported in Fig. 4(b). Fig. 4(c) and 4(d) show the Fast Fourier Transform of the original and the demodulated signal, respectively. Accurate comparisons and measurements between the two sources of signal have been made in order to check the adequacy of the modulation strategy, leading to the conclusion that the information is not lost in the whole sampling and modulation process.

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Fig. 4. (a) Trend of x variable. (b) The demodulated signal. (c) Fourier Fast Transform (FFT) of the signal in (a). (d) FFT of the signal in (b).

In order to perform a more in-depth analysis of the phenomenon, the attractors reconstructed by embedding the x state variables coming from the Chua's circuit and the demodulator are compared.

In Fig. 5 the one-step delay embedding [7] of the original signal is represented.

Fig. 5 – The one-step delay embedding of the original Chua's attractor.

The reconstructed embedded signal (coming from the demodulator) is shown in Fig. 6, where the phase plane $x_n \rightarrow x_{n+1}$ is illustrated.

Fig. 6. Reconstructed attractor $x_1 \rightarrow x_4$ of the demodulated signal.

4. CONCLUSIONS

In this paper an extension of the Chaotic Pulse Position Modulation to the case in which the generator of chaos is a real continuous chaotic circuit has been examined. In particular a Chua's circuit has been used as chaos generator. This can be very useful when a simpler modulation circuitry is required. Experimental results show that chaos is maintained even after the modulation scheme introduced in this paper.

5. REFERENCES