Hyperchaotic dynamic generation via SC-CNNs for secure transmission applications

R. Caponetto†, L. Fortuna‡, L. Occhipinti§, M.G. Xibilia‡

†SGS-Thomson Microelectronics SC&MT Group Catania, Stradale Primosole 50, 95121 Catania Italy, E-mail: riccardo.caponetto@st.com

‡University of Catania, D.E.E.S. System and Control Group Viale A. Doria 6, 95100 Catania Italy, E-mail: xibi@dees.unict.it

Abstract

In this paper it is proposed a CNNs based circuit for secure communication applications. An hyperchaotic Saito oscillator has been designed by using a configuration of cellular neural network named State-Controlled CNNs. A secure communication system, based on chaotic inverse system synchronization, is described and the results, relative to a prototype circuit realization, are reported.

1. Introduction

Recently an increasing interest is devoted to chaotic communications due to their robustness, low power consumption and spread spectrum features [1-3].

The basic principle of chaotic communication is to hide the signal to be transmitted inside a chaotic carrier, in the transmitter circuit, and then, in the receiver, to reconstruct the original message by using appropriate synchronization techniques [4-5].

Chua’s circuit and other chaotic signals have been used in secure transmission application, but the security of the system can be increased by adopting hyperchaotic whereas chaotic signal [6],[7].

Saito circuit [8], showing an hyperchaotic dynamic, well fit this kind of approach.

On the other hand it was proved that a particular class of cellular neural networks named SC-CNNs can be easily adopted to generate chaotic and hyperchaotic signal [9],[10].

The idea to use SC-CNNs for chaotic communication application is justified by the fact that SC-CNNs can be digitally programmed, so the enciphering keys, of the communication system, can be easily selected/changed [9].

In the following it will be described the proposed circuit. In particular in section 2 the SC-CNNs structure is introduced, in 3 the Saito circuit is described and designed by using SC-CNNs and finally, in section 4, the transmission chain and the results, obtained implementing a discrete components board, are reported.

2. State-Controlled CNNs

In [10] the unfolded Chua’s circuit has been obtained from an appropriate collection of three generalized CNN cells. A generalized State-Controlled-CNN cell is described by the following dimension-less non-linear state equation:

\[
\begin{align*}
\tau \frac{dx_j}{dt} &= -x_j + \sum_i a_{ij}x_i + \sum_i b_{ij}x_i + i_j \\
y_j &= f(x_j) = 0.5 \ast \left( |x_j + i| - |x_j - i| \right)
\end{align*}
\]  

where \(x_j\) is the state of the generic cell \(j\) of the CNN, \(y_j\) is the output of the cell, \(i_j\) is the bias current and \(\tau=RC\) is the time-constant of the circuit. Finally \(\{a_{ij}\}\) and \(\{b_{ij}\}\) are the SC-CNN templates.

As described in [11] using a 3-cells, fully interconnected, SC-CNN is possible to obtain high complex dynamic behavior, which corresponds to a gallery of chaotic attractors containing also all possible attractors of the Chua’s circuit.

For example the following templates correspond to the same attractors of the Chua’s circuit:
where $\alpha$, $\beta$, $\gamma$, $m_0$ and $m_1$ are the Chua's circuit parameters.

Analogously other chaotic attractors can easily be obtained like Colpitts oscillator, $\pi$-double scroll, coupled oscillators, Canards and, like described in the following section, Saito hyperchaotic circuit.

3. Saito Hysteresis Chaos Generator via SC-CNNs

The dimensionless state equations of the Saito Hysteresis Chaos Generator (SHCG) are:

\[
\begin{align*}
\dot{x} &= -z - w \\
\dot{y} &= \gamma (2\delta y + z) \\
\dot{z} &= \rho (x - y) \\
\dot{w} &= x - h(w)
\end{align*}
\]

with: $h(w) = w - (w + 1)(w - 1)$

$x$, $y$, $z$ and $w$ being the state variable and $\gamma$, $\delta$, $\rho$ and $\varepsilon$ the system parameters.

The SHCG is shown in Figure 1, while in Figure 2 is reported a state space representation of the hyperchaotic attractors in the $x$-$w$ plane, with $\gamma = 1$, $\delta = 1$, $\rho = 14$, and $\varepsilon = 10^{-2}$ obtained by simulating the system using SIMULINK.

Using (1) it is possible to map (2) by appropriate setting the coefficients of (1) the obtained values of the $a_{ij}$, $s_0$ and $i_0$ are reported in the following:

\[
\begin{align*}
a_{11} &= a_{12} = a_{22} = a_{23} = a_{33} = a_{34} = a_{44} = 0; \\
a_{11} &= \alpha (m_1 m_0); \\
a_{12} &= s_0 = 0; \\
s_2 &= s_3 = l; \\
s_3 &= s_4 = l; \\
\beta &= \gamma.
\end{align*}
\]

From a practical point of view it is possible to implement a single cell of a SC-CNN by using the circuit configuration reported in Figure 3 and introduced in [10].

![Figure 3: Circuit realization a single SC-CNN](image)

Taking into account that the SHCG is a fourth order circuit it is necessary to arrange four cell in the following configuration:

![Figure 4: Scheme of four SC-CNNs arranged to obtain a SHCG](image)

The adopted resistor and capacitor values of the four cells utilized to realized the SHCG circuit are:

**Cell #1**

$R_1 = R_2 = R_3 = R_4 = 100\, \text{K}\Omega$; $R_5 = 1\, \text{K}\Omega$; $R_6 = 200\, \text{K}\Omega$; $R_7 = 100\, \text{K}\Omega$; $C_1 = 140\, \text{nF}$;

**Cell #2**

$R_1 = 100\, \text{K}\Omega$; $R_2 = 400\, \text{K}\Omega$; $R_3 = 1\, \text{K}\Omega$; $R_4 = 500\, \text{K}\Omega$; $R_5 = 100\, \text{K}\Omega$; $R_6 = 100\, \text{K}\Omega$; $C_2 = 140\, \text{nF}$;
A SPICE simulation of the described circuits is reported in Figure 5, while in Figure 6 the measured (by using a Tektronix TDS 544 A) state space variables $x_2$ vs. $x_4$ of the implemented circuit are showed.

![Figure 5: SPICE simulation of SHCG (x-w plane).](image)

![Figure 6: SHCG x-w plane measured values](image)

### 4. Secure transmission application

The adopted transmission chain is reported in Figure 7 and the inverse system synchronization has been adopted.

The analog input signal to be enciphered is converted from voltage to current and then is added to the hyperchaotic Saito circuit generated using SC-CNNs. The obtained current is injected into the capacitor of the $X_1$ node of the Saito circuit. The choice of a particular state variable depends on the voltage amplitude. The higher the amplitude, better the signal enciphering.

It must be outlined that due to the fact that the system is hyperchaotic two state variables have been transmitted through the channel.

Though recently it has been proved that it is possible to synchronize master (transmitter) and slave (receiver) by using only one state variable [12] the proposed circuit use two signals for systems synchronization.

Once the receiver synchronizes with the transmitter; the original signal is extracted by using a current to voltage converter that sense the current of the synchronized circuit and transform it in to a voltage proportional to the original message.

![Figure 7: Block scheme of the adopted transmission chain.](image)

Synchronization was accomplished not for every state variables, but only with the following:

1. $CH_1 \to X_1$  $CH_2 \to X_2$ AND VICEVERSA
2. $CH_1 \to X_2$  $CH_2 \to X_3$ AND VICEVERSA
3. $CH_1 \to X_2$  $CH_2 \to X_4$ AND VICEVERSA

In other cases the two circuits are not synchronized.

In the realized circuit the variables $X_2$ and $X_3$ has been utilized for synchronization.

One of the drawback of chaotic communication is the fact that the transmitted signal has a broadband spectrum. It means that, theoretically, it is necessary a very large band for transmission. In order to reduce the band occupancy it was introduced pass-band filter and by trial and error it was proved that it is possible to synchronize the circuit within the following band; low cut-off frequency 200Hz, high cut-off frequency 16KHz. It means that a 16KHz band is sufficient for transmission.

As a first example the transmission of a sinusoidal signal with 1Vpp amplitude and 10KHz frequency is described.
In Figure 8 four signals are reported representing: the input sinusoidal signal, its frequency spectrum, the chaotic transmitted signal and its frequency. It is possible to see that a noise like signal flows through the channel allowing a secure transmission against undesired listeners.

In Figure 9 four signal of the receiver are reported. The first represent the received message and its frequency spectrum while the third and the fourth represent the reconstructed sinusoidal signal and its frequency spectrum.

A second experiment was done enciphering a square signal. The results are reported in Figure 10 and Figure 11. The signals have the same means of those reported in the first example.

5. Discussion and conclusion

Other tests have been done changing the frequency of the sinusoidal signal from 1KHz to 30KHz. The circuit continues to synchronize and encipher the sinusoidal signal. This allows to transmit voice signals.

In both the reported examples the transmitted signal is hyperchaotic assuring a secure transmission system but the reconstructed signals are noise affected. This problem can be overcome by adding a filter to the receiver to perfectly rebuild the original message.

Next step consists in redesign the circuit taking into account that only one signal is necessary for synchronization. This approach being more feasible for commercial application will be deeply studied.

6. References

