On the dynamics of hyperchaotic circuits: a new synchronization approach

Michele Brucoli, Leonarda Carnimeo, Giuseppe Grassi
Dipartimento di Elettrotecnica ed Elettronica
Politecnico di Bari
Via E. Orabona, 4 - 70125 Bari (Italy)
Phone (39) 80-5460482 Fax (39) 80-5460410 E-Mail: brucoli@poliba.it

Abstract **- In this paper a technique for synchronizing hyperchaotic circuits under parameter or initial condition changes is illustrated. Each hyperchaotic circuit is constituted by two coupled Chua's circuits. Conditions for achieving synchronization are discussed in detail in both the cases of bidirectional and unidirectional coupling between two Chua's circuits. Several numerical examples are carried out to show the capability of the suggested approach.**

I.INTRODUCTION

Recently there has been an increasing interest in the problem of synchronizing chaotic circuits [1-3]. This because chaotic synchronization has become very important in speech and image processing [4] as well as in secure communications [5, 6]. Very recently an attempt of synchronizing hyperchaotic systems has been developed in [3] by extending the approach of Carroll and Pecora illustrated in [2]. The problem of obtaining two real hyperchaotic circuits oscillating in a synchronized way is not trivial. Namely, as the same initial conditions cannot be exactly reproduced and identical system parameters cannot be realized in real circuits, synchronization seems not to be a reachable objective. In fact, in this case any small difference in initial conditions and parameters would be exponentially amplified [4]. The aim of this paper is to provide a technique to overcome this problem. In particular, since it has been demonstrated the existence of several hyperchaotic attractors, such as the "double-double scroll" [7], in this paper the theoretical results obtained in [3] are used to achieve the synchronization between two hyperchaotic circuits under parameter or initial condition changes, each constituted by two coupled Chua's circuits. The capability of the suggested approach is demonstrated by carrying out some numerical examples.

II. DYNAMICS AND SYNCHRONIZATION OF HYPERCHAOTIC CIRCUITS

The hyperchaotic circuit considered in this work is formed by two bidirectionally coupled Chua's circuits as shown in Fig.1. The state equations for this circuit can be expressed as [7]:

$$\dot{x}_1 = \alpha(x_2 - x_1 - f(x_1)) \tag{1}$$
$$\dot{x}_2 = x_1 - x_2 + x_3 + K(x_5 - x_2) \tag{2}$$
$$\dot{x}_3 = -\beta x_2 \tag{3}$$
$$\dot{x}_4 = \alpha(x_5 - x_4 - f(x_4)) \tag{4}$$
$$\dot{x}_5 = x_4 - x_5 + x_6 + M(x_2 - x_5) \tag{5}$$
$$\dot{x}_6 = -\beta x_5 \tag{6}$$

0-7803-31-09-5/96/$5.00 © 1996 IEEE

where

$$f(x_1) = bx_1 + (a-b)(|x_1+1| - |x_1-1|)/2 \tag{7}$$
$$f(x_4) = bx_4 + (a-b)(|x_4+1| - |x_4-1|)/2 \tag{8}$$

and α, β, a and b are constants.

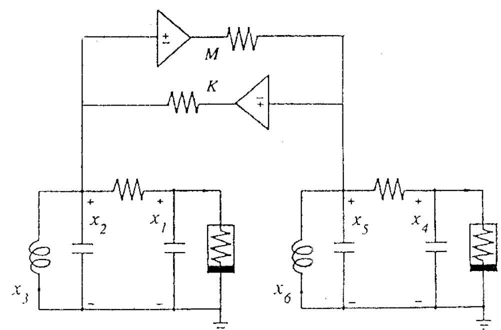

Fig.1 Bidirectionally coupled Chua's circuits

If the parameters K and M are both different from zero, the two Chua's circuits are bidirectionally coupled, whereas if only one of these parameters is equal to zero, the remaining one individualizes the unidirectional coupling between the two circuits.

The proposed approach consists in evaluating the Lyapunov exponents of this network, which has to be decomposed into two subcircuits of dimensions k and $(6-k)$ respectively, where k is the number of positive Lyapunov exponents. These subcircuits are then duplicated and cascaded [3]. Following this procedure, the original circuit (1)-(8), called "drive system", is arranged to generate two "response" or "driven" subcircuits which have to be driven by proper signals [3]. The first response subsystem is represented by the following equations:

$$\dot{x}_2' = x_1 - x_2' + x_3' + K(x_5' - x_2') \tag{9}$$
$$\dot{x}_3' = -\beta x_2' \tag{10}$$
$$\dot{x}_5' = x_4 - x_5' + x_6' + M(x_2' - x_5') \tag{11}$$
$$\dot{x}_6' = -\beta x_5' \tag{12}$$

and is driven by the hyperchaotic signals $x_1(t)$ and $x_4(t)$; the latter response subsystem is expressed by the equations:

$$\dot{x}_1'' = \alpha(x_2' - x_1'' - f(x_1'')) \tag{13}$$
$$\dot{x}_4'' = \alpha(x_5' - x_4'' - f(x_4'')) \tag{14}$$

and is driven by the variables $x_2'(t)$ and $x_5'(t)$. A block diagram illustrating the cascaded synchronizing technique is shown in Fig.2.

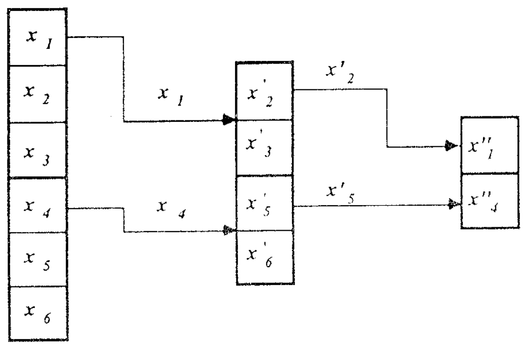

Fig.2 Block diagram illustrating the proposed synchronizing technique

It can be shown [3] that if all conditional Lyapunov exponents, i.e., the Lyapunov exponents of the driven subcircuits, are negative, then the subcircuits are stable and the response variables will synchronize with the corresponding variables of the original circuit (1)-(8).

III. APPLICATION TO A PAIR OF COUPLED CHUA'S CIRCUITS

In this application all numerical computations have been performed using the software INSITE [9].

A. Bidirectional coupling

The following parameter values have been chosen for the two bidirectionally coupled Chua's circuits shown in Fig.1: $K=M=0.02, \alpha=10.00$, $\beta=14.87$, $a=-1.27$, $b=-0.68$ with the following initial conditions: $x_1(0)=0.010$, $x_4(0)=0.011$, $x_2(0)=x_3(0)=x_5(0)=x_6(0)=0$. The following values of the Lyapunov exponents have been found: $\lambda_1 = 0.463$; $\lambda_2= 0.397$; $\lambda_3= 0$; $\lambda_4= 0$; $\lambda_5= -3.483$; $\lambda_6= -3.925$.

In this case, the system evolution is hyperchaotic. The corresponding double-double scroll attractor behaviour is depicted in Fig.3. The method illustrated in the previous section has been applied assuming that the response subsystems parameters have the same values of the corresponding parameters of the drive system, whereas the initial conditions are: $x_1''(0) = 0.011$, $x_4''(0)= 0.012$, $x_2'(0) = x_3'(0) = x_5'(0) = x_6'(0) = 0$. The Lyapunov exponents of the drive system become: $\lambda_1 = 0.454$; $\lambda_2= 0.416$; $\lambda_3= 0$; $\lambda_4= 0$; $\lambda_5= -3.627$; $\lambda_6= -3.937$, whereas the values of the conditional Lyapunov exponents of both the response subsystems have been calculated and are: $\lambda_7 = -0.491$; $\lambda_8= -0.504$; $\lambda_9= -0.522$; $\lambda_{10} = -0.559$; $\lambda_{11}= -2.275$; $\lambda_{12}= -2.345$. It can be observed that the response subsystems are stable, as the conditional Lyapunov exponents are negative. Moreover, since there are two positive Lyapunov exponents for the original circuit (1)-(8), the hyperchaotic signals $x_1(t)$ and $x_4(t)$ have been used as drive variables to synchronize the coupled circuits.

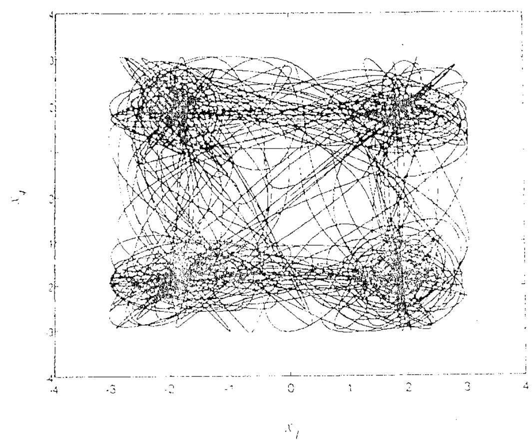

Fig.3 The double-double scroll attractor

Simulation results show that all the state variables of subsystems (9)-(14) synchronize with the corresponding variables of system (1)-(8). As an example, Fig.4 shows the time waveform of the synchronization error $e_5(t) =x_5(t) - x_5'(t)$.

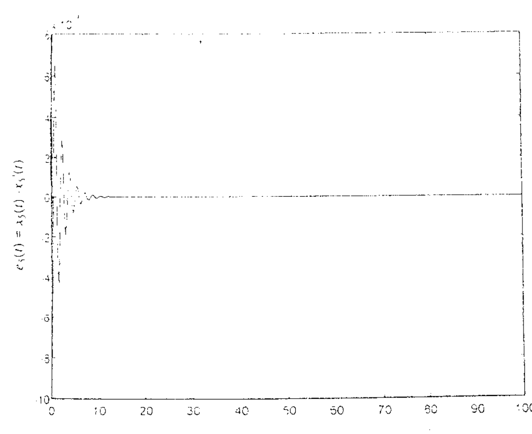

Fig.4 Time waveform of the synchronization error $e_5(t) =x_5(t) - x_5'(t)$

B. Unidirectional coupling

In this case the same parameters and initial conditions of the previous case have been chosen for the two unidirectionally coupled circuits with the only exception of the parameter M which has been considered equal to zero. The following values of the Lyapunov exponents have been calculated: $\lambda_1 = 0.411$; $\lambda_2= 0.415$; $\lambda_3= 0$; $\lambda_4= 0$;

$\lambda_5 = -3.766$; $\lambda_6 = -3.849$. The two unidirectionally coupled Chua's circuits exhibit hyperchaotic behaviours.

Now, by applying the suggested synchronization method, the following Lyapunov exponents for the drive circuit have been found: $\lambda_1 = 0.388$; $\lambda_2 = 0.434$; $\lambda_3 = 0$; $\lambda_4 = 0$; $\lambda_5 = -3.682$; $\lambda_6 = -3.882$. Moreover, the values of the conditional Lyapunov exponents of both the response subsystems are: $\lambda_7 = -0.500$; $\lambda_8 = -0.509$; $\lambda_9 = -0.510$; $\lambda_{10} = -0.535$; $\lambda_{11} = -2.255$; $\lambda_{12} = -2.491$. The response subsystems are stable also in this case. Moreover, since there are two positive Lyapunov exponents for the original circuit (1)-(8), the hyperchaotic signals $x_1(t)$ and $x_4(t)$ have been used as drive variables to synchronize the coupled circuits. Simulation results show that all the state variables of subsystems (9)-(14) synchronize with the corresponding variables of system (1)-(8) also in the case of unidirectional coupling. As an example, Fig.5 clearly shows the time waveform of the synchronization error $e_3(t) = x_3(t) - x_3'(t)$.

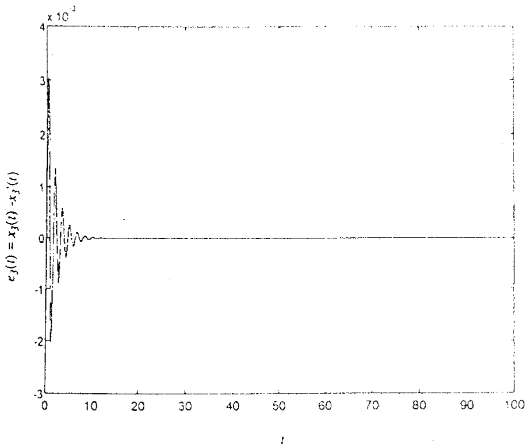

Fig.5 Time waveform of the synchronization error $e_3(t) = x_3(t) - x_3'(t)$

C. Robust synchronization under parameter variations

The aim of this section is to investigate the robustness properties of the proposed synchronization technique with respect to parameter changes. In the preceding section the robustness of the suggested technique in the presence of 10% changes of the initial conditions for the response circuits with respect to the drive one has been considered. However, as is well known, unavoidable fabrication tolerances make impossible to realize identical parameters in real circuits. As a consequence, if a robust synchronization is not guaranteed in this case too, there is a serious risk that the motions of the drive and response systems would be uncorrelated.

Table I shows all the Lyapunov exponents of the bidirectional coupled circuits under a 5% parameter variation of the response subsystems with respect to the nominal values. The perturbed parameters have been chosen both one at a time and grouped.

Table I shows that the conditional Lyapunov exponents of the driven circuit are negative. This guarantees that parameter values need not be exactly the same to perform correctly hyperchaotic synchronization. These results have been confirmed by considering the time behaviours of the synchronization errors.

Analogous behaviour has been found when unidirectionally coupled Chua's circuits are considered. The corresponding Lyapunov exponents under a 5% parameter variation with respect to the nominal values have been reported in Table II.

IV. CONCLUSIONS

In this paper a method for synchronizing two circuits exhibiting hyperchaotic behaviours has been illustrated. To this purpose, an original circuit constituted by two coupled Chua's circuits has been duplicated to generate two cascaded response subsystems. Synchronization is achieved by means of proper synchronizing signals. Several numerical examples have also shown that the suggested technique assures robustness to the synchronization under both parameter and initial condition changes.

TABLE I

LYAPUNOV EXPONENTS OF THE BIDIRECTIONALLY COUPLED CIRCUITS UNDER 5% PARAMETER VARIATIONS WITH RESPECT TO THE NOMINAL VALUES

Perturbed parameter	drive system	response subsystems
1) α	*0.456; 0.406; 0; 0; -3.701; -3.958*	*-0.489; -0.505; -0.523; -0.568; -2.479; -2.556*
2) β	*0.443; 0.414; 0; 0; -3.700; -3.978*	*-0.489; -0.506; -0.522; -0.571; -2.365; -2.440*
3) a	*0.433; 0.407; 0; 0; -3.694; -4.021*	*-0.488; -0.504; -0.524; -0.579; -2.606; -2.633*
4) b	*0.446; 0.399; 0; 0; -3.646; -3.913*	*-0.490; -0.505; -0.522; -0.566; -2.058; -2.144*
5) α, β, a, b	*0.457; 0.425; 0; 0; -3.612; -3.919*	*-0.491; -0.504; -0.524; -0.565; -2.522; -2.698*

TABLE II
LYAPUNOV EXPONENTS OF THE UNIDIRECTIONALLY COUPLED CIRCUITS UNDER 5% PARAMETER VARIATIONS WITH RESPECT TO THE NOMINAL VALUES

Perturbed parameter	drive system	response subsystems
1) α	*0.461; 0.397; 0; 0; −3.651; −3.917*	*−0.499; −0.509; −0.510; −0.544; −2.354; −2.653*
2) β	*0.453; 0.434; 0; 0; −3.694; −3.830*	*−0.499; −0.510; −0.510; −0.535; −2.332; −2.444*
3) a	*0.432; 0.405; 0; 0; −3.671; −3.815*	*−0.500; −0.510; −0.510; −0.545; −2.556; −2.591*
4) b	*0.435; 0.403; 0; 0; −3.623; −3.862*	*−0.499; −0.509; −0.510; −0.544 −2.035; −2.212*
5) α, β, a, b	*0.433; 0.396; 0; 0; −3.732; −3.915*	*−0.499 −0.510; −0.511; −0.545; −2.613; −2.706*

REFERENCES

[1] T. L. Carroll and L. M. Pecora, 'Synchronizing chaotic circuits', *IEEE Trans. Circ. Syst.* , vol.38, no.4, pp. 453-456, 1991.

[2] T. L. Carroll and L. M. Pecora, 'Cascading synchronized chaotic systems', *Physica D* , vol.67, pp. 126-140, 1993.

[3] M. Brucoli, L. Carnimeo, and G. Grassi, 'A method for the synchronization of hyperchaotic circuits', *Int. J. of Bifurcation and Chaos,* in press.

[4] M. J. Ogorzalek, 'Taming Chaos-Part I: Synchronization', *IEEE Trans. Circ. Syst., Part I*, vol.40, no.10, pp. 693-699, 1993.

[5] L. Kocarev, K. S. Halle, K. Eckert, L. O. Chua, and U. Parlitz, 'Experimental demonstration of secure communications via chaotic synchronization', *Int. J. of Bifurcation and Chaos*, vol.2, no.3, pp. 709-713, 1992.

[6] K. M. Cuomo, A. V. Oppenheim and S. H. Strogatz, 'Synchronization of Lorenz-Based chaotic circuits with applications to communications, *IEEE Trans. Circ. Syst., Part II* , vol.40, no.10, pp. 626-633, 1993.

[7] V. S. Anishchenko, T. Kapitaniak, M.A. Safonova and O. V. Sosnovzeva, 'Birth of double-double scroll attractor in coupled Chua circuits', *Phys. Lett. A* , vol.192, pp. 207-214, 1994.

[8] K. Pyragas, 'Predictable chaos in slightly perturbed unpredictable chaotic systems', *Phys. Lett. A* , vol.181, pp. 203- 210, 1993.

[9] T.S. Parker and L.O. Chua, *Practical numerical algorithms for chaotic systems*. New York: Springer-Verlag, 1989.