SELF-ORGANISATION IN ARRAYS OF NONLINEAR SYSTEMS INDUCED BY CHAOTIC PERTURBATION: AN EXPERIMENTAL APPROACH

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ABSTRACT

In this work, the effects of diversity in large arrays of chaotic systems are studied. It is here shown how very regular spatio-temporal patterns are achieved by perturbing a system parameter with a suitable spatial disturbance. Moreover, it is shown that synchronisation and order is better achieved if the variation of the parameter is generated by a chaotic system rather than by a random generator based on a stochastic process with uniform distribution. Our conjecture is then validated by several simulation experiences on long chains of both mechanical (pendula) and electrical (Chua's circuits) systems.

1. INTRODUCTION

Natural systems are often constituted by several nonlinear units connected in complex topologies, in which the information flux is exchanged by local connections. It can be easily observed how natural complex systems are intrinsically adaptive and co-operative. In particular, synchronisation emerges as one of the main issues concerning adaptation and co-operation: objects tend to acquire a common operation regime, seeking to achieve order and harmony in their behaviour, clearly showing their natural tendency to self-organisation.

It is well known that diversity plays a fundamental role in evolution and adaptation in nature. Perhaps, it is this idea that inspired scientists to investigate how to exploit diversity to improve the capability of systems to achieve spatial and temporal regularity. The collective behaviour of an N-dimensional array of identical nonlinear systems has recently become a very interesting matter of investigation, especially concerning Cellular Neural Networks [1,2], and spatio-temporal phenomena [3,4], with applications on several fields, like information processing [5], and biological excitable media [6]. Because of the high dimensionality of such systems, a few analytical studies on synchronisation have been carried out [7]. Nevertheless, many experimental studies are currently in progress, and also a classification of the several phenomena occurring in these systems has been performed [8].

Organisation emerges in the simplest way, showing *coherent* attractors, in which two generic variables denote an identical trend in time. When coherent attractors correspond to groups of neighbouring units, the phenomenon of *clustering* takes place. When organisation does not take place, the most interesting phenomenon is the *spatio-temporal chaos,* in which chaotic trends appear both in time and in space.

The effects of diversity in long chains of oscillators have been recently investigated [3]: an array of 128 identical pendula, denoting spatio-temporal chaos, shows regular spatio-temporal patterns when pendula lengths are perturbed by a uniform, random noise.

In this work, it is shown that diversity generated by chaotic perturbations can improve the capabilities of self-synchronisation of long chains of array of nonlinear systems. Self-organisation is achieved in chaotic system arrays, from an initial state of disorder, providing a chaotic space-variant perturbation to one of the parameters of the system. This assumption has often given better results, in terms of global synchronisation of the array, than the application of a random noise. Moreover, chaos is often removed from the oscillators behaviour. It is worth noting that the capability of removing chaos from a system is the first step towards the possibility of extracting information from chaos, which is a very important issue to build communication systems based on chaotic carriers.

Two examples in which the single element of the array is a well known nonlinear system (pendulum, Chua's circuit) are here reported in order to validate the proposed approach.

2. COLLECTIVE BEHAVIOUR OF AN ARRAY OF PENDULA

In this section we assume that the generic element of the array is a simple forced pendulum, a second order nonlinear system whose dynamic equation is easily derivable from Newton's law:

$$ml_k^2 \frac{d^2\vartheta_k}{dt^2} + \gamma \frac{d\vartheta_k}{dt} = mgl_k sin(\vartheta_k) + \tau sin(\omega t) + \tau' \qquad (1)$$

Equation (1) describes the behaviour of an isolated damped oscillator forced by a bias and a sinusoidal input. If we link together N pendula through elastic springs acting on the rotational axis with constant k in a cascade structure, and assume that $\theta_0=\theta_1$ and $\theta_N=\theta_{N-1}$, the generic equation describing this system is:

$$ml_k^2 \frac{d^2\vartheta_k}{dt^2} + \gamma \frac{d\vartheta_k}{dt} = mgl_k sin(\vartheta_k) + \\ + \tau sin(\omega t) + \tau' + k(\vartheta_{k-1} - \vartheta_k) - k(\vartheta_k - \vartheta_{k+1}) \qquad (2)$$

or , in terms of state equations:

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$$\frac{dx_k}{dt} = y_k$$

$$\frac{dy_k}{dt} = -\frac{\gamma}{ml_k^2} y_k + \frac{g}{l_k} sin(x_k) + \qquad (3)$$

$$+ \frac{k}{ml_k^2}(x_{k+1} - 2x_k + x_{k-1}) + \frac{\tau}{ml_k^2} sin(\omega t) + \frac{\tau'}{ml_k^2}$$

where $x_k=\theta_k$ and $y_k=d\theta_k/dt$.

Fixing $l_k=1$ for each pendulum, and assuming $m=1$, $\gamma=0.75$, $\tau'=0.7155$, $\tau=0.4$, $\omega=0.25$, $k=0.5$, $g=1$, the behaviour of the array is chaotic, as depicted in fig. 1(a), in which angular velocities are coded versus time by a grey scale for each pendulum. Grey scale codes pendula velocities ranging from black (lowest) to light grey (highest). As it can be seen, as time increases, more and more disordered spatio-temporal patterns emerge, denoting a chaotic behaviour, which is confirmed by a positive Lyapunov exponent (0.207290). The prevalence of the dark grey in the grey scale map is due to the d.c. term in the forcing torque.

It has been shown [3] that introducing a random, symmetrical disorder in the length of each pendulum ($l_k \in [0.9, 1.1]$), periodic spatiotemporal patterns can be observed (fig. 1(b)). In this work our analysis deals with the effects induced by the introduction of chaotic disorder. Let us consider the following well known nonlinear system (Chua's system):

$$\dot{x} = \alpha(y - f(x))$$

$$\dot{y} = x - y + z \qquad (4)$$

$$\dot{z} = -\beta y$$

where $f(x) = m_0 x + 0.5(m_0 - m_1)[|x+1| - |x-1|]$.

For suitable values of the parameters α and β the behaviour of the system is chaotic [9], and state variables assume unpredictable values. Taking N samples of $x(t)$ and normalizing their amplitudes to obtain a signal varying into the interval [-0.1, 0.1], it is possible to build a vector of lengths l_k having 10% of "chaotic" noise around the constant value l=1. The parameters of system (4) have been fixed to α= -4.08685, β= -2, m_0=-1.142857, m_1=-0.7142857. The results of simulations show (see fig. 1(c)) that the chaotic variation of this parameter leads the array towards a collective organisation. Pendula in the central region of the array are shyncronised both in space and time, oscillating with the same frequency of the forcing torque, while in the external bands a regular spatial wave is propagated. Therefore, chaotic attractors of the system have been transformed in periodic ones by simply perturbing the system with chaos, but the most important result is that chaos has been removed. This fact is supported by the decreasing trend of the mean value of the leading Lyapunov exponent which reaches the value of 0.016866 for a disorder ranging in the 10% of pendula length. Another parameter that has been taken into account to evaluate the rate of self-organisation of the array is the average Shannon entropy: a high value of this parameter means that the signal is strongly correlated with itself. Table I resumes the significant parameters obtained from experiments, definitely validating our results.

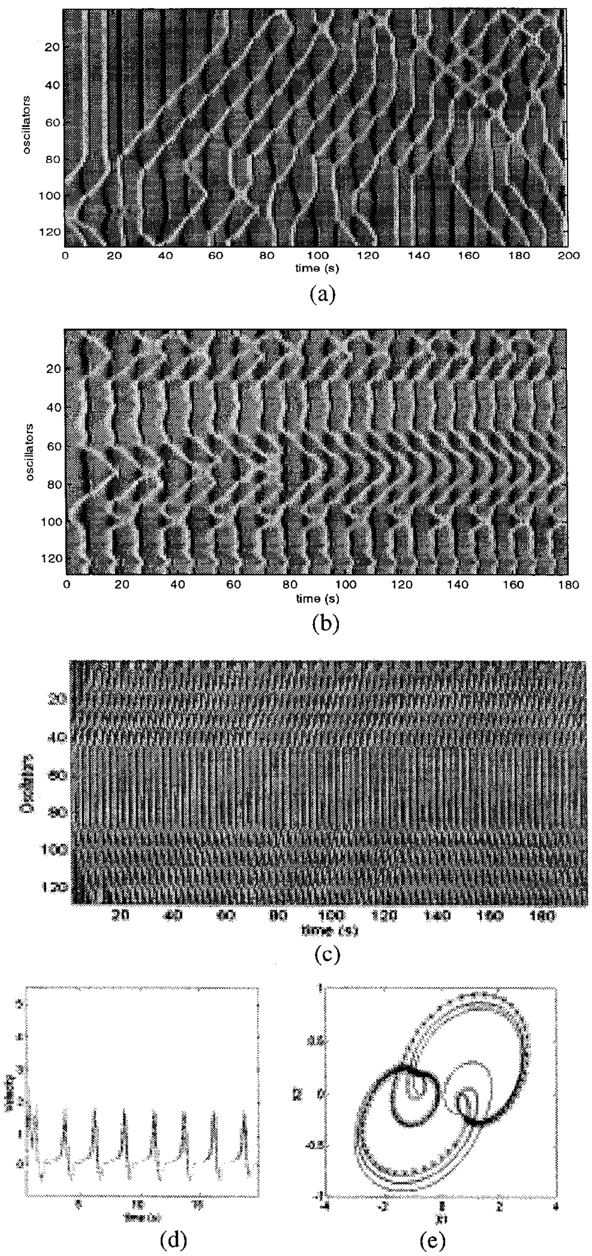


Fig.1: Pendula array experiment. (a) Chaotic behaviour of identical pendula; (b) application of 10% random noise; (c) application of 10% chaotic disorder; (d) single pendula velocities from 80th to 85th pendulum related to experiment (c), (e) portion of the Chua's attractor used to generate chaotic variation on pendula lengths.

Disorder	Avg Leading Lyapunov Exponent	Shannon Entropy
0%	0.2073	5.4869
10% Random	0.0211	8.1840
10% Chaotic	0.0168	9.2388

Table I: Significant experimental parameters versus kinds of disorder introduced for pendula array (2).

3. COLLECTIVE BEHAVIOUR OF AN ARRAY OF CHUA'S CIRCUITS

Many studies about chains of Chua's circuits have been carried out in the last years, concerning several aspects, like investigation on chaotic behaviour [12, 13], propagation of impulsive information [10], and formation of spiral waves in a 2D circuit matrix [14]. All the works performed agree on the fact that, for arrays constituted by identical circuits, global behaviour is strongly affected by changes in the connection coefficient, denoting in a spatio-temporal context all the phenomena that can be observed in the single circuit: equilibrium states, limit cycles, and, obviously, spatio-temporal chaos. In this section, we repeat the same experience performed with pendula array: a Chua's circuit is used as a basic cell to build a linear array in which adjacent units are coupled through linear resistors, and both a random and a chaotic variation on a circuit parameter are introduced. As it can be observed, very regular spatio-temporal patterns emerge only when the imposed perturbation is chaotic, confirming the conjecture that chaos can help systems to achieve order and synchronisation. The following equations describe the behaviour of the k-th unit, in the well known dimensionless form [9]:

$$\begin{aligned} \dot{x}_k &= \alpha(y_k - h(x_k) + D[x_k - y_{k+1}]) \\ \dot{y}_k &= x_k - y_k + z_k + D[y_k - x_{k-1}] \\ \dot{z}_k &= -\beta y_k \end{aligned} \quad (5)$$

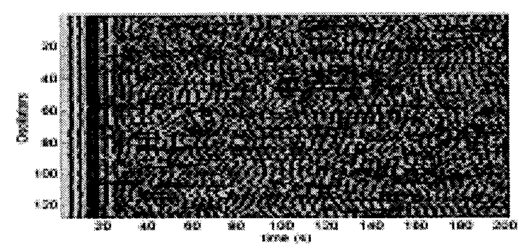
According to [11], the nonlinearity of Chua's circuit can be approximated by the cubic:

$$h(x) = \frac{-mx + kx^3}{G} \quad (6)$$

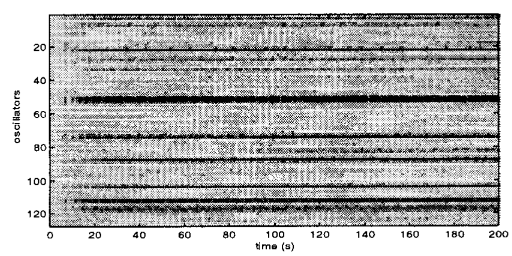
Fixing α=6, β=8, m=0.8, k=0.04, G=0.7, the single, uncoupled unit shows a chaotic behaviour. Coupling the circuits with a coefficient D=0.05, the collective behaviour of the array denotes a chaotic trend, as it is shown in figure 2(a), in which the value of the first state variable is coded versus time by the usual grey scale map (cf. section 2).

The first step to achieve the synchronisation of the array is choosing the parameter to perturb, by making a formal comparison between (3) and (5). A suitable double transform, discrete in space and continuous in time, has been applied in order to reduce the two arrays to scalar Lur'e systems [12]. The first Markov's parameters of both the linear parts of the two systems have then been computed, and a formal correspondence between l in pendula array and α in Chua's circuits array has been found. Thus, it has been conjectured that a suitable chaotic noise introduced in the α parameter would have produced the same synchronising effects as in the case of pendula.

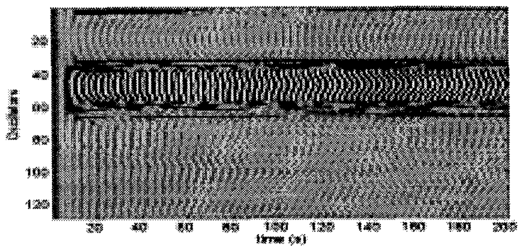
In order to investigate how the collective behaviour of the array is affected by chaotic changes in a circuit parameter, the α parameter in array (5) has been changed according to a chaotic temporal series provided by system (4), in which the parameters have been fixed to α=35, β=75. The output of the control circuit is normalised in order to obtain a maximum change of ±20% with respect to the nominal value. In figure 2(b-e), colour maps of state variable x, together with the portion of chaotic attractor used to generate the variation of α, are shown. Some synchronised state variable are also reported. The regular spatio-temporal patterns denote two main behaviours that can be clearly distinguished in two different areas of the colour map. As in pendula array, spatio-temporal chaos is removed.



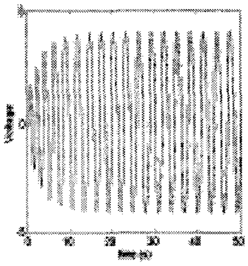
(a)



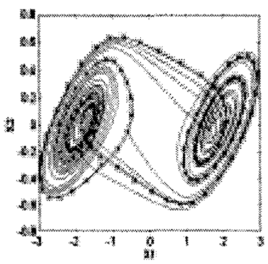
(b)



(c)



(d)



(e)

Fig.2: Chua's circuits array experiment. (a) Chaotic behaviour of identical chua's circuits; (b) application of 20% random noise; (c) application of 20% chaotic disorder; (d) single state variables from 44^{th} to 48^{th} circuit related to experiment (c), (e) portion of the Chua's attractor used to generate chaotic variation on α parameter.

5. CONCLUSIONS

In this work, a novel approach to the control of highly dimensional spatio-temporal systems has been depicted. The considered systems are long arrays of both mechanical (pendula) and electrical (Chua's circuits) chaotic oscillators. It has been shown how diversity can help in achieving self-organisation in large arrays of oscillators, by introducing a slight spatial perturbation in their parameters. Several simulation experiments and numerical characteristic parameter (Lyapunov exponents, Shannon entropy) evaluations have been carried out, confirming the main result that synchronisation is dramatically improved when diversity is generated by a deterministic chaotic system, indicating that chaos itself could be a fundamental paradigm for controlling complex chaotic dynamics.

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