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Design of globally stable controllers for a class of chaotic systems

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A simple method to design chaos suppressors for a class of chaotic systems that can be decomposed into a linear subsystem and a nonlinear subsystem whose states can be decoupled is presented. The method yields a constant state feedback controller and global stability of the controlled system. The proposed methodology is demonstrated on three well-known chaotic systems, the third-order Chua circuit, the fourth-order hyperchaotic Chua circuit and the hyperchaotic Rössler oscillator.

1. Introduction

Many natural and mechanistic systems as well as certain systems that have been modelled by a reduced set of nonlinear differential equations have been shown to exhibit chaotic behaviour (Genesio *et al*. 1995, Alvarez *et al*. 1997). Controlled chaotic systems are being used as encoders/decoders in communications. It is not surprising, therefore, that the control of chaos has been receiving considerable attention of late. In designing chaos controllers (Torres and Aquirre 1999), emphasis is not focused on the temporal behaviour of the system but on its stability properties and the manner in which convergence to desired non-chaotic attractors is achieved (Lai 1996). From a theoretical viewpoint, the suppression of chaos is equivalent to feedback stabilization and is related to synchronization (Kocarev and Parlitz 1995).

Hwang *et al*. (1996) and Hsieh *et al*. (1999) (see also the bibliography therein) have proposed a method for suppressing chaos in the Chua circuit and the hyperchaotic Rössler oscillator using linearization around their point attractors, an approach that guarantees only *local* asymptotic stability of the controlled system. Similarly, Torres and Aguirre (1999) focus on the Chua circuit exclusively and *local* stability through the use of a manifold consisting of equilibrium points of the controlled system. No evolution takes place on this manifold while the basic objective is to drive every initial state outside the equilibrium manifold to a point contained on it.

Referring to synchronization of chaotic systems and not their stabilization, Kocarev and Parlitz (1995) decompose chaotic systems into active and passive subsystems. Active/passive decomposition is a reformulation of the original autonomous system state equations into a form in which the system is partitioned into two subsystems, the original chaotic system (the *passive* subsystem) one parameter of which is made time varying. The manner in which this parameter (or *active* subsystem) is made to vary with time, constitutes the control law. The object here is to force the trajectories of the controlled system from any two initial states to become identical by eliminating of the error between them asymptotically.

An entirely different approach was taken by Tian (1999) who proposed driving the controlled system trajectory towards an invariant manifold and thereafter to a desired point attractor. In their approach, the number of control inputs to the chaotic system must equal the difference between the order of the initial chaotic system and the dimension of the invariant manifold (i.e. the co-dimension of the invariant manifold). Furthermore, the state feedback used alters the nature of the original chaotic systems drastically (Yu 1996).

The present paper proposes a new systematic approach to the design of controllers capable of suppressing chaos for a wide class of chaotic systems that includes most of the well-known paradigms. In the proposed approach, the original chaotic system is partitioned into two subsystems: a *nonlinear subsystem* and a *linear subsystem* whose states can be disassociated from the nonlinear component. As in Tian (1999), the primary objective of the proposed approach is to drive

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the trajectory of the controlled system from *any* arbitrary initial state towards an invariant manifold and thereafter asymptotically to some desired stable point attractor. Unlike Tian, however, the number of inputs required to suppress chaos is less than the co-dimension of the invariant manifold. No assumptions are made on the functional derivatives of the system, while the nonlinear nature of the original chaotic system is preserved. Moreover the resultant controllers are *linear* and involve constant state feedback (Kailath 1980, Friedland 1995, Antsaklis and Michel 1997, Sarachik 1997, Chen 1998) and guarantee *global stability*.

The proposed method is demonstrated on three benchhmark chaotic paradigms: the classical Chua circuit, the hyperchaotic four dimensional Chua circuit and the hyperchaotic Rössler oscillator. The procedure is applicable to any set of system parameters. In all three paradigms, one or at most two inputs are shown to be sufficient to drive the system from any initial state to an invariant manifold of co-dimension two or three asymptotically and thereon to stable equilibrium points asymptotically, thus assuring global asymptotic stability.

1.1. *Proposed controller design methodology*

Consider the class of chaotic systems (S) whose behaviour is governed by the state equations

$$\dot{x} = f(x, u), \tag{S}$$

where $x \in \mathbf{R}^n$ is the state vector of the system, $u \in \mathbf{R}^m$ is the control vector and $f : \mathbf{R}^n \times \mathbf{R}^m \rightleftarrows \mathbf{R}^n$ is a smooth mapping, and that the chaotic system can be decomposed into a linear subsystem (1) and a nonlinear subsystem (2) involving the partial states of the system as follows:

$$\dot{\xi}_1 = f_1(\xi_1, \xi_2, u) = [A_1 | A_2] \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + Bu \tag{1}$$

$$\dot{\xi}_2 = f_2(\xi_1, \xi_2, u), \tag{2}$$

where $\xi_1 = (x_1, \ldots, x_r)^{\mathrm{T}}$, $\xi_2 = (x_{r+1}, \ldots, x_n)^{\mathrm{T}}$ are the partial state vectors of the linear and nonlinear subsystem respectively. To cast the controlled system into the linear/nonlinear form, it may prove necessary to make appropriate transformations on the original state variables. A number of classical paradigms of chaotic systems that belong, or can be transformed, to this class of systems are considered in the subsequent application examples.

The first objective in the proposed design methodology is to disassociate the state equations of the linear subsystem from the partial state equations of the nonlinear subsystem. To this end, a linear law is proposed in which the control vector u is made equal to a constant u_0 plus a term proportional to the state of the system, i.e.

$$u(\xi_1, \xi_2) = Kx + u_0 = [K_1 | K_2] \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + u_0.$$

Substituting in (1), it follows that the linear subsystem is described by:

$$\dot{\xi}_1 = f_1(\xi_1, \xi_2, u) = [A_1 + BK_1 | A_2 + BK_2] \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + Bu_0.$$

To achieve the first objective, the elements of the submatrix K_2 must be chosen so that the following condition is met:

$$A_2 + BK_2 = 0.$$

The state equations of the system thus reduce to:

$$\dot{\xi}_1 = (A_1 + BK_1)\xi_1 + Bu_0 = \hat{A}\xi_1 + Bu_0$$

$$\dot{\xi}_2 = f_2(\xi_1, \xi_2, u(\xi_1, \xi_2)).$$

Recalling the definition of a submanifold.

Definition 1: A submanifold M of the state space is an invariant manifold of the system (S) if every trajectory of (S) starting at an arbitrary point on M remains on it for all time.

It is evident that a hyperplane E defined by $\xi_1 = \xi_{1e}$, where ξ_{1e} is an equilibrium point of (1), is an invariant manifold of (S). Thus, provided that ξ_{1e} is globally asymptotically stable, then every initial state of the controlled system (S) tends to E asymptotically and every trajectory of the system converges to a trajectory of the partitioned subsystem 2 asymptotically (Isidori 1989). Indeed, following asymptotic convergence to the submanifold E, the subsequent motion of the controlled system is uniquely specified by the restricted system

$$\dot{\xi}_2 = f_2(\xi_{1e}, \xi_2, u(\xi_{1e}, \xi_2)), \tag{3}$$

which evolves on E entirely. Thus if every initial state of three converges to a point attractor, then the same will be true for the overall system (S).

The next step in the proposed approach involves selection of the elements of the constant feedback matrix K_1 so that the controlled linear subsystem is asymptotically stable [12]. This is possible provided the linear subsystem is controllable (or at least stabilizable). In this case, the eigenvalues $\lambda(\hat{A})$ can be placed arbitrarily through suitable choice of the elements of K_1 using established pole-placement techniques. The procedure on selecting the eigenvalues is well known and is covered in a host of texts on linear systems (Kailath 1980, Friedland 1995, Antsaklis and Michel 1997, Sarachik 1997, Chen 1998) which also refer to the relationship between the eigenvalues and the control effort required.

Obviously, the speed of response of the controlled system is inversely related to the control effort.

The final step in the design of the controller is to select the constant u_0 that yields the desired equilibrium points of the controlled system. Although the dimensions of the partial state vectors ξ_1, ξ_2 can be arbitrary, in all three paradigms that follow, the dimension of the invariant manifold (i.e. the order of the nonlinear subsystem 2 is equal to unity, thereby considerably simplifying the stability analysis of the nonlinear subsystem. Clearly stability analysis increases in complexity as the order of the nonlinear subsystem increases.

Remark 2: In cases where the chaotic system (S) cannot be cast in the desired form (1), (2) then the concept of partitioning the system into subsystems that lead to stable invariant manifolds can still be useful. In this case however, the resultant controller may be more complex, possibly involving nonlinear feedback.

2. Chua circuit

The Chua circuit, shown in figure 1, is a classical paradigm of a simple, low-order nonlinear system consisting of passive and active components (Zhong and Ayron 1985) which exhibits chaotic behaviour (Chua *et al.* 1986, Madan 1993). Suppression of chaotic behaviour can be affected by simply placing a controlled current source u in parallel with the linear resistor. The controlled Chua circuit is now governed by the state equations:

$$\begin{aligned} C_1\dot{v}_1 &= -v_1 + v_2 - \phi(v_1) - u \\ C_2\dot{v}_2 &= v_1 - v_2 - i + u \\ L\dot{i} &= v_2, \end{aligned}$$

which are expressed in terms of the capacitor voltages v_1, v_2 and the inductor current i. Without loss of generality, the linear resistor is assigned a value of unity. Without loss of generality, the five-segment piecewise-linear characteristic of the nonlinear resistor used in the original Chua circuit has been replaced by a current voltage characteristic defined by a cubic polynomial $\phi(v) = av^3 - bv$, where $a, b > 0$, which has been shown not to alter the qualitative behaviour of the circuit significantly (Hwang *et al.* 1996, and references therein).

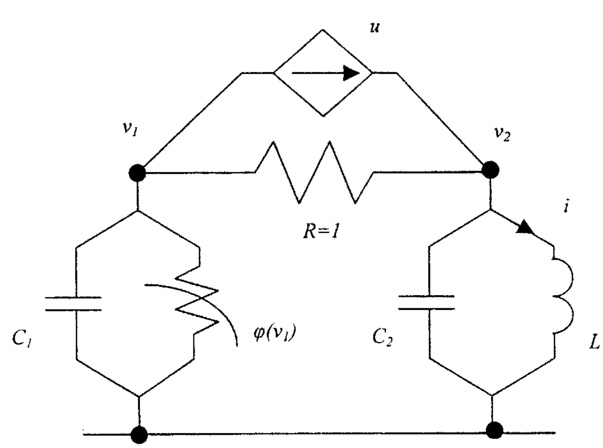

Figure 1. Chua circuit.

Let the control variable u be equal to the linear combination of the states of the circuit plus a constant, i.e.

$$u = \alpha_1 v_1 + \alpha_2 v_2 + \rho i + u_0.$$

This control law can be implemented by means of a constant current source in parallel with two voltage controlled current sources (VCCS) and a current controlled current source (CCCS) or alternatively, a summing amplifier or ADA-LINE. The weights α_1 and α_2 are non-dimensional amplification factors and ρ is the trans-resistance of the controlled source. The state equations of the controlled circuit thus become:

$$\begin{aligned} C_1\dot{v}_1 &= -(1+\alpha_1)v_1 - \phi(v_1) + (1-\alpha_2)v_2 - \rho i - u_0 \\ C_2\dot{v}_2 &= (1+\alpha_1)v_1 - (1-\alpha_2)v_2 - (1-\rho)i + u_0 \\ L\dot{i} &= v_2. \end{aligned} \tag{4}$$

Following the foregoing design procedure, we first disassociate the state equations of the linear subsystem comprising the states (v_2, i) from the state equation for v_1. This can be achieved simply by letting $1 + \alpha_1 = 0$, whereupon the controlled system reduces to

$$\begin{aligned} C_1\dot{v}_1 &= -\phi(v_1) + (1-\alpha_2)v_2 - \rho i - u_0 \\ C_2\dot{v}_2 &= -(1-\alpha_2)v_2 - (1-\rho)i + u_0 \\ L\dot{i} &= v_2. \end{aligned} \tag{5}$$

The characteristic polynomial of the linear subsystem is simply:

$$\lambda^2 + \frac{(1-\alpha_2)}{C_2}\lambda + \frac{(1-\rho)}{LC_2} = 0.$$

The straight line λ defined by $v_2 = 0$, $i = i_e$ is an invariant manifold of (5). It is readily shown that the linear subsystem is completely controllable, implying that the eigenvalues of the controlled system can be placed arbitrarily through suitable choice of the controlled source parameters ($\alpha_2, \rho < 1$). Assignment of stable eigenvalues implies that the equilibrium state $(0, i_e)$, which is dependent on the bias current u_0 (since $i_e = u_0/1 - \rho$). The linear subsystem is thus globally asymptotically stable. Hence every trajectory of (5) with initial conditions anywhere on the set $\mathbf{R}^3 - \gamma$ tends to a trajectory on γ asymptotically. Restraining the system (5) to the invariant manifold γ thus yields the simple scalar state equation of the nonlinear subsystem:

$$C_1\dot{v}_1 = -\phi(v_1) - i_B. \qquad (6)$$

The number of equilibrium points and their location clearly depend on the constant $i_B = \rho i_e + u_0$. The critical values of i_e at which bifurcation occurs are given by

$$i_{ec} = \pm 2a\left(\frac{b}{3a}\right)^{3/2}.$$

For each i_e the phase portrait of (6) is one of the four shown in figure 2. It follows that for any arbitrary equilibrium state i_e, the trajectories of (6) are either constant or converge to a point attractor, implying that the temporal responses of the controlled chaotic system (5) will also be constant and will converge to the point attractor $(v_{1e}, 0, i_e)$.

In summary, the controlled Chua circuit has either one or two coexisting asymptotically stable equilibrium states and for any arbitrary initial state, the trajectory converges asymptotically to one of the desired point attractors. The proposed linear controller affects both the number and location of the point attractors. It is obvious that the rate of convergence of the trajectory can be assigned arbitrarily.

Remark 3: It is noted that where more than one attractor exists, the basin of attraction of each attractor cannot be specified. A local estimate can be obtained using Lyapunov functions for the linearized system in the steady state. In general, even though every trajectory is known to converge, the specific attractor to which a particular trajectory will converge cannot be predetermined in a global sense.

Remark 4: In implementing the control law, we can make use of the fact that

$$i = \frac{1}{L}\int_0^t v_2\,\mathrm{d}\tau + i_0$$

whereupon the control law has the familiar proportional plus integral (PI) plus a constant form:

$$u = -v_1 + \left(\frac{\rho}{L}\int_0^t v2\,\mathrm{d}\tau + \alpha_2 v_2\right) + \left(\frac{\rho}{L}\,i_0 + u_0\right).$$

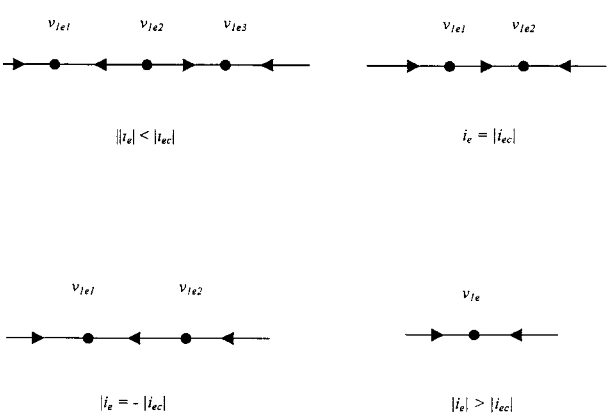

Figure 2. Phase portraits and bifurcation points for the Chua circuit.

2.1. *Application example*

By way of example of the proposed design procedure, consider the Chua circuit shown in figure 1 with $C_1 = 0.1$ F, $C_2 = 1$ F and $L - \frac{7}{100}$ H. Assume that the characteristic of the nonlinear resistor is given by a cubic polynomial with coefficients $a = \frac{2}{7}$, $b = \frac{8}{7}$. The eigenvalues of the linear subsystem are placed at -2 and -3 by setting the controller weights $\alpha_2 = -4$ and

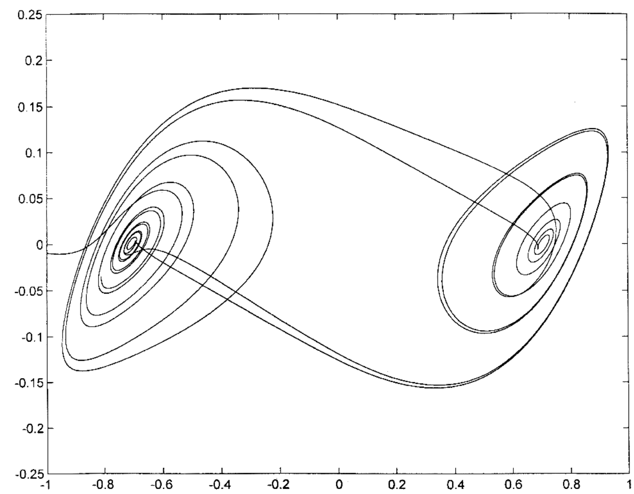

Figure 3a. State trajectory of the uncontrolled Chua circuit with initial states (-3, 0.1, -1) in the (v_1, v_2) plane.

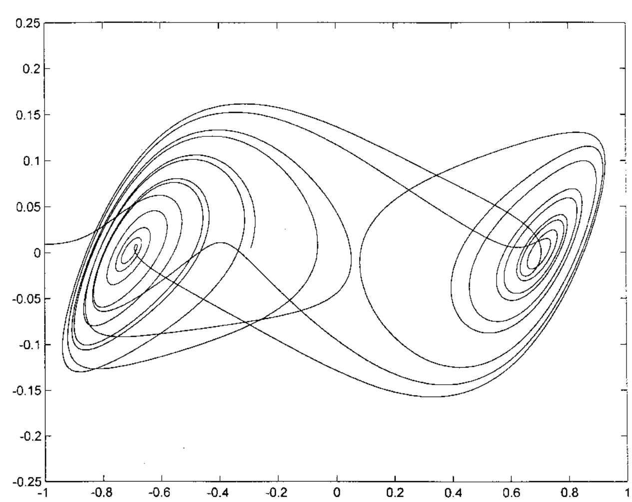

Figure 3b. State trajectory of the uncontrolled Chua circuit with initial states (-3, 0.1, -1.1) in the (v_1, v_2) plane.

$\rho = 0.58$. Letting $u_0 = 0.37$, the equilibrium state is placed at $|i_{ec}| = |u_0/\rho - 1| = 0.88$. Thus the linear law

$$u = -v_1 - 4v_2 + 0.58i - 0.37$$

guarantees both suppression of chaotic behaviour as well as asymptotic convergence to the desired equilibrium point.

Figure 3a portrays the trajectory in the (v_1, v_2) plane of the uncontrolled Chua circuit with initial states (−3, 0.1, −1) while figure 3b shows the trajectory when the initial state is perturbed slightly to (−3, 0.1, −1.1). Figure 4a shows the states of the circuit and the corresponding control law (or control policy) $u(t)$ required to control chaos, starting from an initial state (−3, 0.1, −1) while figure 4b shows the case for an initial state (−3, 0.1, −1.1). A slight perturbation in the initial inductor current $i(0)$ is thus seen to affect the final value $v_1(\infty)$ dramatically.

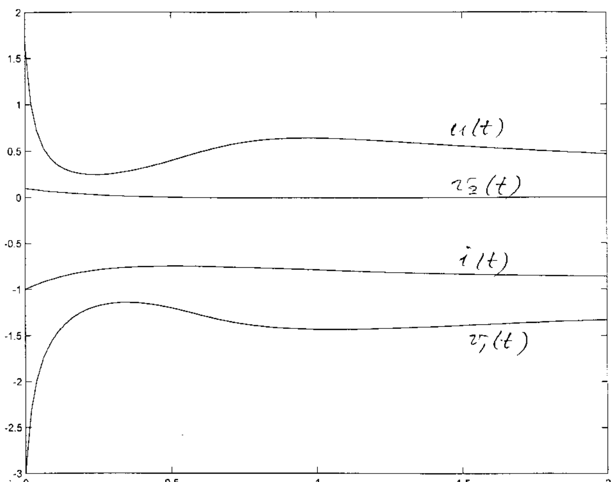

Figure 4a. Temporal responses of the controlled Chua circuit with initial states (−3, 0.1, −1).

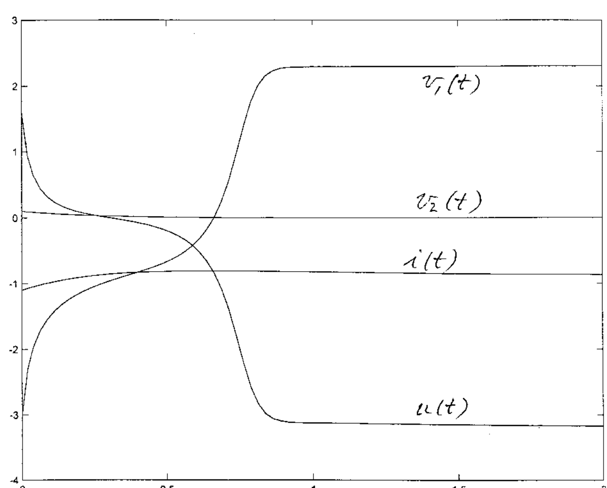

Figure 4b. Temporal responses of the controlled Chua circuit with initial states (−3, 0.1, −1.1).

3. Hyperchaotic Chua circuit

Figure 5 shows the fourth-order hyperchaotic Chua circuit (Matsumoto *et al.* 1986, Galais 1999), which possesses a chaotic attractor with two positive Lyapunov exponents. Unlike the first paradigm, this circuit contains *two* controlled voltage sources u_1 and u_2. The dynamic behaviour of the circuit is governed by the four state equations:

$$C_1\dot{v}_1 = \phi(v_2 - v_1) - i_1$$
$$C_2\dot{v}_2 = -\phi(v_2 - v_1) - i_2$$
$$L_1\dot{i}_1 = v_1 + i_1 + u_1$$
$$L_2\dot{i}_2 = v_2 + u_2.$$

As in the previous example, the objective is to drive the trajectory of the system to a point attractor asymptotically from any arbitrary initial state.

In establishing the required control law, we let $C_2 = \mu C_1$ and apply the coordinate transformations $w_1 = -v_1 + v_2$ and $w_2 = v_1 + \mu v_2$ so that the nonlinear function $\phi(.)$ appears in only one of the state equations. The behaviour of the transformed hyperchaotic Chua circuit can then be expressed in terms of the transformed state equations:

$$\begin{aligned} C_2\dot{w}_1 &= -(1+\mu)\phi(w_1) + \mu i_1 - i_2 \\ C_1\dot{w}_2 &= -i_1 - i_2 \\ L_1\dot{i}_1 &= -\frac{\mu}{1+\mu}w_1 + \frac{1}{1+\mu}w_2 + i_1 + u_1 \\ L_2\dot{i}_2 &= \frac{1}{1+\mu}w_1 + \frac{1}{1+\mu}w_2 + u_2, \end{aligned} \quad (7)$$

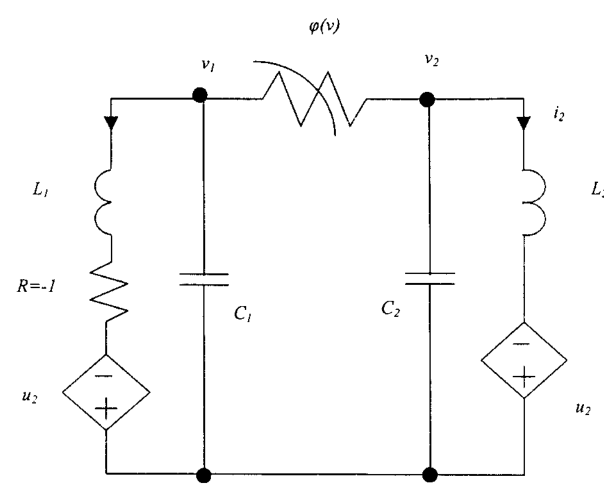

Figure 5. Hyperchaotic Chua circuit.

where the state vector now has components $(w_1\ w_2\ i_1\ i_2)$.

Consider the linear multivariable control law:

$$u_1 = \alpha_{11}w_1 + \alpha_{12}w_2 + \rho_{11}i_1 + \rho_{12}i_2 + u_{01}$$

$$u_2 = \alpha_{21}w_1 + \alpha_{22}w_2 + \rho_{21}i_1 + \rho_{22}i_2 + u_{02},$$

which can be implemented using two VCVS, two CCVS and a constant voltage source. Setting

$$\alpha_{11} = \frac{u}{1+\mu} \quad \text{and} \quad \alpha_{21} = -\frac{1}{1+\mu}$$

leads to an independent linear subsystem involving the last three state equations, which can easily be shown to be completely controllable for any set of system parameters. Controllability implies that through suitable choice of the remaining parameters of the controlled sources, it is possible to place the eigenvalues of the linear subsystem at any desired location. The voltages of the constant sources u_{01}, u_{02} directly effect the location of the attractor (w_{2e}, i_{1e}, i_{2e}). The line defined by $(w_2, i_1, i_2) = (w_{2e}, i_{1e}, i_{2e})$ is an invariant manifold of (7) on which the temporal behaviour of the circuit is determined solely by the nonlinear differential equation:

$$C_1\dot{w}_1 = -(1+\mu^{-1})\phi(w_1) + i_0,$$

where

$$i_0 = (i_{1e} - \mu^{-1}i_{2e} \tag{8}$$

The stability and bifurcation analysis presented in the previous paradigm can be applied here with minor modifications. The behaviour of (8) is clearly related to i_0. Figure 2 shows the four possible phase portraits for the system, while the critical values of i_0 are given by:

$$i_{0c} = \pm 2a\left(\frac{b}{3a}\right)^{3/2}\left(\frac{1+\mu}{\mu C_1}\right).$$

It is concluded that the proposed control law drives *every* initial state of the overall system to a point attractor asymptotically. Note that in this case the codimension of the invariant manifold is three whereupon two controlled sources are required to suppress the chaotic behaviour of the circuit.

3.1. *Application example*

Consider the case where $C_1 = 0.5$ F, $C_2 = 0.05$ F (whence $\mu = 0.1$), $L_1 = 1$ H, $L_2 = 2/3$ H and $\phi(v) = v^3 - 3v$. It follows that the critical values of $i_0 = 2(i_{1e} - 10i_{2e})$ are $i_{0c} = \pm 44$. Assume the transformations $w_1 = v_2 - v_1$ and $w_2 = v_1 + \mu v_1$ the multivariable control law becomes:

$$u_1 = 0.0909w_1 - 2.9091w_2 - 28i_1 + 14i_2 + 123$$

$$u_2 = -0.9090w_1 - 2.9091w_2 - 28i_1 + 14i_2 + 126.$$

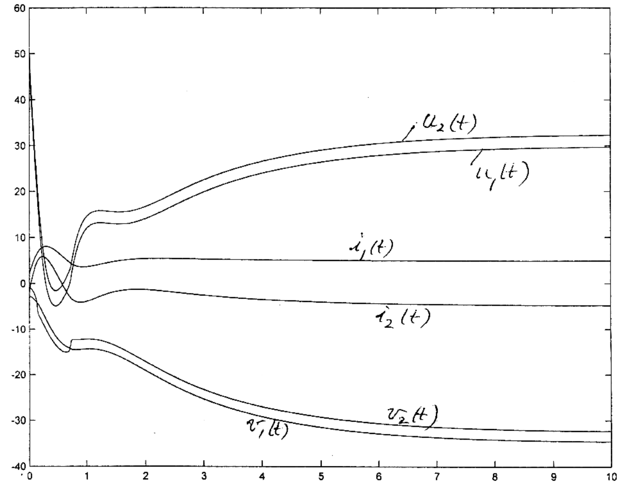

Figure 6. Temporal responses of the controlled hyperchaotic Chua circuit.

The eigenvalues of the linear subsystem are placed arbitrarily at -1, -2, -3 and the globally asymptotically stable equilibrium point is at (0, 3, -3) for which $i_0 = 66 > |i_{0c}|$. This implies that a single globally asymptotically stable point attractor exists for the overall system. In implementing the controller, the concept used in the first paradigm can also be used, leading to a multivariable proportional plus integral law. Figure 6 shows typical temporal responses of the controlled circuit, starting from arbitrary initial states.

4. Hyperchaotic Rössler oscillator

The Rössler oscillator is a fourth-order system that possesses a chaotic attractor with two positive Lyapunov exponents. See Hwang *et al.* (1996) and Rössler (1979) and the references therein for applications and analysis of the oscillator as well as chaos control techniques that have been proposed for this system.

Shown below is that the chaotic behaviour of the oscillator can be controlled by using only one control variable u. The state equations of the oscillator can then be stated as:

$$\dot{x} = -y - z + 2u$$

$$\dot{y} = x + 0.25y + w$$

$$\dot{z} = 3 + xz$$

$$\dot{w} = -0.5z + 0.05w + u.$$

Following the procedure outlined above, consider the linear control law:

$$u = \alpha_z z + \alpha_x x + \alpha_y y + \alpha_w + u_0.$$

Setting $\alpha_z = 0.5$ yields the desired state decomposition:

$$
\begin{aligned}
\dot{x} &= 2a_x x - (1 - 2a_y)y + 2a_w w + 2u_0 \\
\dot{y} &= x + 0.25y + w \\
\dot{z} &= 3 + xz \\
\dot{w} &= a_x x + a_y y(0.05 + a_w)w + u_0.
\end{aligned}
$$

The linear subsystem involving the partial state variables (x, y, w) can be shown to be completely controllable, implying that the controlled system eigenvalues can be arbitrarily placed through appropriate choice of the triplet parameters $(\alpha_x\ \alpha_y\ \alpha_w)$. Stable eigenvalues lead the trajectory of the subsystem from *any* arbitrary initial state to the invariant manifold E defined by $x = x_e$, $y = y_e$, $w = w_e$, where (x_e, y_e, w_e) is the steady-state of the linear subsystem. Once on the invariant manifold E the subsequent evolution of the system is determined by the scalar state equation:

$$\dot{z} = x_e z + 3.$$

Thus, provided that $x_e < 0$, it is evident that only one globally asymptotically stable equilibrium can exist. The bias term u_0 must thus be selected so that the inequality is satisfied. Subject to these conditions, every trajectory of the controlled system converges asymptotically to the attractor at $(x_e, y_e, -3/x_e, w_e)$. It is noted once again that the proposed controller preserves the nonlinear nature of the uncontrolled system. Here, a single control parameter clearly suffices to ensure global asymptotic stability using an invariant manifold of co-dimension three.

4.1. *Application example*

Consider the linear control law

$$u = 0.5z + 0.0455x - 3.8527y - 6.3909w + 3$$

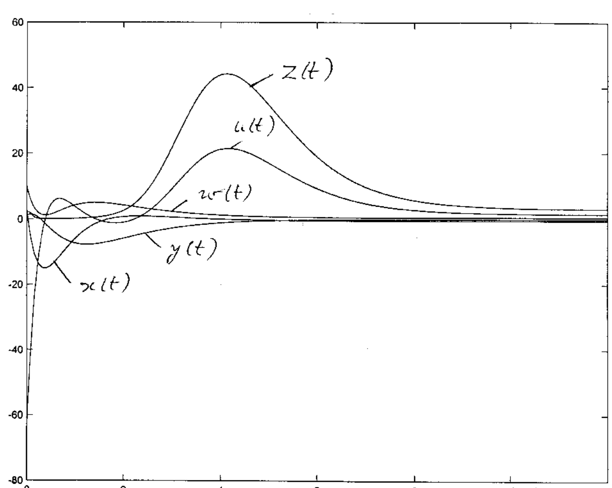

Figure 7. Temporal responses of the controlled hyperchaotic Rössler circuit.

whose weighting coefficients (gains) have been selected so that the eigenvalues of the closed linear subsystem are placed at -1, -2, -3. The constant $u_0 = 3$ has been selected so that the globally asymptotically stable equilibrium point of the linear subsystem is at $(-0.4875, -0.05, 0.5)$. Since $x_e = -0.4875 < 0$ it follows that the closed system has a single globally asymptotically stable point attractor at $(-0.4875, -0.05, 6.1538, 0.5)$. Finally, figure 7 shows temporal responses for the controlled case for an arbitrary initial state.

5. Conclusions

A simple and systematic procedure was presented for the design of controllers for a class of chaotic systems that can be decomposed into a linear and a nonlinear subsystem. The first objective was to drive the trajectory of the chaotic system towards an invariant manifold from which subsequent asymptotic behaviour can be guaranteed. This behaviour is achieved through the use of constant state feedback. The design procedure assures that any arbitrary initial state will converge to a specified point attractor, implying *global asymptotic stability* while the rate of convergence to the attractors is dependent on the choice of the controller parameters. The number and location of the equilibrium points of the controlled system can be predefined. The proposed design procedure is illustrated by way of three paradigms of well-known chaotic systems.

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