A Study of the Chaotic Phenomena in Chua's Circuit

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ABSTRACT

An extensive research is made on the chaotic phenomena in Chua's circuit. The double scroll can be obtained not only by changing one of R, C_1, C_2 and L, but also by changing the op-amp's voltage of the two-terminal nonlinear resistor. Moreover, in Chua's circuit there exists a rule: period-chaos-period plus (minus) 1. This is confirmed by adjusting the op-amp's voltage, resistance, capacitance or inductance monotonously. Besides, as a comparison, the author presents the other simplest 3-order circuit following the period plus (minus) 1 rule from starting period 1.

I. INTRODUCTION

Chua's circuit is made of 4 linear passive elements (L,R,C_1 and C_2) and 1 nonlinear active 2-terminal resistor characterized by a 5-segment piecewise linear v-i characteristic, as shown in Fig.1 [1], [2].

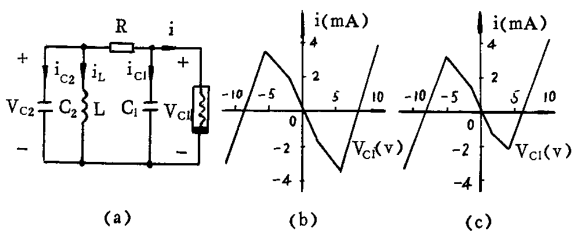

Fig.1. (a) Chua's circuit
(b) symmetric v-i characteristic
(c) asymmetric v-i characteristic

Three chaotic attractors had been observed by Matsumoto via computer simulation [1], their experimental confirmation had already been reported [2], and some bifurcation and chaotic phenomena had been measured from Chua's circuit [3]. Later the authors of [4] confirmed that these attractors have a macroscopic "double scroll" structure. The author of [5] discussed Chua's Circuit Family.

The author in this paper further discovered that all these attractors can also be obtained by adjusting the op-amp's voltage of the 2-terminal nonlinear resistor. Moreover, in Chua's circuit there exists a rule: period-chaos-period plus (minus) 1. This is confirmed by adjusting the op-amp's voltage, R,C_1,C_2 or L monotonously.

As a comparison, the author presents a new circuit following the rule.

II. THE NONLINEAR RESISTOR & 10 DOUBLE SCROLLS

The nonlinear resistor uses 2 op-amps, 8 linear resistors and 2 regulated power supplies for the op-amps [2]. Fig.1(b) and (c) are the v-i characteristics at different +Vcc {in (b)+Vcc=11v, in (c) +Vcc=7v} and the same -Vcc=-11v.

Consider Fig.1(a). Select i_L, i_{ci} , i_{c2}, v_{c1} and v_{c2} as independent variables. The combinations are c_5^2=10, for example: v_{c1}-i_{c1}.

First of all, we use Fig. 1(b) and select the nominal value C_1=4050pf, C_2=0.1μf, L=7.3mH and R=1.52k. When we adjust C_1,C_2, L or R to these value, 10 double scrolls were observed as shown in Fig.2-11. To save the space, the chaotic spectrums were not shown.

By taking all the 2 variables combinations, C_8^2=28, one may find out 28 double scrolls. For example, the double scrolls between V_R-V_{C1}, i_R-V_{C1}, V_R-i_{c1} and i_R-V_{c1} were observed. All these results further demonstrated the macroscopic structure of the double scroll.

III. PERIOD PLUS (MINUS)1 RULE

Now we report the whole process of formation of the double scroll. The R is adjustable. We take V_{c1}-i_{c1} for example.

When R=2kΩ, a dot appeared on the oscilloscope. A decrease in R leads to the sequence of period doubling, as shown in Fig.12-15. A slight decrease in R leads to the chaotic motion with a large hole in the middle, as shown in Fig.16. Then it leads to 3 loops, chaos, 4 loops, ..., as shown in Fig.17-19. As we continue to decrease R in small amount it changes rapidly and becomes a chaotic motion with a very small hole, as shown in Fig.20. An asymmetric double scroll appeared with a minute decrease in R as shown in Fig.21. During the course of the further decrease in R, the double scrolls with upper 9 lower 5 (9-5),8-6,7-7 loops were observed as shown in Fig.22-24. The chaotic motion exists between them. Here, they follow the period plus (minus) 1 rule. When R decreases, it shrinks correspondingly. Sometimes the double scrolls with 4-4, 3-3 loops appeared. When R=1277Ω, it becomes a double scroll with 2-2 loops, as shown in Fig.25. Then it is chaotic. As R decreases 2Ω, the double scroll becomes 2 loops appeared only in the upper, as shown in Fig.26. Then it is chaotic again. Then 4,2,1 limit cycle appeared with the decrease in R, as shown in Fig. 27-29. It shrinks to a dot at R=1.27 kΩ. But when R decreased to 1.25kΩ it

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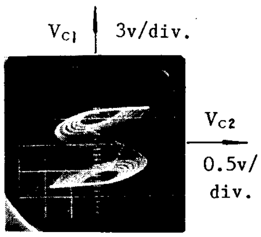
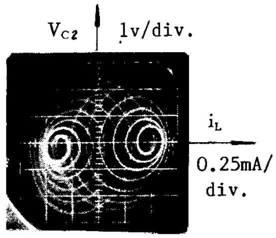


Fig. 2



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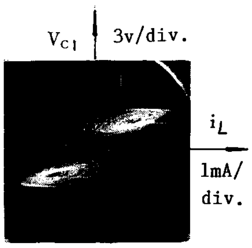

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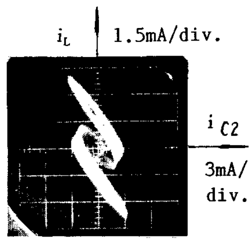

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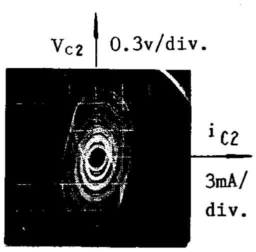

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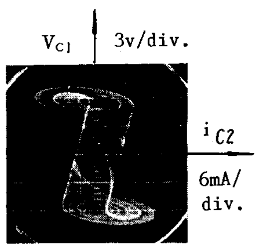

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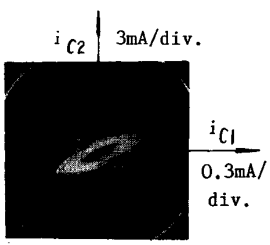

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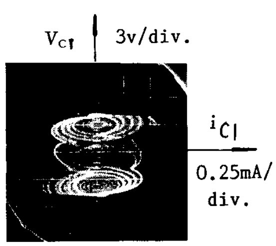

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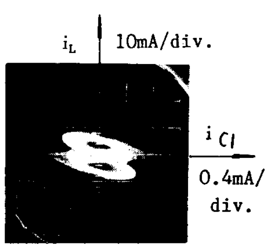

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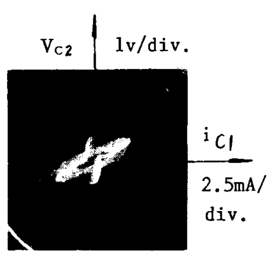

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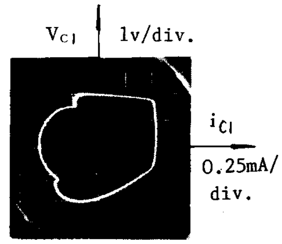

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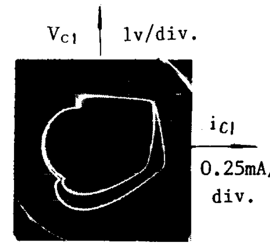

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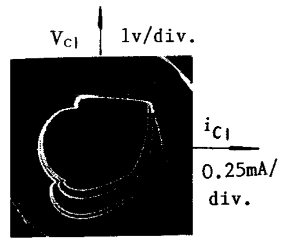

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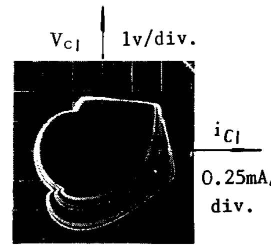

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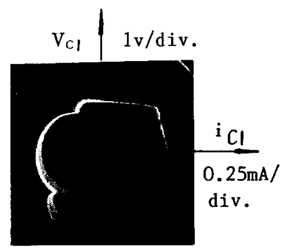

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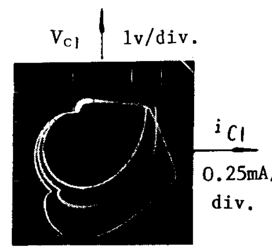

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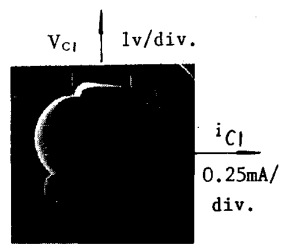

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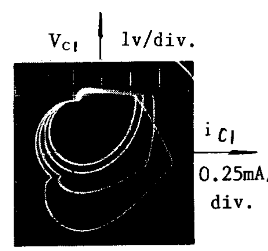

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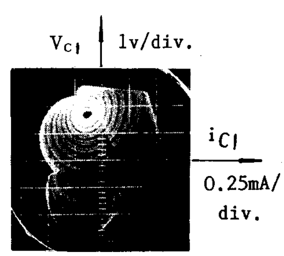

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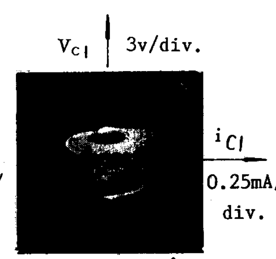

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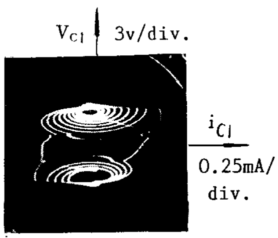

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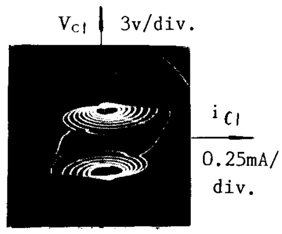

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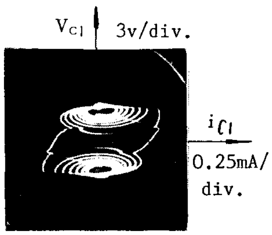

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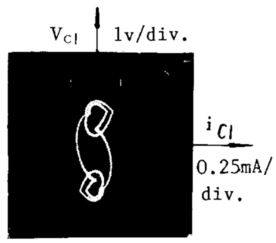

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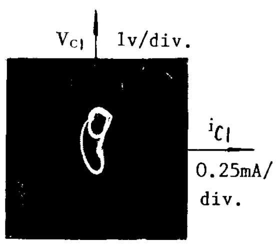

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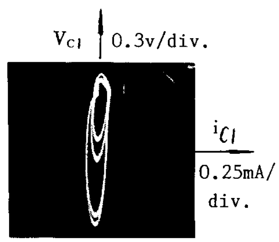

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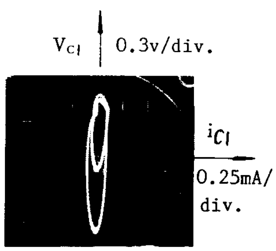

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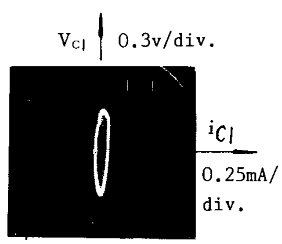

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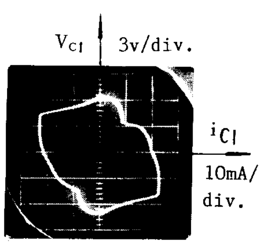

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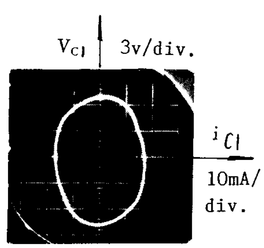

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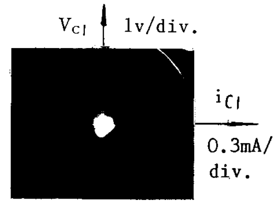

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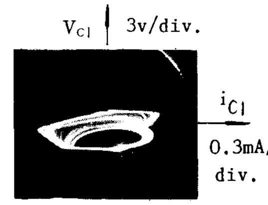

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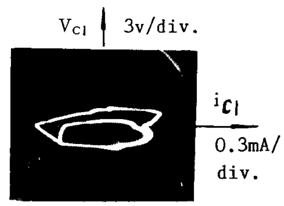

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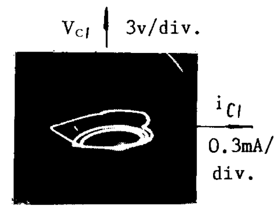

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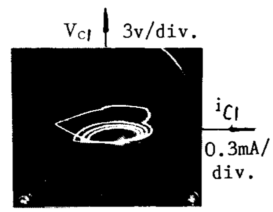

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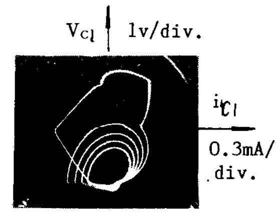

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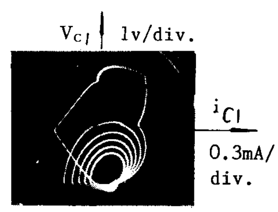

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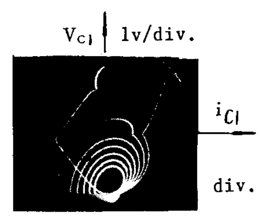

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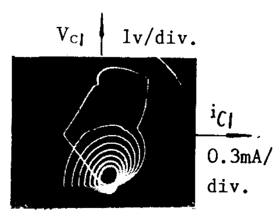

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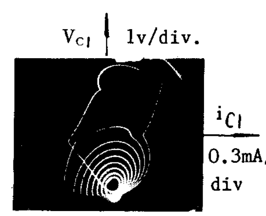

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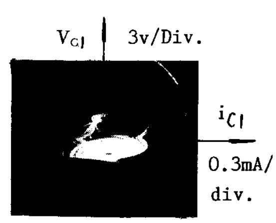


Fig.42

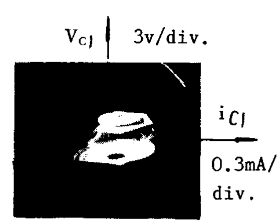


Fig.43

becomes a big circle, as shown in Fig.30. Then its shape changes slightly. When R=0, it becomes an approximate ellipse, as shown in Fig.31.

On the contrary, when R increases from 0, the double scroll appeared at R=1.6k abruptly, then the phenomena is opposite to the above.

As mensioned above, there exists a rule: period-chaos-period plus (minus) 1 in addition to the period doubling to chaos and typical reverse bifurcation.

As a matter of fact, the same phenomena were observed by changing C_1,C_2, or L.

IV. THE DOUBLE SCROLLS OBTAINED BY ADJUSTING THE VOLTAGE OF THE OP-AMPS

In Fig.1 (a), we select the nominal value as before. The -Vcc of the op-amp is set to -11v. When +Vcc was increased from 0 to 11V, 10 double scrolls were observed, as shown in Fig. 2-11.

At +Vcc=0, a dot was observed. When it was increaseed to 1.47v, a very small loop appeared abruptly. Then 2-, 4-loop limit cycles, ..., chaos, 3 loops, chaos, 4 loops... The small chaotic figure was shown in Fig.32 when the voltage = 1.96v. The phenomena of the period doubling to chaos and period plus 1 from one chaotic state to the other were similar to the Fig.12-20 when the voltage increases from 1.47v to 1.96v.

As soon as the voltage is slightly over 1.96v a upside down chaotic figure which is much larger appeared abruptly, as shown in Fig.33. When the voltage increases to 2.3v, it becomes 2 loops, as shown in Fig.34. Then it is chaotic. Later they follow the rule: period-chaos-period plus one. The voltages appeared in 3-9 loops had been increased to 2.85,3.60,4.23,4.62,4.78,5.14,5.40v and their Lissajous figures were shown in Fig.35-41 respectively. Thereafter it is very difficult to observe 10 and more than 10 loops. Then the figure observed was always chaotic. When the voltage approaches 5.5v, its Lissajous figure is shown in Fig.42. When the voltage is equal to 5.5v, an asymmetric double scroll appeared, as shown in Fig.43. When the voltage increases to 11v, it becomes a symmetric double scroll. In the course of changing from asymmetric to symmetric double scrolls, we observed the stable double scrolls. And they change in the same way as the Fig.22 changed to Fig.24 but with lower loop minus one and upper loop plus one. It remains to point out that, when the voltage is over 11v, it becomes an asymmetric double scroll with upper larger and lower smaller structure.

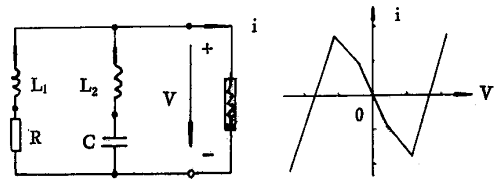
On the contrary, when +Vcc decreases from +11v the phenomena is opposite to the above except that a further decrease in voltage after 2 loops at 2.3v leads to a big circle. Then when we continue to decrease the voltage it shrinks to a dot until the voltage +Vcc = 0.

As for other combinations, there exists the same phenomena when the voltage of the op-amp changed.

V. DISCUSSION

The chaotic attractor can also be obtained by changing the op-amp's voltage, i.e., by changing the asymmetric v-i characteristic to symmetric.

And during the whole process, the author discovered the period plus(minus) 1 rule in Chua's circuit. However, the starting period is different. It is 3 when R decreases, and it is 2 when +Vcc increases. The author guesses that this rule exists in some other nonlinear circuits or systems, especially in 3-order autonomous and reciprocal circuits. And in 1986 the author designed a circuit, as shown in Fig.44, which follows the rule from starting period 1 [6].



(a) (b)

Fig.44 (a) 3-order circuit
(b) v-i characteristic

Now the main task is being carried out in order to demonstrate the chaotic phenomena in Chua's circuit. Of course, the ultimate goal is to develop a theory to explain the obtained results and the rule.

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