Non Integer Order Integration by using Neural Networks

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Abstract—In this paper a new method for non integer order integration is presented. It is based on a Multi-layer Perceptron modelization that allows an easy circuit design and a wide range of integration order. The neural network approach allows to overcome some problems related to practical implementation of the circuit. An example of application regarding the control of a chaotic system is also reported in order to validate the proposed Neural Network modeling technique.

Index Terms—Nonlinear systems, Non Integer Order Systems, Neural Networks.

I. Introduction

In recent literature, increasing interest has been devoted to non-integer-order systems, because of the wide variety of their application fields [1-3]. Referring in particular to [4-6] fractional-order systems with complex dynamics have been studied in a theoretical way. However their realization is not always very exact, due to frequency limitations and approximations error. In particular analog design implies the realization of a time varying capacitors usually difficult to modulate according to desired trend [7] and digital circuitry needs a large memory area to store function samples. A neural network modelization may guarantee, at the same time, a good degree of approximation and the possibility, by keeping a fixed structure, of changing the integration order by well-defined configuration of weights.

II. Overview of Fractional Systems

Fractional systems (or more Non Integer Order Systems) can be considered as a generalization of integer order systems. The most common example of a fractional system transfer function is given by:

$$H(s)=\frac{1}{s^{m}} \qquad m\in\Re^{+} \tag{1}$$

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which is called a "fractional integrator" and can be found in studying many physical phenomena [8-9]. The study of fractional systems may be approached in the time domain by using the following non integer order integration operator [10-11]:

$$\frac{d^{-m}h(t)}{dt^{-m}}=\frac{1}{\Gamma(m)}\int_{0}^{t}(t-y)^{m-1}h(y)dy \tag{2}$$

where $\Gamma(m)$ is the factorial function. It must be noted that $h(t)$ may be any function for which the integral in (2) exists. To build the numerical algorithm, we need a discrete approximation of relation (2). Taking T as the sample time and supposing h to be constant during this time interval, after some calculation results [10]:

$$\begin{gathered}\frac{d^{-m}h(0)}{dt^{-m}}=h(0)\\ \frac{d^{-m}h(k)}{dt^{-m}}=\frac{T^{m}}{\Gamma(1+m)}\sum_{j=0}^{k-1}\frac{h(j)+h(j+1)}{2}\left[(k-j)^{m}-(k-j-1)^{m}\right]\\ (k>1)\end{gathered} \tag{3}$$

Sampling time T must be chosen in a suitable way, according to precision requirements (which need a small T) and speed performances (for which the number of intervals considered must be not so high and therefore T not too small). For example, for our purposes it has been used the value T =0.006, which fits quite well with the previous considerations.

In the frequency domain high-order model approximations are used. In fact, an exact representation of a fractional system would require an infinite number of linear time-invariant systems represented by an alternative sequence of poles and zeros [2]. The choice of the approximation degree depends on the desired bandwidth, which is strictly related to the model order.

In terms of frequency response, the order m modulates the slope of the magnitude plot, being for $G(s)=1/(s+1)^{m}$ equal to –20m dB/dec, like reported in figure 1.

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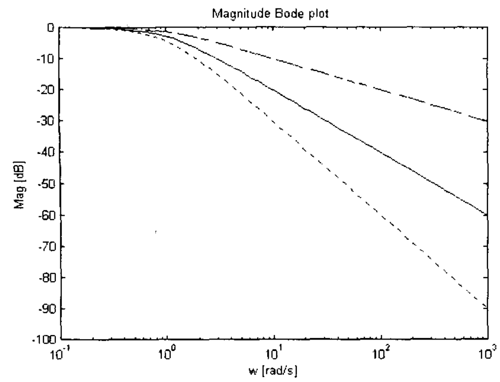

Fig.1- Magnitude Bode Plot of fractional systems $F(s)=1/(s+1)^m$ with m=1 (solid), m=0.5 (dashed), m=1.5 (pointed).

III. PROPOSED STRATEGY

The proposed approach has been structured in the following steps:

a) Structure of the network

As explained in [2], non integer order systems can be approximated by suitable relaxation systems. Some examples are reported in the table I:

$$\frac{1}{s^{0.9}} \approx \frac{1.766(s+0.127)(s+21.79)}{(s+0.0013)(s+0.2155)(s+35.93)}$$

$$\frac{1}{s^{0.7}} \approx \frac{4.849(s+0.0064)(s+0.0578)(s+5.179)(s+46.41)}{(s+0.0139)(s+0.1245)(s+1.116)(s+10)(s+89.61)}$$

$$\frac{1}{s^{0.5}} \approx \frac{15.97(s+0.1269)(s+1.777)(s+35.232)}{(s+0.0316)(s+0.5013)(s+7.945)(s+125.8)}$$

Table I.

However, due to the few poles representation, these approximations are not sufficient for a large bandwidth simulation of non integer order systems behavior. This fact leads us to consider higher order approximations for our fractional integrators because the more integer order poles are used for obtaining the desired approximation, the more it results to be accurate. To this aim it is chosen a multi-layer perceptron with a sufficiently large number of inputs in order to simulate a high order system.

The integrator in (1) is supposed to be modelized by the following NARMAX equation:

$$y(k)=F[y(k-1),\ldots,y(k-19),u(k-1),\ldots,u(k-19)] \quad (4)$$

The order set to 19 resulted good compromise between accuracy and simplicity of the network to be designed. In fact, according to the theory of black box identification [12], the Neural Network must have 38 inputs (19 real inputs, 19 feedback of the output), and 1 output.

The use of only one hidden layer resulted to be sufficient for our purpose, providing very good results.

The number of neurons in the hidden layer was set initially to 20 and then moved to 25 so the total number of weights is 950.

An example of structure for the network is reported in figure 2.

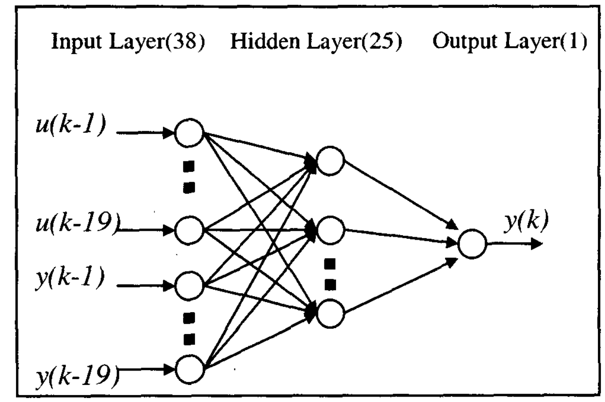

Fig. 2 The Multi-layer Perceptron adopted for the simulation of the m-order integrator (38-25-1).

b) Learning Phase

It has now to be considered the type of integrator we wish to realize. Due to several applications in the field of control [13-14], we focus our attention to order 0.3, 0.4, 0.5, 0.7. Furthermore it must be noted that, with a suitable combination of them, it is possible to obtain a universal integro-differentiator whose order can be changed in the range of [-1,1] by means of steps of 0.1 (see Fig. 3).

To this aim, four different neural networks are trained with a chirp signal containing frequencies from 1 to 100 Hz (Fig. 4) and with their m-order integrals (Fig. 5). The number of learning patterns used for the training phase has been fixed to 10000.

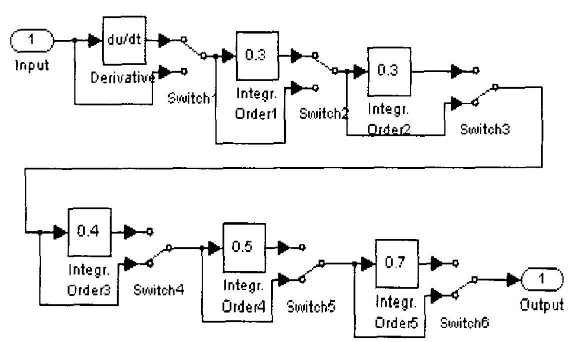

Fig. 3 Switch configuration for a derivative of order 0.7.

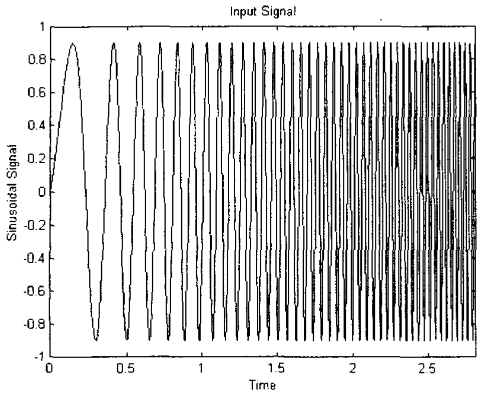

Fig. 4 Input signal for the learning phase.

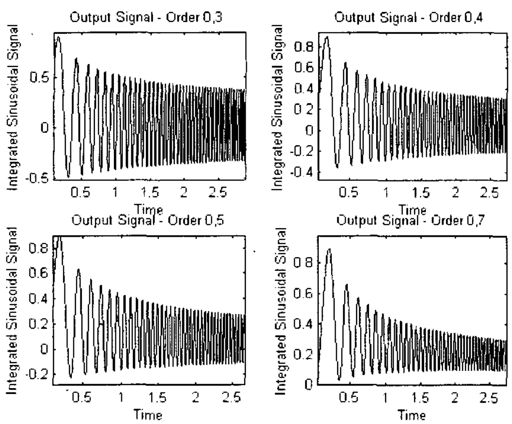

Fig. 5 Output signals used in the training of the four networks. Nineteen samples are feed backed at each step.

c) Test Phase

The obtained networks are then tested with a square wave signal at the frequency of 10 Hz. This choice has been made since a square wave is a signal with a good number of harmonics. It can be represented by the following equation:

$$u(t)=\begin{cases}0.9 & n/10<t<(n+1)/10 \quad n \ \ even \\ -0.9 & n/10<t<(n+1)/10 \quad n \ \ odd\end{cases}$$

The frequencies contained in the signal are the odd ones (10, 30, 50, 70 Hz) and have appreciable amplitude until 190 Hz.

The assumption that the networks have been trained with a signal having, as maximum frequency 100 Hz does not affect the final results.

In fact, every network seems to fit quite well the expected results (see Fig. 6) and therefore it can be concluded that they perform exactly the desired integrations.

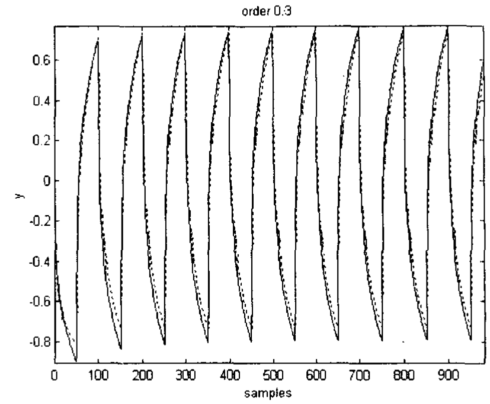

Fig. 6 Comparison between the output of a 0.3 integrator (solid) and the final network which approximate it (dashed). The input signal is a square wave.

IV. An Example of Application: Non Integer Order Control of Chua's Circuit

Chua's circuit [15] is a third order non-linear system that is able to display, for a well-defined set of parameters, a chaotic behaviour. Its state equations are

$$\begin{aligned}\dot{x} &= \alpha(y-x-f(x)) \\ \dot{y} &= x-y+z \\ \dot{z} &= -\beta y\end{aligned} \tag{4}$$

in which $f(x)$ is a piecewise linear function having the form

$$f(x)=bx+\frac{1}{2}(a-b)\left[|x+1|-|x-1|\right] \tag{5}$$

A typical parameters set that leads to the onset of chaos is $a=-1/7$, $b=2/7$, $\alpha=9$, $\beta=14.286$. The relative attractor plotted in the phase space is a well-defined double scroll (Fig. 7)

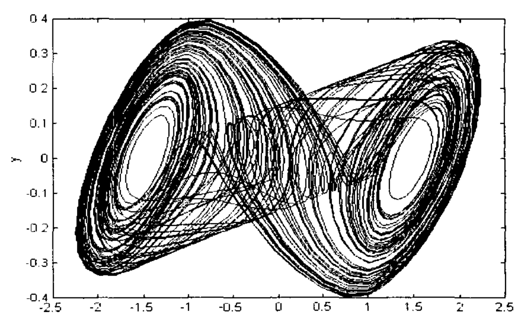

Fig. 7 Chaotic attractor of the Chua's System (parameters values are $a=-1/7$, $b=2/7$, $\alpha=9$, $\beta=14.286$).

The control of a chaotic system is a quite difficult task to achieve and for this reason it does not exist a general theory to follow for the design of the controller. Non-standard regulators have been proposed for the control of nonlinear systems. In fact industrial controllers like PID, commonly adopted due to their simple structure, not always are able to achieve good performances. To this aim the idea of a non integer order PID (called $PI^{\lambda}D^{\mu}$) has been introduced in [14] in order to realize a good compromise between simplicity and efficiency provided by the addiction of further tuning parameters like non integer order λ and μ.

Our aim is to build a PD^{μ} with neural networks in order to obtain a circuital implementation. It follows the scheme reported in fig. 8 in which the μ-order derivative is performed using an integrator of order $1-\mu$ ($0<\mu<1$) realized by the Neural Network.

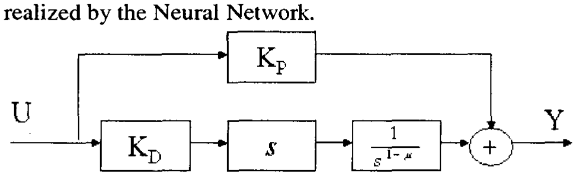

Fig. 8 Block scheme of the non integer order controller PD^{μ}.

This non integer controller has been used on-line for the control of Chua's circuit, according to the scheme in reported in Fig. 9.

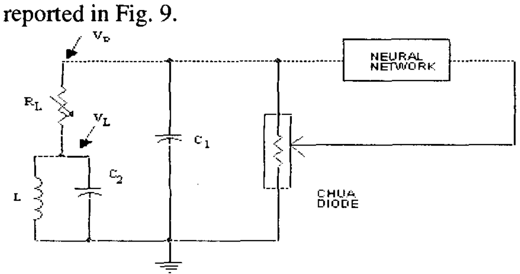

Fig. 9 Scheme of the Chua's Circuit controlled by a PD^{μ} neural network.

The best performances for the controlled system have been reached with the addition to a classical PID, that is not able to stabilize the system, of a 0.7 non integer order derivative using the scheme still reported in Fig. 3. The stabilization of the y(t) state variable around an equilibrium point is clearly depicted in Fig. 10

V. CONCLUSIONS

An approach based on neural network modelling for non integer order integrators has been presented and validated with an example. The black box method aims to overcome realization problems due to frequency limitations and approximations error. Further works are in progress to obtain an hardware prototype for real world applications.

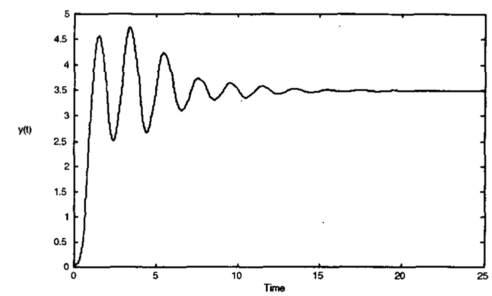

Fig. 10 Time behaviour of y(t) when PD^{μ} neural network controller is applied.

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