Node Voltage Analysis

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1. Basic Procedure

Node voltage analysis is a systematic method for finding the voltage at all nodes in an electrical circuit. We explain the steps of the basic procedure for the example shown in the Figure 1.
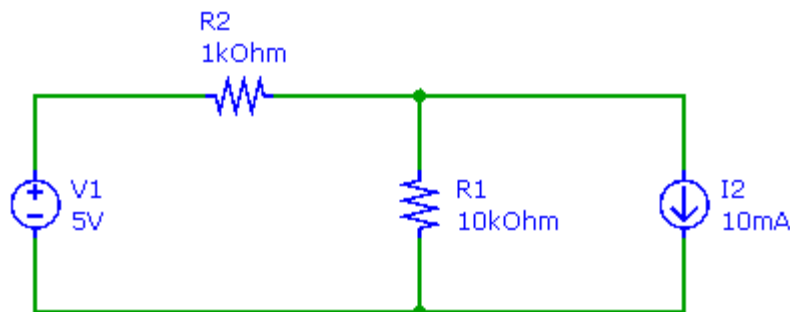

Figure 1 Example circuit for node voltage analysis.

a) Identify all nodes in the circuit

The wiring connections between circuit elements are called "nodes". In Figure 1, circuit elements are shown in blue and the wires are green. All the wires that are joined together form a single node, regardless of the number of elements joined in the node. The example circuit has 3 nodes: one joining V1 and R2, one joining R1, R2, and I2, and a final node that joins elements V1, R1, and I2.

Note that dots are only to clarify electrical connectivity. A node may have zero, one, or more dots. The number of dots has no relationship with the number of nodes in a circuit.

b) Assign a reference node

Node voltage analysis determines the voltages at the nodes in a circuit with respect to a reference at which the voltage is defined to be zero. Usually the negative terminal of the supply or a terminal shared by the supplies in circuits with multiple voltage supplies is chosen as the reference, but any other terminal could be chosen as reference also. Several names are used interchangeably for the reference node including common node and ground node.
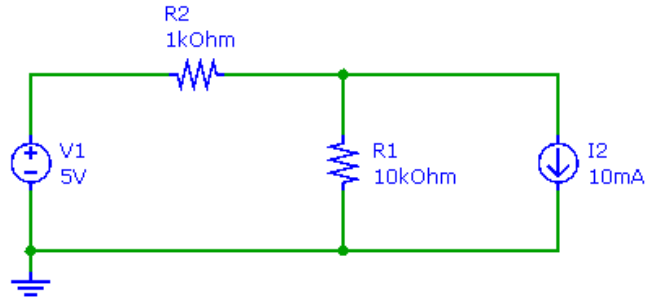
Figure 2 Example circuit with reference node.

The basic node voltage analysis technique described here assumes that all voltage sources have one terminal connected to the reference node. A slightly modified technique is used for circuits containing floating voltage sources (i.e. voltage sources with neither terminal tied to the reference node).

c) Assign node voltages

Assign names to all node voltages. No label is needed for the reference node. In the example, the voltage at one node is set by voltage source V1 and conveniently labeled V1 also. A different name would also be acceptable, albeit at the risk of introducing unnecessary confusion.
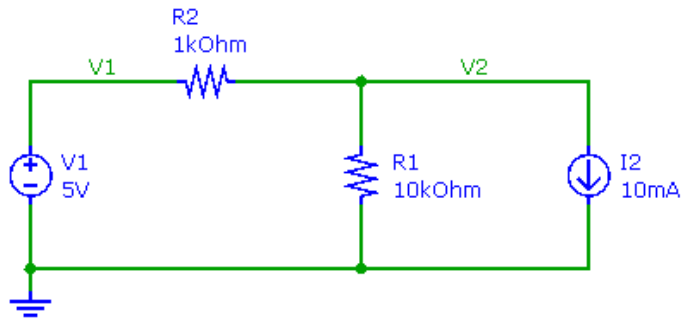

Figure 3 Example circuit with node voltage labels.

d) Identify unknown node voltages

The voltage at the reference node is zero by definition and does not need to be calculated. No equations are needed at nodes for which the voltage is set by a voltage source, as is the case for node V1 in the example. In the example only unknown node voltage is V2.

e) Write KCL equations

Write Kirchhoff's current law for every node with unknown voltage. For clarification, draw an arrow for each current leaving the node. There will be arrows for each element connected to the node. Express these currents in terms of element values and node voltages. Kirchhoff's law states that the sum of these currents must be zero, i.e.

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_1} + I_2 = 0$$

f) Solve for unknown node voltages

Verify that you have one equation for each unknown node voltage and solve. In the example there is only one unknown node voltage (V2), and hence a single equation. For more complex circuits we obtain several equations. Sometimes we are interested only in a subset of the node voltages. In this case we can save time by solving only for those variables.

g) Calculate currents

Sometimes we are interested in branch currents (currents flowing through elements). These currents can be calculated from the element IV characteristics. For example, the current through R1 is V2/R1. The current I1 flowing through voltage source V1 equals the current through R2, i.e. I1 = (V1-V2)/R2.

2. Ideal Operational Amplifiers

The voltage across the inputs of an ideal operational amplifier is zero (provided that the circuit is stable). Node voltage analysis proceeds like above, but in step (e) no equation is written for the node to which the output is connected.
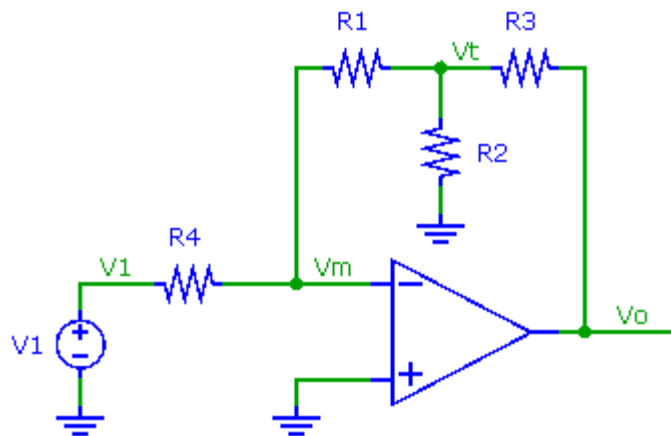

Figure 4 Operational amplifier example.

For example, in the circuit shown in Figure 4 node voltages V_m, V_t, V_o are unknown but equations are written only for nodes V_m and V_t. Since the voltage across the input of the opamp is zero and the positive terminal is connected to the reference, $V_m = 0$. The nodal equations are

$$\frac{-V_1}{R_4} + \frac{-V_t}{R_1} = 0$$

for node V_m (using $V_m = 0$) and

$$\frac{V_t}{R_1} + \frac{V_t}{R_2} + \frac{V_t - V_o}{R_3} = 0$$

for node V_t.

The technique works equally well for configurations where none of the amplifier inputs is grounded. For the circuit in Figure 5 the nodal equations are

$$\frac{V_m - V_1}{R_2} + \frac{V_m - V_o}{R_1} = 0 \qquad \text{for node } V_m \text{ and}$$

$$\frac{V_m - V_1}{R_3} + \frac{V_m}{R_4} = 0 \qquad \text{for node } V_p.$$

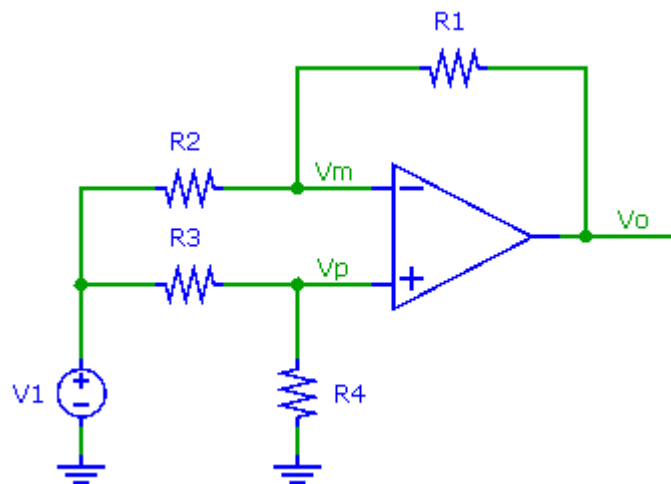In the equation for node V_p, V_p has been replaced by V_m since $V_m = V_p$. No equation is needed for node V_o.

Figure 5 Operational amplifier example: V_p is not grounded.