Reference:W:\Lib\MathCAD\Default\defaults.mcd

1. Objective
   a. Motivation
      . Finite circuit speed, e.g. amplifier - effect on signals
      . E.g. how "fast" an amp do we need for audio? For video? For RF?
      . How do we characterize "fast"?
      . Check response for "every possible" input is not practical
      . Solution: check response for "representative signals"
        that stand in for "every possible" signal
   b. What is Frequency domain analysis?
      . Analysis technique
      . Not a new circuit element, it's applies to the circuit we already know
      . Widely used in engineering (not just circuits)
         - e.g. EE 20 ++
         - Underlying math: Laplace & Fourier transform, (phasors)

2. Analysis with sinusoids
      . cascadable (shape retained)
      . efficient: phasor analysis
      . universal: Fourier analysis
      . Demo: **http://www.falstad.com/fourier** (live!)

3. Phasor / Laplace
   0. Motivation: time domain diff eqs, trigonometry: math problem
   a. Represent sinusoids with complex numbers
   b. Signals are not complex
   c. RC LPF example steady state analysis
   d. Impedance, Admittance
      . RLC
      . generalized

4. Bode plot
   a. Ratios, dB (why logarithmic)
   b. Bode plot
   c. Techniques

5. Power (NR Ch 10)
   . instantaneous, average
   . rms
   . complex

Lecture 25: Sinusoidal Steady State (see also ppt)

Motivation

- circuits 'deform' changing signals
- e.g. digital signals do not retain their "square" shape
- amps also cannot keep up with rapidly changing signals (capacitors inside!)
- how determine e.g. if amplifier is fast enough?
- how do we even characterize how "fast" an amplifier is?
- what's a good metric? We don't want to deal at the component (schematic) level (not very hierarchical)

Problem

Example: RC LPF, (ideal) amp, RC HPF
square wave input (for simplicity)
"exponentials" after LPF
even more complicated waveform after HPF
How to analyze this???

Demo

Conclusion: sinusoids retain shape --> modular!!!

Sinusoidal Response (AC analysis)

Sinusoids:
- sin or cos
- amplitude
- frequency (f or omega)
- phase

Initial transient
Example: RL HPF (see NR text), **calculate V_R**

$V_m := 1V$ $f := 1MHz$ $\omega := 2\pi \cdot f$ $R := 5k\Omega$ $L := 1mH$ $\tau := \frac{L}{R}$ $\tau = 200\,ns$

$T := \frac{1}{f}$

input: $v_s(t) := V_m \cdot \cos(\omega \cdot t)$ for t>=0, 0 otherwise

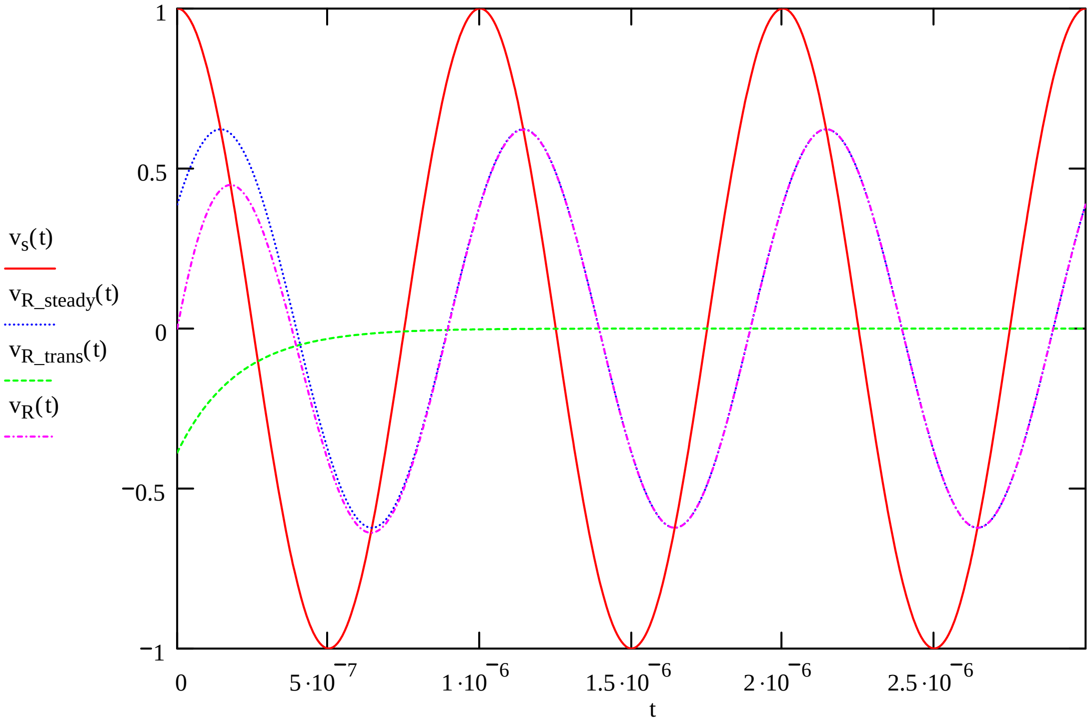
KVL: $L \cdot \frac{di(t)}{dt} + R \cdot i(t) = v_s(t)$

solution: $\phi := \operatorname{atan}\left(\frac{\omega \cdot L}{R}\right)$ $\phi = 51.488\,deg$

$$v_{R\_steady}(t) := \frac{V_m \cdot R}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \cos(\omega \cdot t - \phi) \qquad \frac{R}{\sqrt{R^2 + \omega^2 \cdot L^2}} = 0.623$$

$$v_{R\_trans}(t) := \frac{-V_m \cdot R}{\sqrt{R^2 + \omega^2 \cdot L^2}} \cdot \cos(\phi) \cdot e^{\frac{-t}{\tau}}$$

$$v_R(t) := v_{R\_trans}(t) + v_{R\_steady}(t)$$



Lecture 26: Phasors

Analysis technique:
- . efficiently analyze sinusoidal steady state response
- . linear differential equations --> linear algebraic equations
- . uses complex numbers (signals are real)

Sinusoidal signal:

$$v(t) = V_m \cdot \cos(\omega \cdot t + \phi)$$

$$v(t) = \mathrm{Re}\left[V_m \cdot e^{j\cdot(\omega \cdot t + \phi)}\right] = \mathrm{Re}\left(V_m \cdot e^{j\cdot\phi} \cdot e^{j\cdot\omega\cdot t}\right) = \mathrm{Re}\left(V(j\cdot\omega)\cdot e^{j\cdot\omega\cdot t}\right)$$

Phasor

$$V(j\cdot\omega) = V_m \cdot e^{j\cdot\phi}$$

abbreviation $s := j\cdot\omega$
- more compact equations without j
- complex numbers show up in numeric result
- phasor analysis is special case of Laplace transform,
  --> consistency with other texts on subject

Example: inductor

diff eq analysis

$$i_L(t) = I_{Lm}\cdot\cos(\omega\cdot t)$$

$$v_L(t) = L\cdot\left(\frac{d}{dt}i_L(t)\right) = -L\cdot I_{Lm}\cdot\sin(\omega\cdot t)$$

phasor analysis

$$i_L(t) = \mathrm{Re}\left(I_L(s)\cdot e^{s\cdot t}\right)$$

$$v_L(t) = L\cdot\frac{d}{dt}\mathrm{Re}\left(I_L(s)\cdot e^{s\cdot t}\right) = \mathrm{Re}\left(L\cdot I_L(s)\cdot s\cdot e^{s\cdot t}\right)$$

$$v_L(t) = \mathrm{Re}\left(V_L(s)\cdot e^{s\cdot t}\right)$$

with $V_L(s) = s\cdot L\cdot I_L(s)$

e.g. $L := 1\mathrm{mH}$ $I_{Lm} := 1\mathrm{mA}$ $f := 1\mathrm{MHz}$ $\omega := 2\cdot\pi\cdot f$ $s := j\cdot\omega$

$I_L(s) := I_{Lm}$ $V_L(s) := s\cdot L\cdot I_L(s)$ $V_L(s) = 6.283i\,\mathrm{V}$

Impedance:

$$Z_L(s) = \frac{V_L(s)}{I_L(s)} = s\cdot L$$

- universal! Treat L, C like "resistors"
- complex number
- units: ohm!

Admittance:

$$Y_L(s) = \frac{I_L(s)}{V_L(s)} = \frac{1}{s\cdot L}$$

e.g. $s\cdot L = 6.283i\,\mathrm{k\Omega}$

$$\frac{1}{s\cdot L} = -159.155i\,\mu\mathrm{S}$$

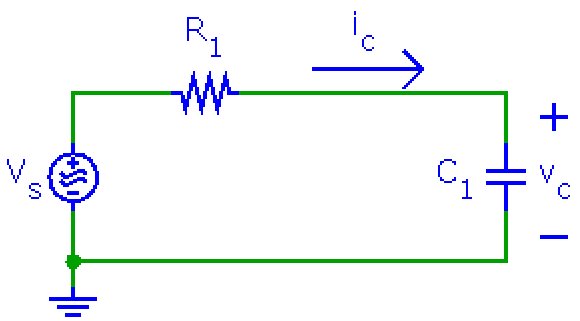
Let class derive admittance of a capacitor.

Lecture 27: Network analysis with phasors

- KVL and KLC still hold (with phasors)
- represent signals with phasors (complex numbers)
- Restance becomes Impedance, generalizable to L, C!
- Ditto Admittance
- Generalized ohm-meters exist also: LCR meters, impedance / network analyzers

$Z_R = R$ $\quad Z_L = s \cdot L$ $\quad Z_C = \dfrac{1}{s \cdot C}$

Example: calculate magnitude and phase of ic, vc



$$Z_{tot}(s) = Z_{R1}(s) + Z_{C1}(s)$$

$$Z_{tot}(s) = R_1 + \frac{1}{s \cdot C_1} = \frac{1 + s \cdot R_1 \cdot C_1}{s \cdot C_1}$$

$$I_c(s) = \frac{V_s(s)}{Z_{tot}(s)} = V_s(s) \cdot \frac{s \cdot C_1}{1 + s \cdot R_1 \cdot C_1}$$

with $\quad V_{sm} := 1V \quad R_1 := 10k\Omega \quad C_1 := 1pF \quad R_1 \cdot C_1 = 10\,ns \quad \tau_1 := R_1 \cdot C_1$

$V_s(s) := V_{sm} \quad I_c(s) := V_s(s) \cdot \dfrac{s \cdot C_1}{1 + s \cdot R_1 \cdot C_1}$

$I_{c1} := I_c(2j \cdot \pi \cdot 10MHz) \quad I_{c1} = 28.304 + 45.048i\ \mu A \quad |I_{c1}| = 0.053\ mA$

$\arg(I_{c1}) = 57.858\ deg$

KVL, KCL, node voltage, divider - whatever, we get:

$V_c(s) = \dfrac{V_s(s)}{1 + s \cdot R_1 \cdot C_1} \quad H_1(s) = \dfrac{V_c(s)}{V_s(s)} = \dfrac{1}{1 + s \cdot R_1 \cdot C_1} \quad H_1(s) := \dfrac{1}{1 + s \cdot R_1 \cdot C_1}$

$H_a := H_1(2j \cdot \pi \cdot 0MHz) \quad |H_a| = 1 \quad \arg(H_a) = 0\ deg$

$H_a := H_1(2j \cdot \pi \cdot 1MHz) \quad |H_a| = 0.998 \quad \arg(H_a) = -3.595\ deg$

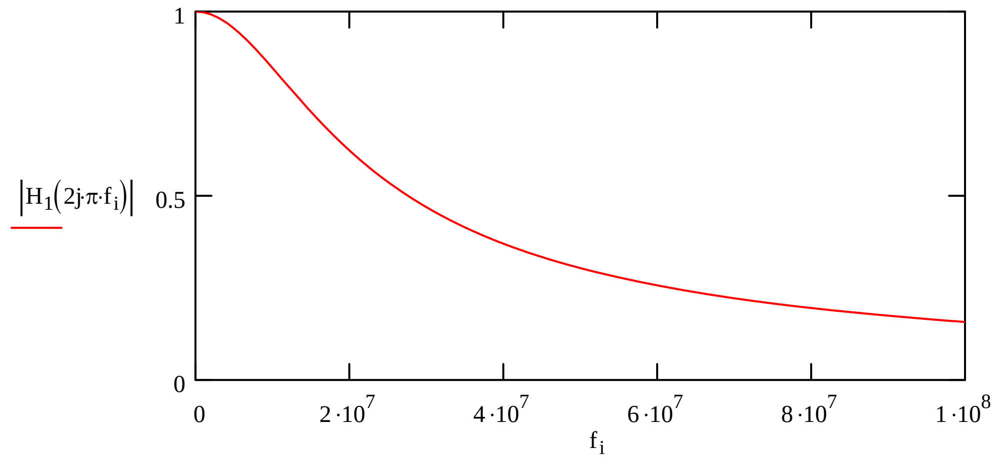
$H_a := H_1(2j \cdot \pi \cdot 10MHz) \quad |H_a| = 0.847 \quad \arg(H_a) = -32.142\ deg$

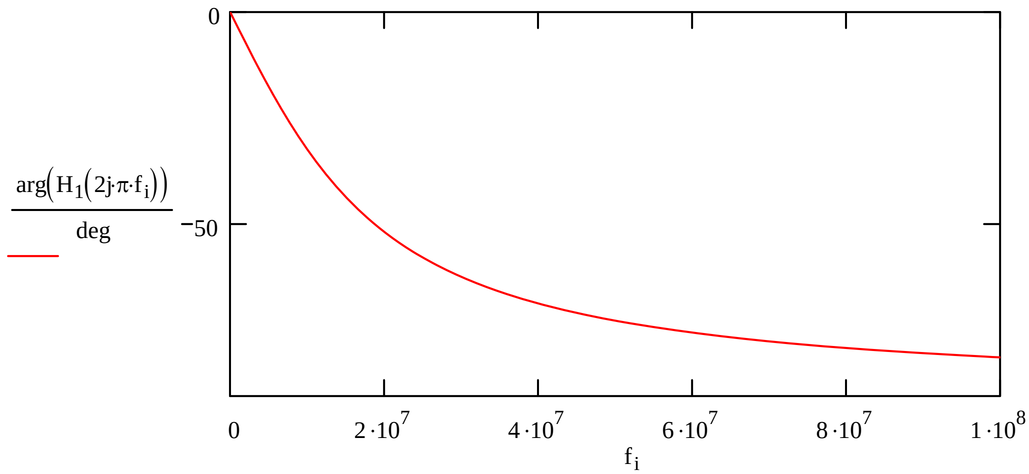
$H_a := H_1(2j \cdot \pi \cdot 100MHz) \quad |H_a| = 0.157 \quad \arg(H_a) = -80.957\ deg$

$H_a := H_1(2j \cdot \pi \cdot 1GHz) \quad |H_a| = 0.016 \quad \arg(H_a) = -89.088\ deg$

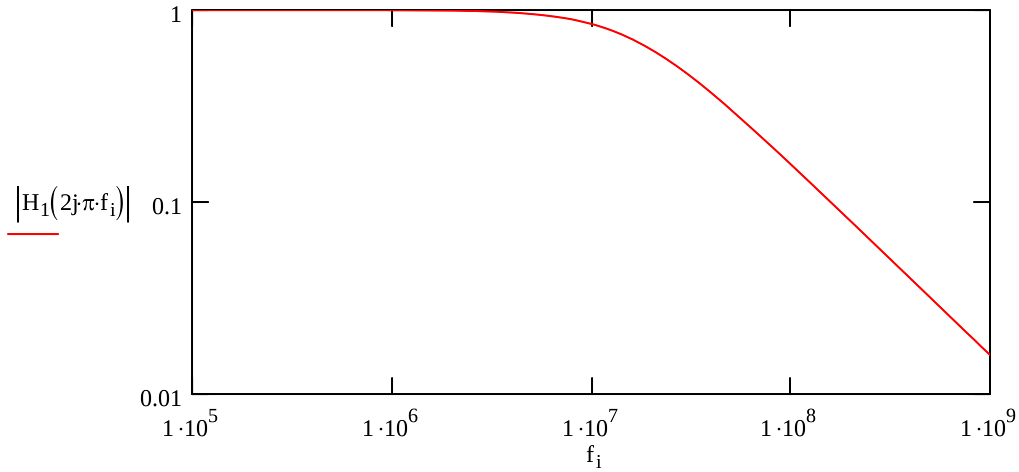
Lecture 28: Frequency Response

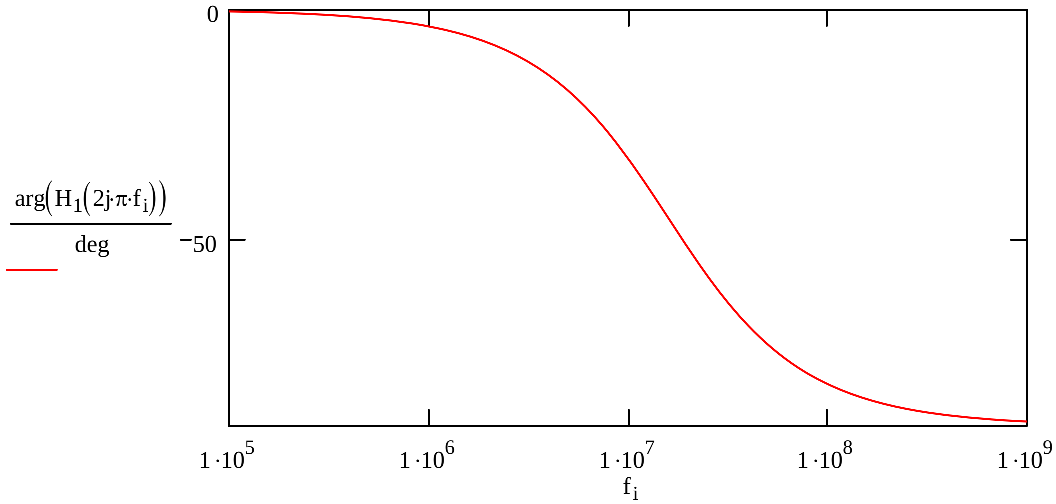
N := 100 i := 0 .. N − 1 f := linrange(0Hz, 100MHz, N)





N := 100 i := 0 .. N − 1 f := logrange(100kHz, 1GHz, N)





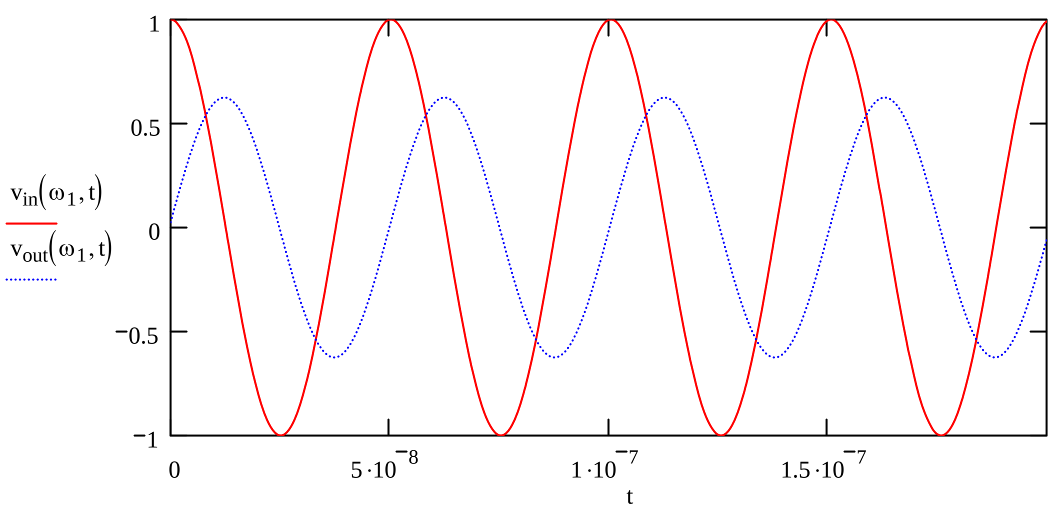
Logarithmic axis:
- more informative:
  . 1Hz resolution at low frequencies, but not at 1GHz!
  . many processes are logarithmic (cf. octave)
- for frequency - beware of label log f; usually the freq and not the log is specified
- amplitude
- phase scale is always linear

Interpretation

$$H_{1abs}(\omega) := \frac{1}{\sqrt{1+(\omega\cdot\tau_1)^2}} \qquad phi_1(\omega) := -\text{atan}(\omega\cdot\tau)$$

$$V_{in} := 1V \qquad v_{in}(\omega,t) := V_{in}\cdot\cos(\omega\cdot t)$$

$$V_{out}(\omega) := V_{in}\cdot H_{1abs}(\omega) \qquad v_{out}(\omega,t) := V_{out}(\omega)\cdot\cos(\omega\cdot t + phi_1(\omega)) \qquad \omega_1 := \frac{25}{\tau}$$



deci-Bel

- for power ratios $dB(r) := 10 \cdot \log(|r|)$

e.g. $P_{antenna} := 1\mu W$ $P_{ref} := 1mW$ $dB\left(\frac{P_{antenna}}{P_{ref}}\right) = -30$ dB (dBm)

$P_{antenna} = \frac{V_{antenna}^2}{R_{antenna}}$ $P_{ref} = \frac{V_{ref}^2}{R_{antenna}}$ $dB\left(\frac{P_{antenna}}{P_{ref}}\right) = dB\left(\frac{V_{antenna}^2}{V_{ref}^2}\right) = 20 \cdot \log\left(\frac{V_{antenna}}{V_{ref}}\right)$

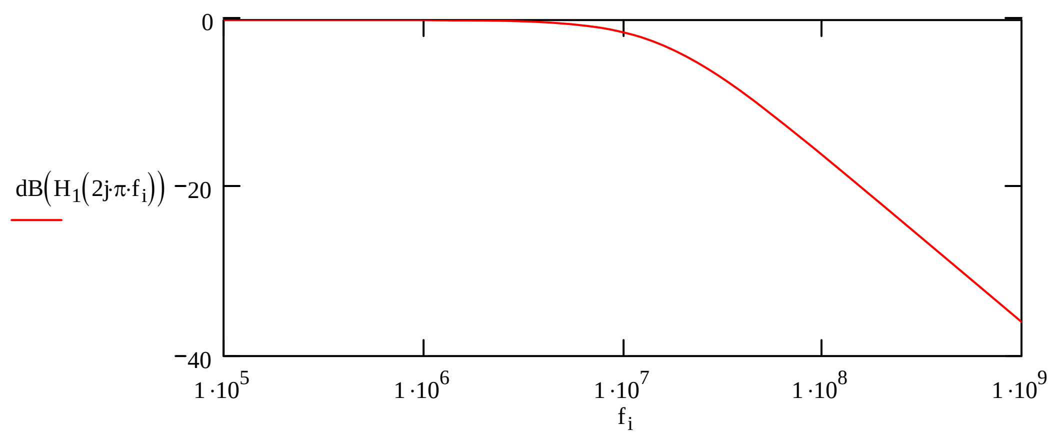
- for voltage ratios $dB(r) := 20 \cdot \log(|r|)$

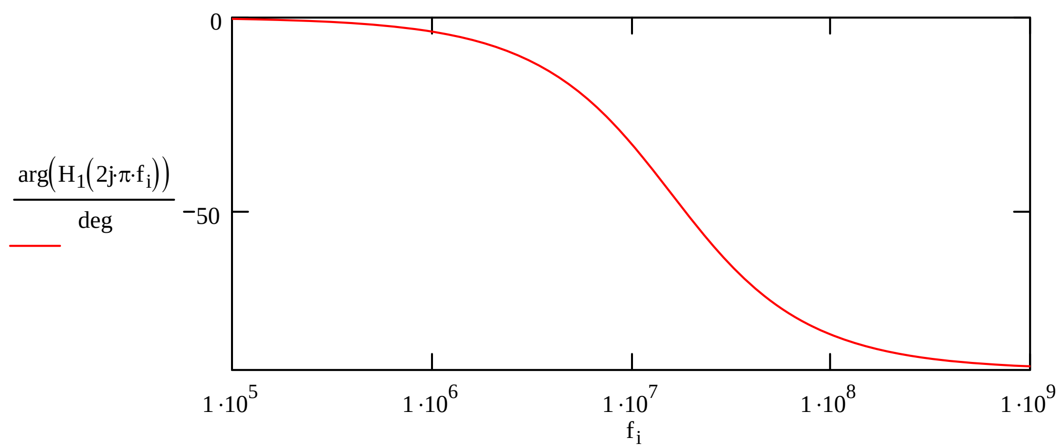
e.g. $H_1(2j \cdot \pi \cdot 100MHz) = 0.025 - 0.155i$ $dB(H_1(2j \cdot \pi \cdot 100MHz)) = -16.072$ dB

$dB(1) = 0$ $dB(10) = 20$ $dB\left(\frac{1}{10}\right) = -20$

$dB(2) = 6.021$ $dB(0.5) = -6.021$ $dB(20) = 26.021$

$N := 100$ $i := 0 .. N - 1$ $f := \text{logrange}(100kHz, 1GHz, N)$





Construct Bode-plot (single pole) from equation. Pole. 3dB Bandwidth.

Filters

LPF, HPF, BPF, BNF ideal, practical (approximate shapes)

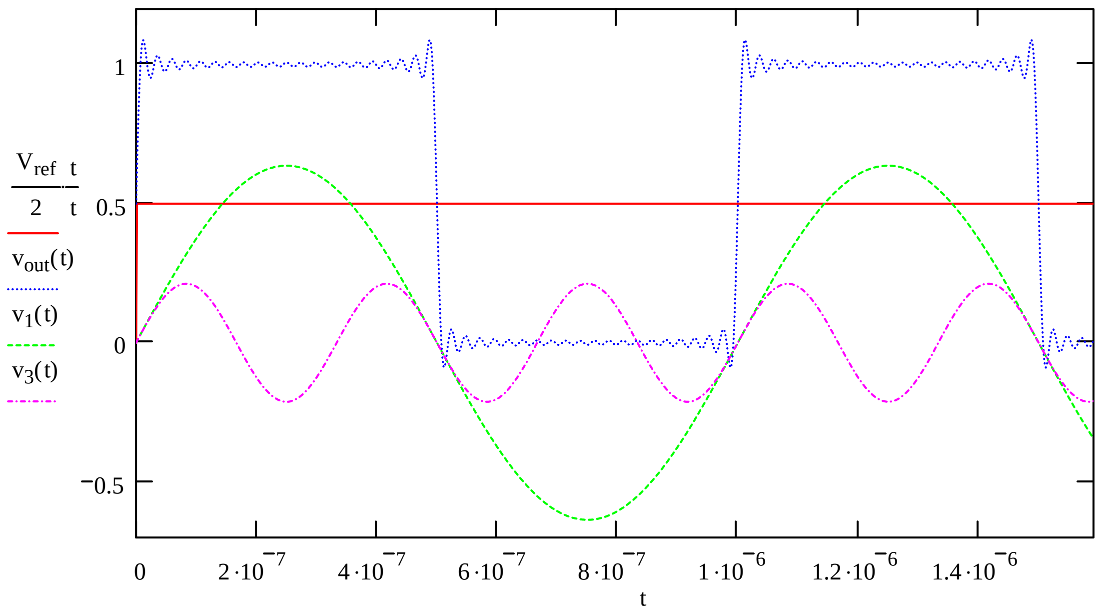
Example: PWM DAC

$V_{dac} = D_{dac} \cdot V_{ref}$ $\quad D_{dac} = \frac{0..255}{256}$ $\quad t_1 = D_{dac} \cdot T$ (time when Vout = Vref)

worst case ripple for $t_1 = \frac{T}{2}$ (50% duty cycle) $\quad V_{ref} := 1V \quad \omega_1 := 2 \cdot \pi \cdot 1MHz$

$$v_{out}(t) := \frac{V_{ref}}{2} + \frac{2 \cdot V_{ref}}{\pi} \cdot \sum_{n=0}^{20} \frac{\sin[(2 \cdot n + 1) \cdot \omega_1 \cdot t]}{2 \cdot n + 1}$$

$$v_1(t) := \frac{2 \cdot V_{ref}}{\pi} \cdot \sin(\omega_1 \cdot t) \qquad v_3(t) := \frac{2 \cdot V_{ref}}{\pi} \cdot \frac{\sin(3 \cdot \omega_1 \cdot t)}{3}$$



Goal: amplitude of ripple < r*Vref $\quad r := 10\%$

Ripple dominated by v1 $\quad A_1 := \frac{2 \cdot V_{ref}}{\pi} \quad \frac{A_1}{V_{ref}} = 0.637$ < r

Attenuation required $\quad a := \frac{A_1}{V_{ref}} \cdot \frac{1}{r} \quad a = 6.366 \quad dB(a) = 16.078$ dB

a) graphical solution (from Bode diagram, determine wp = 1/RC from w1 and a)

b) algebraic

$$\left|H(j \cdot \omega_1)\right| = \frac{1}{\sqrt{1 + (\omega_1 \cdot R \cdot C)^2}} = a \qquad RC := \frac{\sqrt{a^2 - 1}}{\omega} \qquad RC = 1.001\,\mu s$$

$$C := 1nF \qquad R := \frac{RC}{C} \qquad R = 1.001\,k\Omega$$

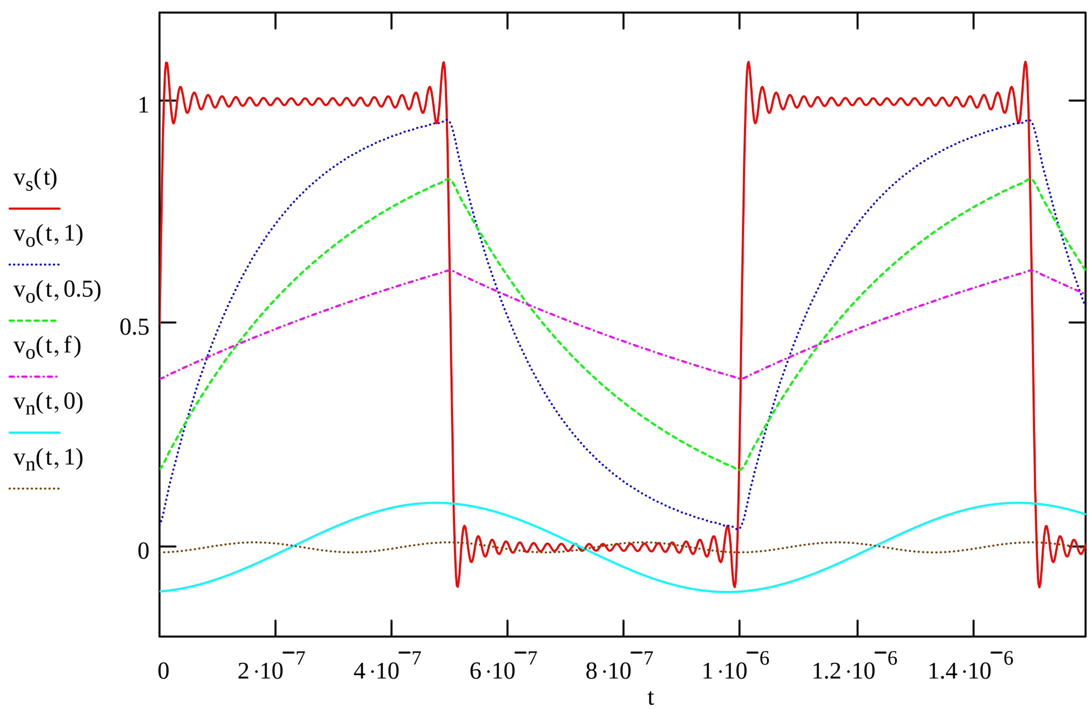
Pole of RC LPF: $\omega_p := \frac{1}{RC}$ $\frac{\omega_p}{2\pi} = 159.054\ \text{kHz}$ $f := \frac{\omega_p}{\omega_1}$ $f = 0.159$

uC output: $$v_s(t) := \frac{V_{ref}}{2} + \text{Im}\left[\frac{2 \cdot V_{ref}}{\pi} \cdot \left[\sum_{n=0}^{20} \left[\frac{e^{j \cdot (2 \cdot n+1) \cdot \omega_1 \cdot t}}{2 \cdot n + 1}\right]\right]\right]$$

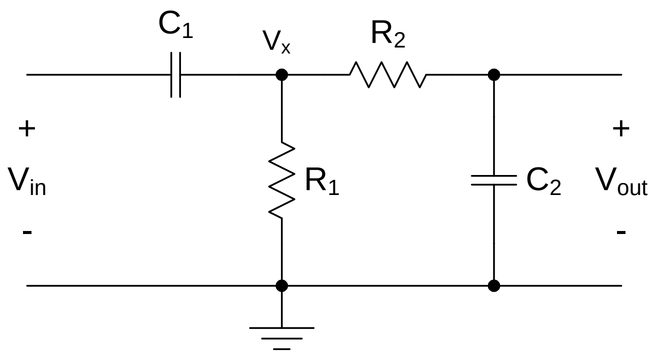
RC LPF: $$H_f(s, f) := \frac{1}{1 + \frac{s}{\omega_1 \cdot f}}$$

Filter output: $$v_o(t, f) := \frac{V_{ref}}{2} + \text{Im}\left[\frac{2 \cdot V_{ref}}{\pi} \cdot \sum_{n=0}^{20} \left[H_f\left[j \cdot (2 \cdot n + 1) \cdot \omega_1, f\right] \cdot \frac{e^{j \cdot (2 \cdot n+1) \cdot \omega_1 \cdot t}}{2 \cdot n + 1}\right]\right]$$

$$v_n(t, n) := \text{Im}\left[\frac{2V_{ref}}{\pi} \cdot \left[H_f\left[j \cdot (2 \cdot n + 1) \cdot \omega_1, f\right] \cdot \frac{e^{j \cdot (2 \cdot n+1) \cdot \omega_1 \cdot t}}{2 \cdot n + 1}\right]\right]$$



Lecture 29: Bode Plot



Given

$$\left(V_x - V_{in}\right) \cdot s \cdot C_{1.} + \frac{V_x}{R_{1.}} + \frac{V_x - V_{out}}{R_2} = 0$$

$$\frac{V_{out} - V_x}{R_2} + V_{out} \cdot s \cdot C_2 = 0$$

$$\mathrm{Find}\left(V_x, V_{out}\right) \rightarrow \begin{bmatrix} s \cdot C_{1.} \cdot R_{1.} \cdot \mathrm{volt} \cdot \dfrac{\left(1 + s \cdot C_2 \cdot R_2\right)}{\left(s \cdot C_{1.} \cdot R_{1.} + s^2 \cdot C_{1.} \cdot R_{1.} \cdot R_2 \cdot C_2 + 1 + s \cdot C_2 \cdot R_2 + R_{1.} \cdot s \cdot C_2\right)} \\ s \cdot C_{1.} \cdot R_{1.} \cdot \dfrac{\mathrm{volt}}{\left(s \cdot C_{1.} \cdot R_{1.} + s^2 \cdot C_{1.} \cdot R_{1.} \cdot R_2 \cdot C_2 + 1 + s \cdot C_2 \cdot R_2 + R_{1.} \cdot s \cdot C_2\right)} \end{bmatrix}$$

e.g.

$f_1 := 100\mathrm{Hz}$ $\quad f_2 := 10\mathrm{MHz}$ $\quad \omega_{p1} := 2\pi \cdot f_1$ $\quad \omega_{p2} := 2\pi \cdot f_2$

$R_1 := 100\mathrm{k}\Omega$ $\quad C_1 := \dfrac{1}{2 \cdot \pi \cdot f_1 \cdot R_1}$ $\quad C_1 = 15.915\,\mathrm{nF}$

$R_2 := 1\mathrm{k}\Omega$ $\quad C_2 := \dfrac{1}{2 \cdot \pi \cdot f_2 \cdot R_2}$ $\quad C_2 = 0.016\,\mathrm{nF}$

Given

$$1 + s \cdot \left(R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2\right) + s^2 \cdot R_1 \cdot R_2 \cdot C_1 \cdot C_2 = 1 + \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1} \cdot \omega_{p2}}$$

$$\mathrm{Find}\left(\omega_{p1}, \omega_{p2}\right) = \begin{pmatrix} 99.899 + 2.583i \times 10^{-6} \\ 1.001 \times 10^7 \end{pmatrix} \mathrm{Hz} \cdot 2\pi$$

$$H(s) = \frac{s \cdot R_1 \cdot C_1}{\left(1 + \dfrac{s}{\omega_{p1}}\right) \cdot \left(1 + \dfrac{s}{\omega_{p2}}\right)} = \frac{s \cdot R_1 \cdot C_1}{1 + \dfrac{s}{\omega_{p1}} + \dfrac{s^2}{\omega_{p1} \cdot \omega_{p2}}}$$

Pole splitting: wp1 << wp2

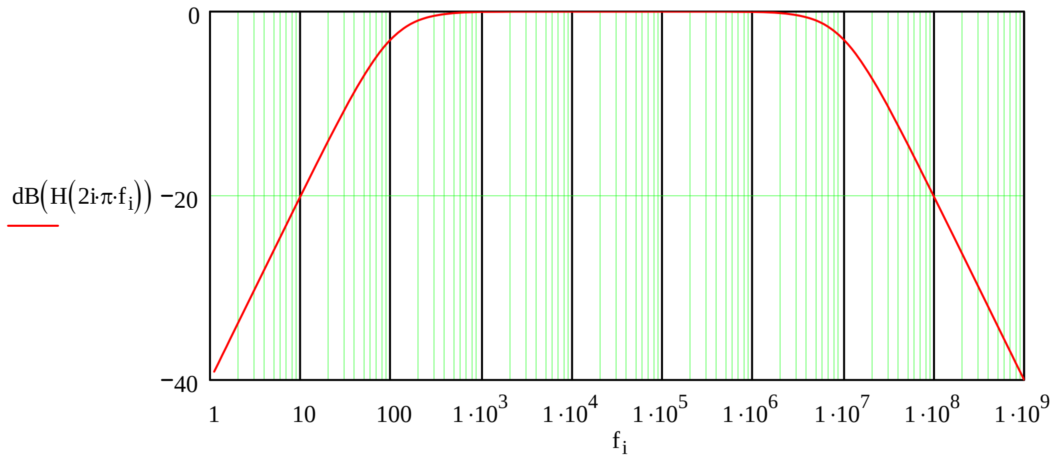
$$H(s) := \frac{s \cdot R_1 \cdot C_1}{1 + s \cdot \left(R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2\right) + s^2 \cdot R_1 \cdot R_2 \cdot C_1 \cdot C_2}$$

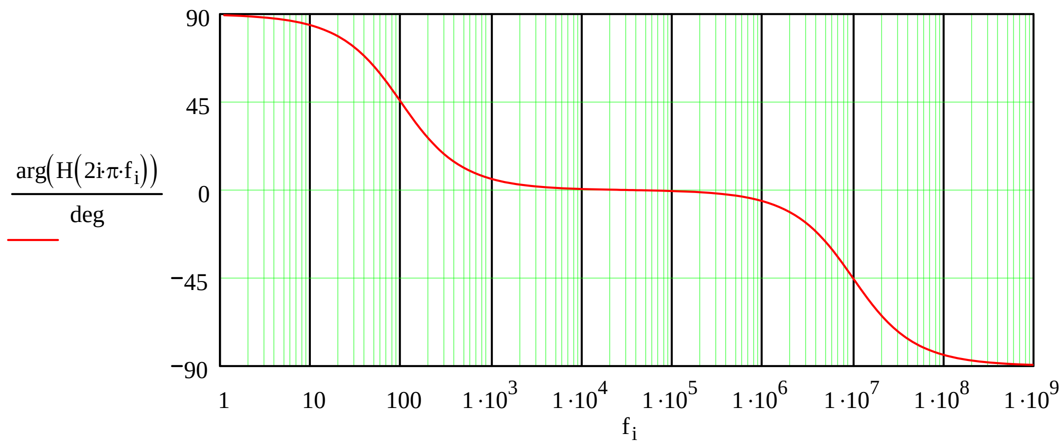
$$H(s) := \frac{s \cdot R_1 \cdot C_1}{\left(1 + \dfrac{s}{\omega_{p1}}\right) \cdot \left(1 + \dfrac{s}{\omega_{p2}}\right)}$$

Bode Plot

$N := 200 \qquad i := 1 .. N \qquad f := \mathrm{logrange}\left(\frac{f_1}{100}, 100 \cdot f_2, N\right)$

3dB bandwidth





Bandpass characteristic:
rejects low and high frequency, passes mid-band

Bode plots capture significant information about behavior of circuit.
Need efficient way to construct and draw them.

Application: Phone

Basspand from 300Hz ... 3kHz

Lecture 30: Constructing Bode Plots

Transfer functions: $H(s) = \frac{N(s)}{D(s)}$

Need roots of N (zeros of H) and roots of D (poles of H) for Bode Plot.
Zeros and poles capture many important characteristics of systems (see other courses).

Then rewrite H(s) with poles and zeros explicitly:

1. Standard form

$$H(s) = K \cdot s^r \cdot \frac{\prod_m \left(1 - \frac{s}{z_m}\right)}{\prod_n \left(1 - \frac{s}{p_n}\right)}$$

z_m zeros of H(s)

p_n poles of H(s)

2. Magnitude response

$$H_m(\omega) = dB(|H(j \cdot \omega)|) = 20 \cdot \log\left[\left|K \cdot s^r \cdot \frac{\prod_m \left(1 - \frac{s}{z_m}\right)}{\prod_n \left(1 - \frac{s}{p_n}\right)}\right|\right]$$

$s = j \cdot \omega$

$$H_m(\omega) = 20 \cdot \log(|K|)$$
$$+ 20 \cdot r \cdot \log(\omega)$$
$$+ 20 \cdot \sum_m \log\left(\left|1 + j \cdot \frac{\omega}{z_m}\right|\right)$$
$$- 20 \cdot \sum_n \log\left(\left|1 + j \cdot \frac{\omega}{z_n}\right|\right)$$

sum individual terms (thanks to log!)

3. Phase response

$$\phi(\omega) = 0 + (r \cdot 90deg) + \sum_m \phi\left(\left|1 - j \cdot \frac{\omega}{z_m}\right|\right) - \sum_n \phi\left(\left|1 - j \cdot \frac{\omega}{p_n}\right|\right)$$

sum individual terms

Example 1: $R := 10k\Omega$ $C := 1pF$

$$H(s) = \frac{1}{1 + s \cdot R \cdot C} = \frac{K}{1 - \frac{s}{p_1}} \qquad K = 1 \qquad p_1 := -\frac{1}{R \cdot C} \qquad f_1 := \left|\frac{p_1}{2 \cdot \pi}\right| \qquad f_1 = 1.592 \times 10^4\,\text{kHz}$$

Magnitude response:

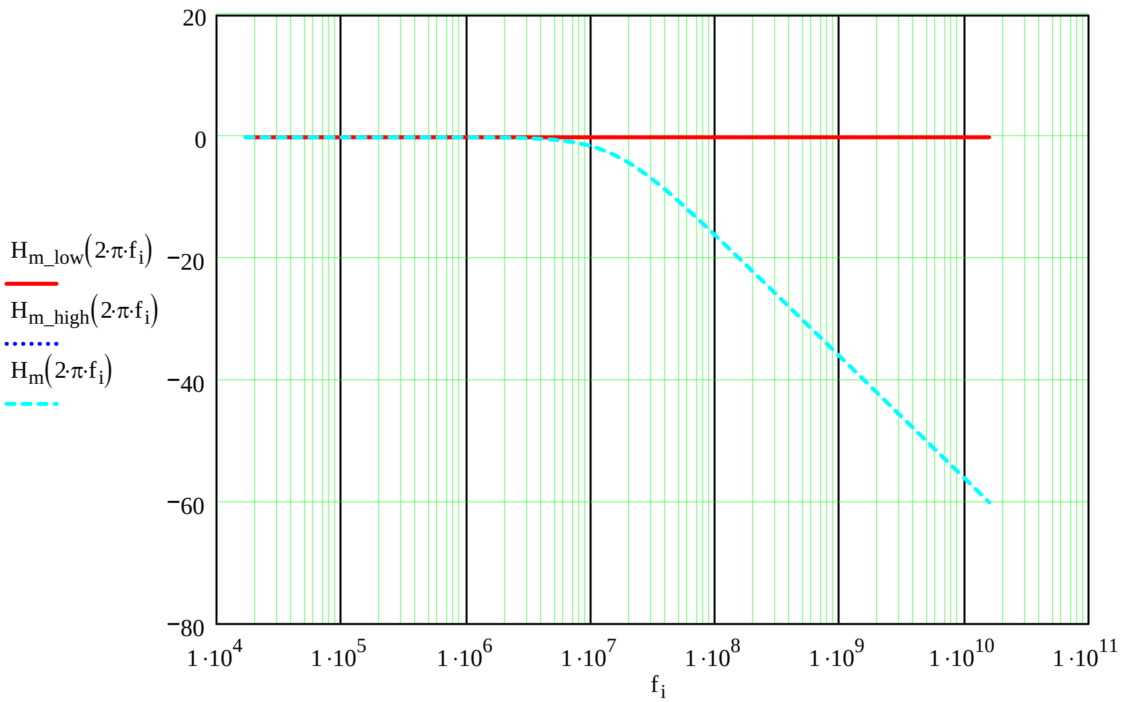
$$H_m(\omega) := -20\log\left(\left|1 - j \cdot \frac{\omega}{p_1}\right|\right) \qquad H_m(\omega) := -10 \cdot \log\left[1 + \left(\frac{\omega}{p_1}\right)^2\right]$$

for $\omega \ll p_1$: $H_{m\_low}(\omega) := 0$

for $\omega >> p_1$: $H_{m\_high}(\omega) := -20\log\left(\frac{\omega}{p_1}\right)$

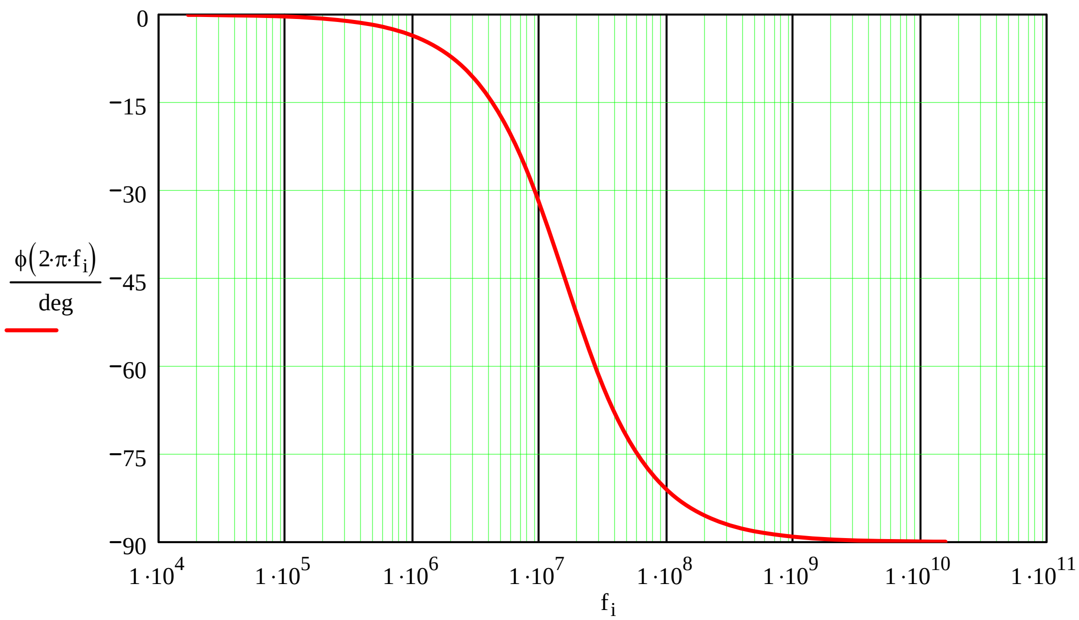
at $\omega = p_1$: $H_m(p_1) = -3.01$

$N := 200$ $i := 1..N$ $f := \text{logrange}\left(\frac{f_1}{1000}, 1000 \cdot f_1, N\right)$



Straight line approximation: lines intersect at $p_1 / 2\pi$

Phase response: $\phi(\omega) := -\arg\left(1 - \frac{j \cdot \omega}{p_1}\right)$ $\phi(\omega) := \operatorname{atan}\left(\frac{\omega}{p_1}\right)$

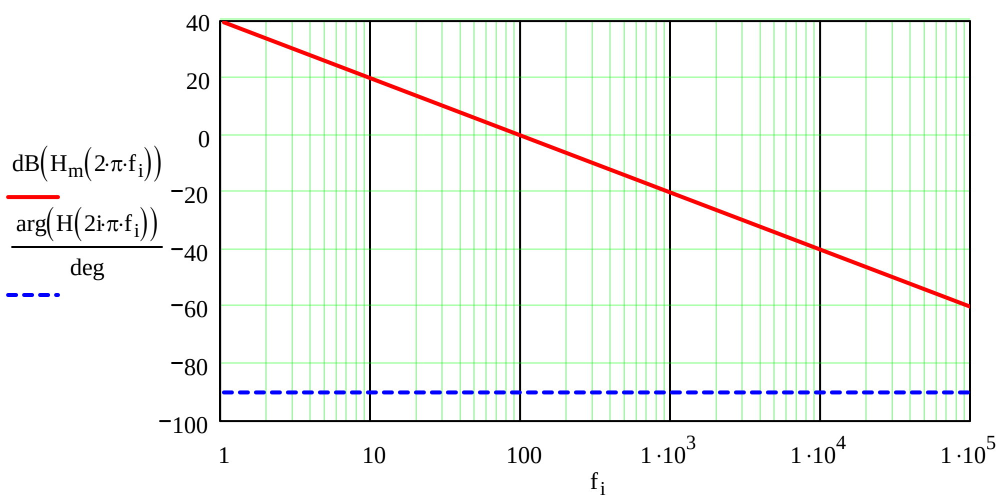


Straight line approximation: 90 deg over 2 decades, -45 deg @ f_1

Example 2: $H(s) := \frac{K}{s}$ $K := 2 \cdot \pi \cdot 100Hz$

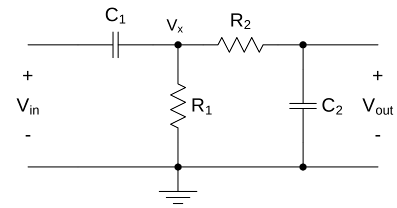
$H_m(\omega) := \frac{K}{\omega}$

$N := 200$ $i := 1 .. N$ $f := \text{logrange}(1Hz, 100kHz, N)$



Example 3: $H(s) := \frac{s \cdot R_1 \cdot C_1}{1 + s \cdot (R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2) + s^2 \cdot R_1 \cdot R_2 \cdot C_1 \cdot C_2}$

Find roots of denominator:



$$1 + s \cdot (R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2) + s^2 \cdot R_1 \cdot R_2 \cdot C_1 \cdot C_2 = 0$$

$a := R_1 \cdot R_2 \cdot C_1 \cdot C_2$ $\quad a = 2.533 \times 10^{-11} s^2$

$b := R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2$ $\quad b = 1.593 \times 10^{-3} s$

$p_1 := \frac{-b + \sqrt{b^2 - 4 \cdot a}}{2 \cdot a}$ $\quad f_1 := \left|\frac{p_1}{2 \cdot \pi}\right|$ $\quad f_1 = 99.9\,\text{Hz}$

$p_2 := \frac{-b - \sqrt{b^2 - 4 \cdot a}}{2 \cdot a}$ $\quad f_2 := \left|\frac{p_2}{2 \cdot \pi}\right|$ $\quad f_2 = 10.01\,\text{MHz}$

check roots:

$$D(s) := 1 + s \cdot (R_1 \cdot C_1 + R_1 \cdot C_2 + R_2 \cdot C_2) + s^2 \cdot R_1 \cdot R_2 \cdot C_1 \cdot C_2$$

$D(p_1) = 1.694 \times 10^{-12}$ should be (nearly) zero

$D(p_2) = -2.91 \times 10^{-11}$

standard form:

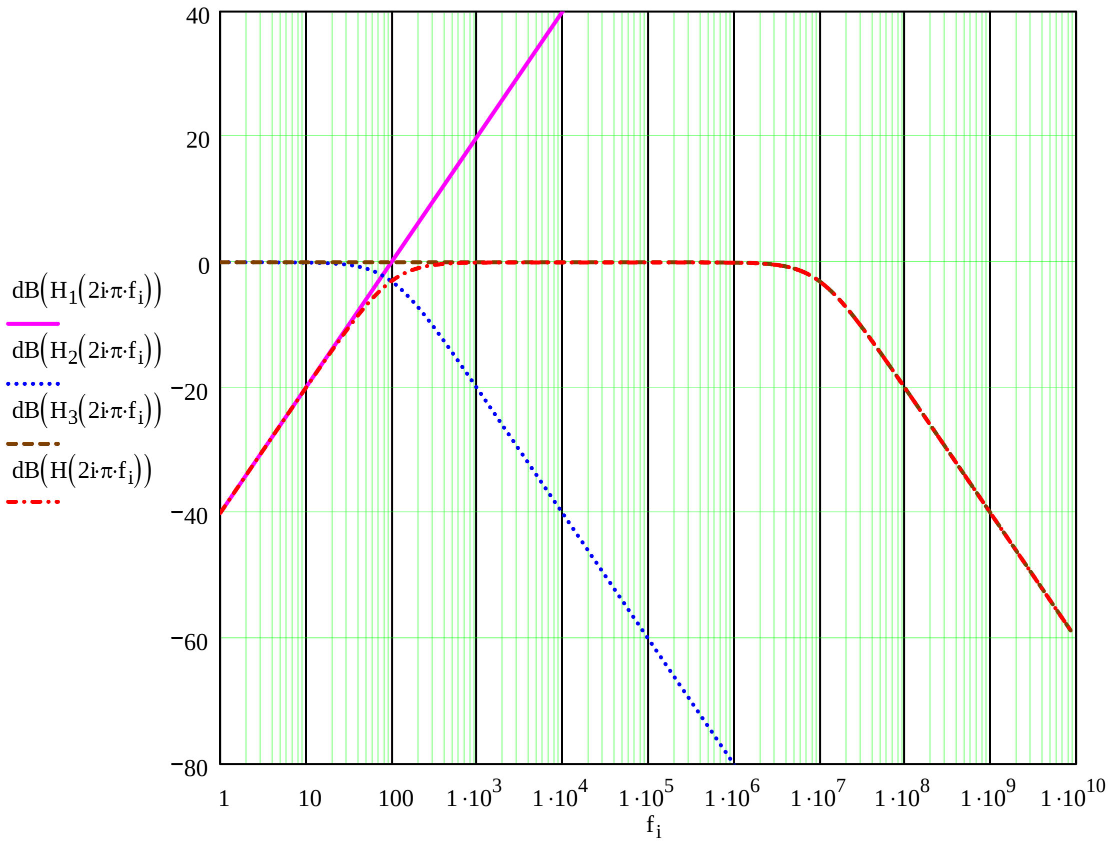
$H(s) := R_1 \cdot C_1 \cdot s \cdot \frac{1}{\left(1 - \frac{s}{p_1}\right)} \cdot \frac{1}{\left(1 - \frac{s}{p_2}\right)}$ $\quad r = 1$

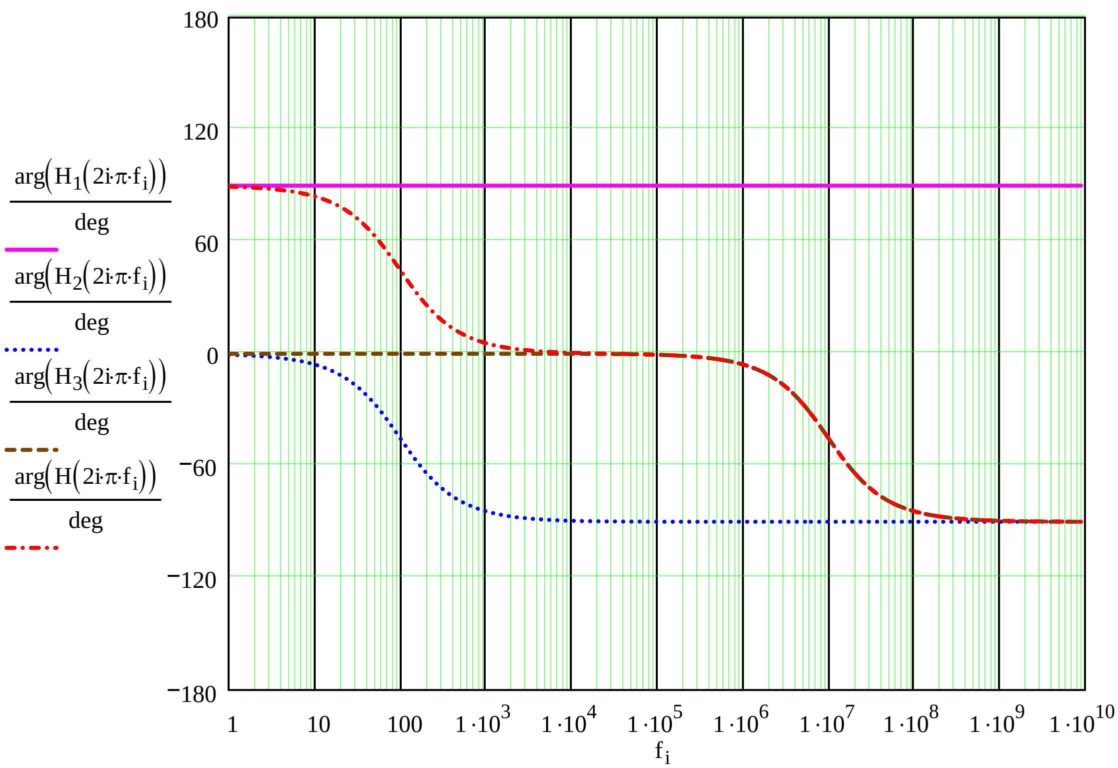
$K = R_1 \cdot C_1$

$H_1(s) := s \cdot R_1 \cdot C_1$ $\quad H_2(s) := \frac{1}{\left(1 - \frac{s}{p_1}\right)}$ $\quad H_3(s) := \frac{1}{\left(1 - \frac{s}{p_2}\right)}$

$\frac{1}{2 \cdot \pi \cdot R_1 \cdot C_1} = 100\,\text{Hz}$

$N := 3000$ $\quad i := 1..N$ $\quad f := \text{logrange}\left(\frac{f_1}{100}, 900 \cdot f_2, N\right)$


dB(H1(2i·π·fi))
dB(H2(2i·π·fi))
dB(H3(2i·π·fi))
dB(H(2i·π·fi))
40
20
0
−20
−40
−60
−80
1
10
100
1·10^3
1·10^4
1·10^5
1·10^6
1·10^7
1·10^8
1·10^9
1·10^10
fi

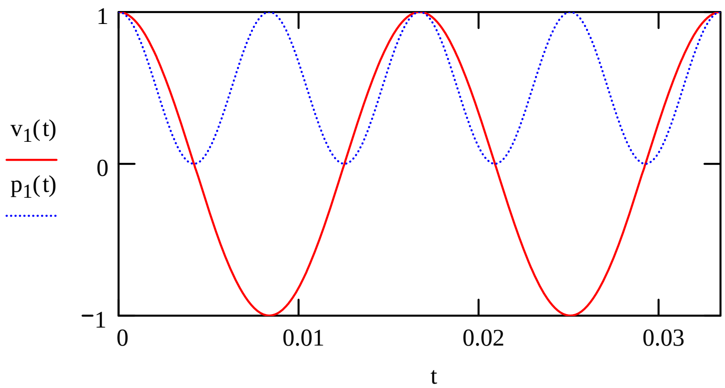

arg(H1(2i·π·fi)) / deg
arg(H2(2i·π·fi)) / deg
arg(H3(2i·π·fi)) / deg
arg(H(2i·π·fi)) / deg
180
120
60
0
−60
−120
−180
1
10
100
1·10^3
1·10^4
1·10^5
1·10^6
1·10^7
1·10^8
1·10^9
1·10^10
fi

Lecture 31: Power

Resistive load

$V_1 := 1V$ $\quad f_1 := 60Hz$ $\quad \omega_1 := 2\pi \cdot f_1$ $\quad R_1 := 1\Omega$ $\quad T_1 := \frac{1}{f_1}$

$v_1(t) := V_1 \cdot \cos(\omega_1 \cdot t)$ $\quad p_1(t) := \frac{v_1(t)^2}{R_1}$



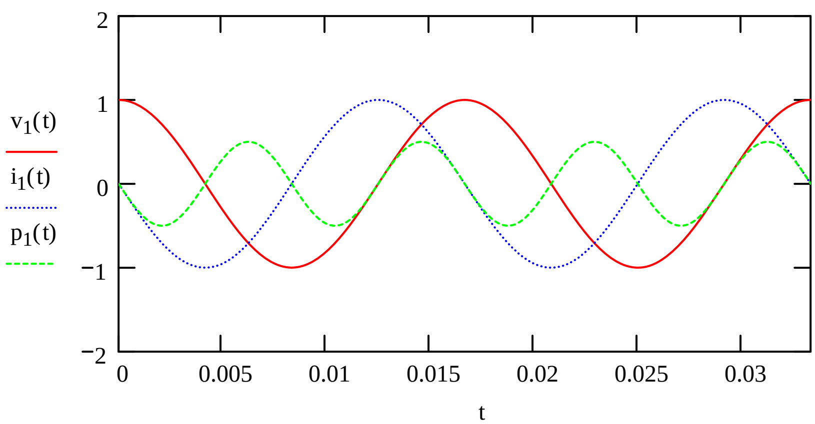
$$P_{rms} = \frac{1}{T_1} \cdot \int_0^{T_1} p_1(t)\,dt = \frac{1}{T_1} \cdot \int_0 \frac{(v_1(t))^2}{R_1}\,dt = \frac{V_{rms}^2}{R_1}$$

$$V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (V_1 \cdot \cos(\omega \cdot t))^2\,dt} = \frac{V_1}{\sqrt{2}} \qquad V_{rms} := \frac{V_1}{\sqrt{2}} \qquad P_{rms} := \frac{V_{rms}^2}{R_1}$$

$P_{rms} = 0.5\,W$

Capacitive Load

$C_1 := \frac{farad}{\omega_1}$ $\quad i_1(t) := C_1 \cdot \frac{d}{dt} v_1(t)$ $\quad p_1(t) := v_1(t) \cdot i_1(t)$



Power Analysis

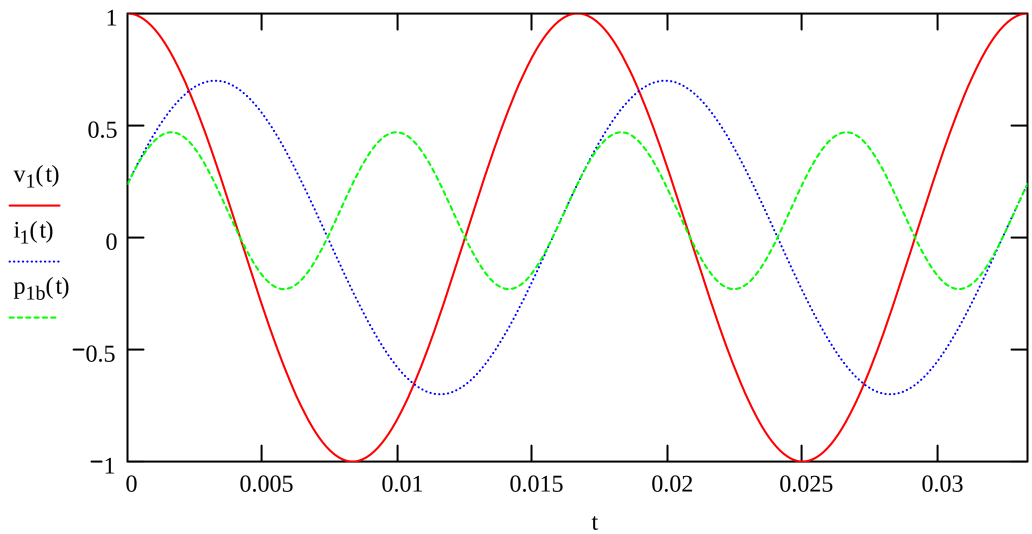
$V_1 := 1V \quad I_1 := 0.7A \quad \phi_1 := 70deg$ Phase: $\phi_1 = \Delta\phi = \phi_v - \phi_i$

$v_1(t) := V_1 \cdot \cos(\omega_1 \cdot t) \qquad i_1(t) := I_1 \cdot \cos(\omega_1 \cdot t - \phi_1)$

$p_{1a}(t) := V_1 \cdot I_1 \cdot \cos(\omega_1 \cdot t - \phi_1) \cdot \cos(\omega_1 \cdot t)$

after some work ...

$$p_{1b}(t) := \frac{V_1 \cdot I_1}{2} \cdot \left[\cos(\phi_1) \cdot (1 + \cos(2 \cdot \omega_1 \cdot t)) + \sin(\phi_1) \cdot \sin(2\omega_1 \cdot t)\right]$$



Nomenclature

Average (real) power: $P_1 := \frac{V_1 \cdot I_1}{2} \cdot \cos(\phi_1)$ power factor $pf := \cos(\phi_1) \quad pf = 0.342$

Reactive power: $Q_1 := \frac{V_1 \cdot I_1}{2} \cdot \sin(\phi_1)$ $rf := \sin(\phi_1) \quad rf = 0.94$

$pf^2 + rf^2 = 1$

$p_1(t) := P_1 \cdot (1 + \cos(2 \cdot \omega_1 \cdot t)) + Q_1 \cdot \sin(2 \cdot \omega_1 \cdot t)$

a) Purely resistive circuits

$\phi_1 := 0deg \qquad P_1 := \frac{V_1 \cdot I_1}{2} \qquad Q_1 = 0$ [W] $P_1 = 350\,mW$

Actually dissipated in load (has a 2w1 ripple on top of average).

b) Purely reactive circuits

$\phi_1 := 90deg \qquad P_1 = 0 \qquad Q_1 := \frac{V_1 \cdot I_1}{2}$ [VAR] $Q_1 = 350\,mW$

Average is zero. Only heats wires (which add a resistive component).

Complex Power

Efficient way to calculate P & Q.

Complex power: $S = P + j \cdot Q$ [VA]

Apparent power: $|S| = \sqrt{P^2 + Q^2}$ [VA]

Power handling capacity of distribution network (e.g. power cord).

... some math ...

$S(s) = \frac{1}{2} \cdot V(s) \cdot \overline{I(s)}$ (complex conjugate of current)

Example 1: computer supply R // C

$V_1 := 110V \cdot \sqrt{2}$ $V_1 = 155.563\,V$ $f_1 := 60Hz$ $\omega_1 := 2 \cdot \pi \cdot f_1$

$R_1 := 1k\Omega$ $C_1 := 20\mu F$ $Z_1(s) := \frac{R_1}{1 + s \cdot R_1 \cdot C_1}$ drops at high freq!!!

$V_1(s) := V_1$ $I_1(s) := \frac{V_1(s)}{Z_1(s)}$ increases at high frequency!

$S_1(s) := \frac{1}{2} \cdot V_1(s) \cdot \overline{I_1(s)}$

$S_1(0Hz) = 12.1\,W$ only resistive part matters

$VA := W$ $mVA := mW$

$kVA := kW$

$VAR := W$

$S_1(j \cdot \omega_1) = 12.1 - 91.232i\,VA$

$|S_1(j \cdot \omega_1)| = 92.031\,VA$ (!!!)

$Re(S_1(j \cdot \omega_1)) = 12.1\,W$ $Im(S_1(j \cdot \omega_1)) = -91.232\,VAR$

$|V_1(j \cdot \omega_1)| = 155.563\,V$ $|I_1(j \cdot \omega_1)| = 1.183\,A$ (!!!) $\frac{V_1(0Hz)}{R_1} = 155.563\,mA$

Current dominated by reactive part

Example 2: loudspeaker with(out) series capacitor

$V_{dc} := 5V \quad V_1 := 2V \quad f_1 := 440Hz \quad \omega_1 := 2\pi \cdot f_1$

$L_1 := \frac{8\Omega}{\omega_1} \quad L_1 = 2.894\,mH \quad R_1 := 1\Omega$

$Z_1(s) := R_1 + s \cdot L_1 \quad I_1(s) := \frac{V_1}{Z_1(s)} \quad S_1(s) := \frac{1}{2} \cdot V_1 \cdot \overline{I_1(s)}$

$S_1(0Hz) = 2\,VA$ just heats speaker! (nobody can hear DC)

$S_1(j \cdot \omega_1) = 30.769 + 246.154i\,mVA$ audio power is much smaller (than heat)

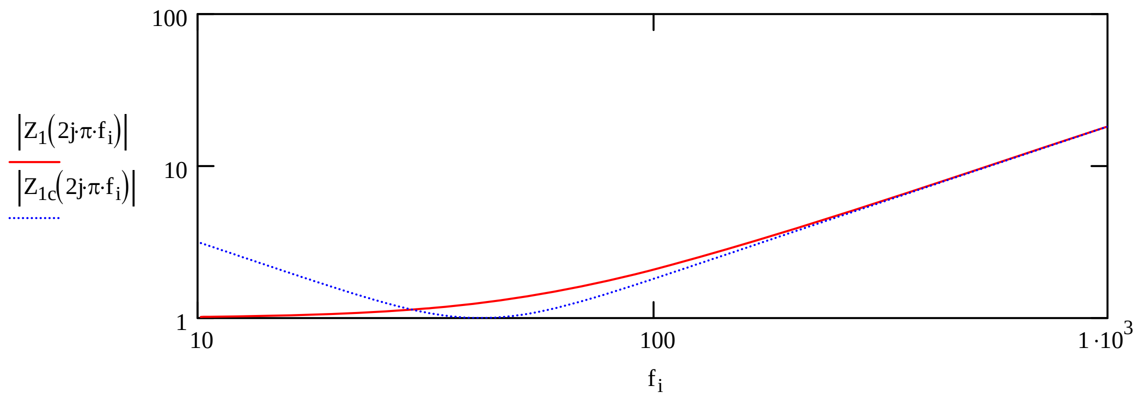
with series capacitor: $C_1 := 5mF$

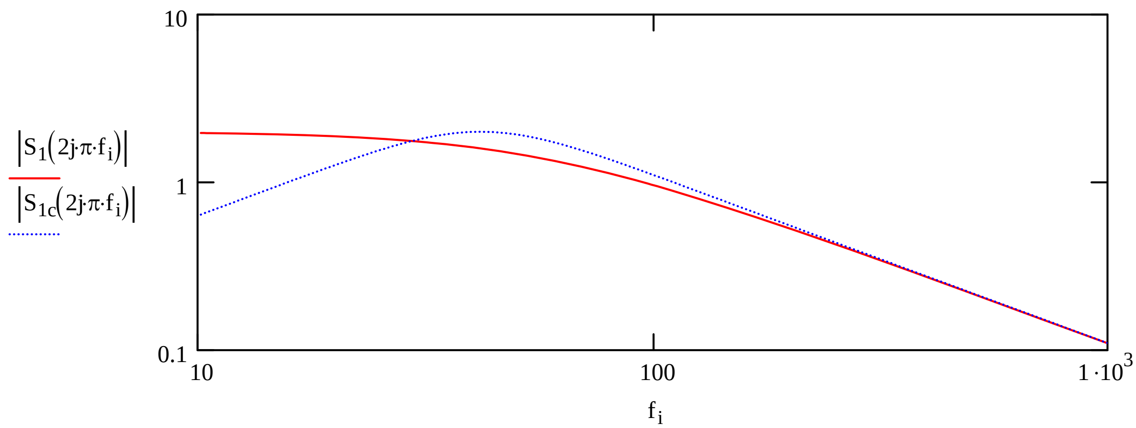
$Z_{1c}(s) := R_1 + s \cdot L_1 + \frac{1}{s \cdot C_1} \quad I_{1c}(s) := \frac{V_1}{Z_{1c}(s)} \quad S_{1c}(s) := \frac{1}{2} \cdot V_1 \cdot \overline{I_{1c}(s)}$

$S_{1c}(j \cdot 10^{-20} \cdot Hz) = 0\,VA$ no heat (DC term)

$S_{1c}(j \cdot \omega_1) = 31.325 + 248.33i\,mVA$ same as w/o C1!

$N := 300 \quad i := 1..N \quad f := logrange(10Hz, 1kHz, N)$





Example 3: correcting the power factor (zero reactive power)

$V_1 := \sqrt{2} \cdot 110V$ $\omega_1 := 2\pi \cdot 60Hz$

Inductive load $Z_L := 40\Omega + j \cdot 20\Omega$

without correction:

$S_a := \frac{V_1^{\,2}}{\overline{Z_L}}$ $S_a = 484 + 242i\,VA$

$I_a := \frac{V_1}{Z_L}$ $I_a = 3.111 - 1.556i\,A$ $|I_a| = 3.479\,A$

with correction:

$Y_c := -\mathrm{Im}\left(\frac{1}{Z_L}\right)$ $Z_c := \frac{1}{Y_c \cdot j}$ $Z_c = -100i\,\Omega$ capacitor!

$Z_{tot} := \frac{Z_L \cdot Z_c}{Z_L + Z_c}$ $Z_{tot} = 50\,\Omega$

$S_b := \frac{V_1^{\,2}}{\overline{Z_{tot}}}$ $S_b = 484\,VA$

Example 4:

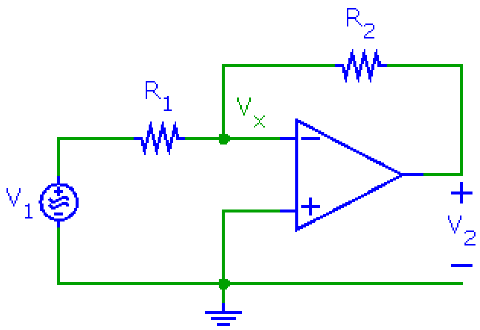
voltage drop in inductive power supply wiring
(model current as sinusoid at fs)

Lecture 32: Finite Opamp Bandwidth

Model: $a(s) = \frac{\omega_b}{s}$ $\qquad |a(j \cdot \omega_b)| = 1$

$|a(0)| = \infty$ like ideal opamp

Note: can model finite gain and bw simultaneously
- more complicated math
- rather use superposition for small errors (system linear)



Given

$$\frac{V_x - V_{1.}}{R_{1.}} + \frac{V_x - V_2}{R_{2.}} = 0$$

$$V_2 = -\frac{\omega_b}{s} \cdot V_x$$

$$\text{Find}(V_2, V_x) \rightarrow \begin{bmatrix} -\omega_b \cdot R_{2.} \cdot \frac{V_{1.}}{(R_{2.} \cdot s + s \cdot R_{1.} + \omega_b \cdot R_{1.})} \\ R_{2.} \cdot V_{1.} \cdot \frac{s}{(R_{2.} \cdot s + s \cdot R_{1.} + \omega_b \cdot R_{1.})} \end{bmatrix}$$

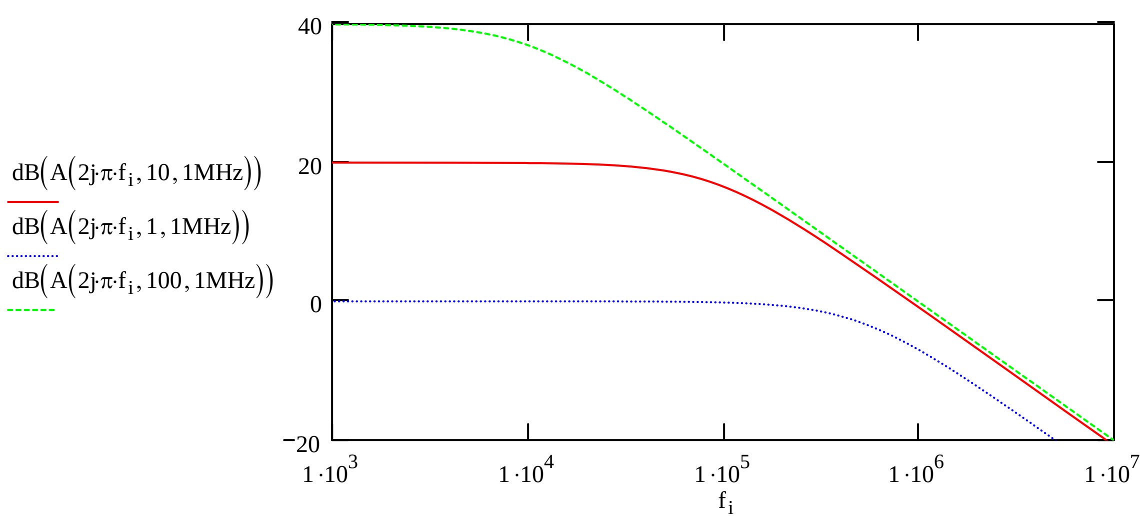
$$A(s) = \frac{V_2(s)}{V_1(s)} = \frac{-\omega_b \cdot R_2}{s \cdot (R_1 + R_2) + R_1 \cdot \omega_b}$$

$$A(s) = -\frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{s}{\omega_b} \cdot \frac{R_1 + R_2}{R_1}} = -A_o \cdot \frac{1}{1 + \frac{s}{\omega_b} \cdot (1 + A_o)} = \frac{-A_o}{1 + \frac{s}{\omega_p}} \qquad \omega_p = \frac{\omega_b}{1 + A_o}$$

$$A(s, A_o, f_p) := \frac{-A_o}{1 + s \cdot \frac{A_o + 1}{2 \cdot \pi \cdot f_p}}$$

$\text{dB}(A(2j \cdot \pi \cdot 100\text{kHz}, 10, 1\text{MHz})) = 16.556$ dB

$N := 300 \qquad i := 1 .. N \qquad f := \text{logrange}(10\text{Hz}, 10\text{MHz}, N)$

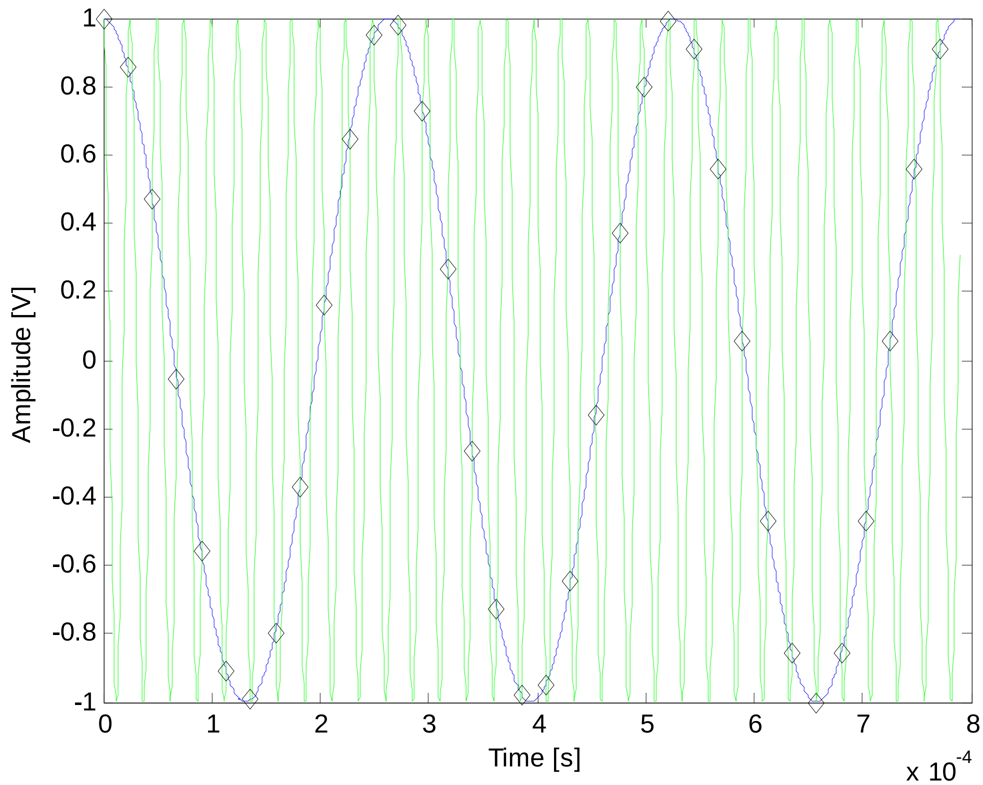


Lecture 33: Electronic Noise

Digital: finite # bits limits resolution
Analog: many errors, interference etc --> shield or use other methods to reduce
ultimate: finite charge of electrons

Lecture 34: Filters

Example: Sampling



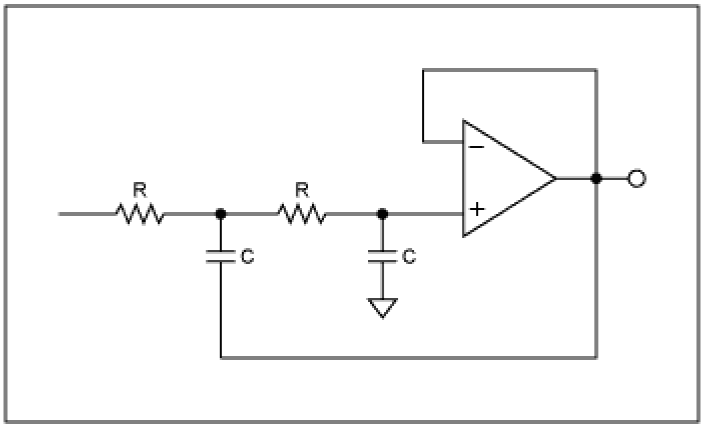
Show spectrum of input signal and alias.
Sampling theorem: frequencies > fs/2 alias --> filter out!

$f_s := 44.1\text{kHz}$ $\quad B := 20\text{kHz}$ $\quad \omega_p := 2 \cdot \pi \cdot B$ $\quad$ 3dB bandwidth

$$H(s) := \frac{1}{1 + \frac{s}{\omega_p}} \qquad \left| H\left(2j \cdot \pi \cdot \frac{f_s}{2}\right) \right| = 0.672 \qquad dB\left(H\left(2j \cdot \pi \cdot \frac{f_s}{2}\right)\right) = -3.455$$

http://www.maxim-ic.com/appnotes.cfm/an_pk/1762 see also MAX 274/5

Sallen Key:



$$H(s) = \frac{1}{1 + s\cdot\tau + (s\cdot\tau)^2}$$

Given

$$1 + s\cdot\tau + (s\cdot\tau)^2 = 0$$

$$\text{Find}(s) \rightarrow \left[\frac{\left(\frac{-1}{2} + \frac{1}{2}\cdot i\cdot\sqrt{3}\right)}{\tau} \quad \frac{\left(\frac{-1}{2} - \frac{1}{2}\cdot i\cdot\sqrt{3}\right)}{\tau} \right]$$