Advanced Analog Integrated Circuits

Operational Transconductance Amplifier I & Step Response

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High-Level View

- Transistors are transconductors
- Some OTA designs consist of >40 transistors
  - Only few (typically 1 … 2) provide the transconductance in the signal path
  - The rest is support, e.g.
    - increasing low frequency gain
    - output voltage range
    - biasing
- Hierarchical design strategies are imperative
  - Unless you like nodal equations for 40 transistors …
Divide and Conquer
In SC Circuit

![Image of SC Circuit Diagram]

B. E. Boser

EE240B – OTA I: Step Response
In SC Circuit

Amplifier is active only in $\phi_2$:
Capacitive Feedback … Simulation

\[ R_{sx} \approx 20 \ \text{G}\Omega \]
Small-Signal Model

Absorb $C_{gd}$, $C_{db}$, etc. in $C_f$, $C_L$, …
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Loop-Gain & Stability

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Model
Loop-Gain Analysis
Loop-Gain $T(s)$

\[
T(s) = \beta \frac{g_m}{s \cdot C_{L_{tot}}}
\]

Feedback factor 
\[
\beta = \frac{v_x}{v_o} = \frac{C_f}{C_f + C_s + C_x} \neq \frac{1}{a_{v_o}}
\]

Effective Load Capacitance 
\[
C_{L_{tot}} = C_L + (1 - \beta)C_f
\]
Frequency Response of $T(s)$

$$T(s) = \frac{\beta g_m}{s \cdot C_{L_{tot}}}$$
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Closed-Loop Response

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Closed-Loop Transfer Function

\[ s_i \xrightarrow{\Sigma} d \xrightarrow{\Sigma} s_o \]

\[ TA_\infty \]

\[ 1/A_\infty \]
$A_{\infty}$
Closed-Loop Gain

\[ A(s) = A_{\infty} \frac{T(s)}{1 + T(s)} + \frac{d}{1 + T(s)} = -\frac{C_s}{C_f} \frac{\beta g_m}{s \cdot C_{Ltot}} + \frac{\beta}{1 + \frac{C_s}{C_{Ltot}}} \]

\[ = -\frac{C_s}{C_f} \frac{1 - s}{g_m} \frac{C_f}{C_{Ltot}} = -\frac{C_s}{C_f} \frac{1 - s}{z} \]

\[ p = -\frac{\beta g_m}{C_{Ltot}} \quad \text{and} \quad z = +\frac{g_m}{C_f} \]
T(s) Verification with Simulator

- Infinite impedance point to break loop usually not accessible
  - E.g. inside transistor

[ Hspice manual ]
Workaround (Don’t do this!)

[ Drawing by B. Murmann ]
General Problem

Any “single loop“ feedback circuit can be represented as:

\[ T(s) = g_m \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} \]

Breakpoint at ideal source is not available.
But there is a breakpoint “between finite impedances“
Middlebrook Double Injection Method

\[ \frac{v_y}{v_x} \equiv T_v = g_m \cdot Z_2 + \frac{Z_2}{Z_1} \]

True Loop Gain:
\[ T = g_m \cdot \frac{Z_1 Z_2}{Z_1 + Z_2} \]

Solving yields:
\[ T = \frac{T_v T_i - 1}{T_v + T_i + 2} \]

- No “DC“ break in the loop, all loading effects covered.
- Measure \( T_v \) and \( T_i \), then calculate actual \( T \)
- Variant implemented in many simulators, e.g. stb-analysis in Spectre
SPICE
Multiple Feedback Loops

- Break all loops at a single point
- If such a point does not exist: [Bode 45]

“If a circuit is stable when all its tubes have their nominal gains, the total number of clockwise and counterclockwise encirclements of the critical point must be equal to each other in the series of Nyquist diagrams for the individual tubes obtained by beginning with all tubes dead and restoring the tubes successively in any order to their nominal gains”

– Suggestion: take a controls class if your circuit depends on this!
Nonlinearities

• Real circuits are nonlinear – frequency response depends on signal amplitude. Bode:

  “... thus the circuit may sing when the tubes begin to lose their gain because of age, and it may also sing, instead of behaving as it should, when the gain increases from zero as power is supplied to the circuit ...“

• Always run a ”large signal” transient analysis for a complete stability check!
  – With realistic (large) signal amplitudes including overload
  – Power supply ramping. Is a special order required?
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Settling

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Step Response

Dynamic Error $\varepsilon_d(t)$

Static Error $\varepsilon_0$

$V_{out}/V_{out \text{ ideal}}$

t/$\tau$
Closed-Loop Gain

- Use favorite analysis method, e.g. return-ratio analysis*, nodal analysis, …

\[ A(s) = \frac{V_o(s)}{V_i(s)} = -\frac{C_s}{C_f} \frac{1 - s \frac{C_f}{g_m}}{1 + s \frac{C_{Ltot}}{\beta g_m}} = -\frac{C_s}{C_f} \frac{1 - \frac{s}{\omega_p}}{1 - \frac{s}{\omega_z}} \]

\[ \omega_p = \frac{\beta g_m}{C_{Ltot}} \approx \omega_{-3dB} \text{ of } A(s) = \omega_u \text{ of } T(s) \]

\[ \omega_z = \frac{g_m}{C_f} \]

\[ \frac{\omega_z}{\omega_p} = \frac{C_{Ltot}}{\beta C_f} \text{ usually } \gg 1 \]

*Note: 2-port analysis ignores the feedforward path and therefore does not get the zero
Dynamic Error (no zero)

Assume switch on-resistance contribution to settling is negligible
Dynamic Settling Error
## Dynamic Settling Error (single pole)

<table>
<thead>
<tr>
<th>$\varepsilon_d$</th>
<th>$t_s/\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>4.6</td>
</tr>
<tr>
<td>0.1%</td>
<td>6.9</td>
</tr>
<tr>
<td>0.01%</td>
<td>9.2</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>13.8</td>
</tr>
</tbody>
</table>
Amplifier Bandwidth versus $f_s$

<table>
<thead>
<tr>
<th>$\varepsilon_d$</th>
<th>$f_{-3dB}/f_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>1.5</td>
</tr>
<tr>
<td>0.1%</td>
<td>2.2</td>
</tr>
<tr>
<td>0.01%</td>
<td>2.9</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>4.4</td>
</tr>
</tbody>
</table>
Static Settling Error
Example

\[ C_s = 4\text{pF} \quad C_f = 1\text{pF} \quad C_x = 1\text{pF} \quad g_m r_o = 6000 \]
Dynamic Error (with zero)

• Instant response to step determined by capacitive feed-forward

\[
A(s) = \frac{V_o(s)}{V_i(s)} = -\frac{C_s}{C_f} \frac{1 - s \frac{C_f}{g_m}}{1 + s \frac{C_{Ltot}}{\beta g_m}}
\]

\[
z = + \frac{g_m}{C_f} \quad \quad \quad p = -\frac{\beta g_m}{C_{Ltot}}
\]
Step Response with Zero

\[ v_{o,\text{step}}(t) = -V_{\text{step}} \cdot G_i \left\{ 1 - \left(1 - \frac{p}{z}\right) e^{-t/\tau} \right\} \]

\[
1 - \frac{p}{z} = \frac{1}{1 - \beta \frac{C_f}{C_f + C_L}}
\]

\[
t_s = -\tau \cdot \ln \left\{ \varepsilon_d \left( 1 - \beta \frac{C_f}{C_f + C_L} \right) \right\}
\]
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Phase Margin

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Cascode
Phase Margin

\[ \frac{\left| T_\delta(f, \infty) \right|}{\left| \frac{T_\delta(f, 5)}{T_\delta(f, \frac{1}{5})} \right|} \]

\[ \text{Phase Margin} \]

\[ \infty \quad 90^\circ \]

\[ 5 \quad \sim 80^\circ \]

\[ 1/5 \quad \sim 28^\circ \]
Relative Settling Error for $f_{p2}/f_u = 3$
Settling Time versus $\frac{f_{p2}}{f_u}$

$$\varepsilon_d = 0.1\%$$
Noise

M2 noise rolls in at high freq.
CT: Noise may be filtered out
SC: Noise will alias

[ B. Murmann ]
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Design Example

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Design Example: Specification

Closed-loop gain (magnitude): \( A_v := 2 \)

Settling time: \( t_s := 5\text{ns} \)

Dynamic settling accuracy: \( \epsilon_d := 0.02\% \)

Static settling accuracy: not specified (later)

Dynamic range at output: \( \text{DR} := 10^7 \)

Supply voltage: \( V_{dd} := 1.8\text{V} \)

Power: minimum

\( f_{s\_max} := \frac{1}{2 \cdot t_s} = 100\text{MHz} \)
**Design: Gain & Feedback Factor**

Sampling capacitance:  
$$C_s := 4 \text{pF}$$  
set by previous stage / iterate ...

Feedback capacitance:  
$$C_f := \frac{C_s}{A_{vo}}$$  
$$C_f = 2 \cdot \text{pF}$$

"Maximum" input capacitance:  
$$C_{x_{\text{max}}} := 0.7 \cdot (C_s + C_f)$$  
$$C_{x_{\text{max}}} = 4.2 \cdot \text{pF}$$

Actual input capacitance (iterate):  
$$C_x := 0.7 \text{pF}$$

Feedback factor:  
$$\beta := \frac{C_f}{C_f + C_s + C_x}$$  
$$\beta = 0.299$$
Available output voltage range:

\[ V_{\text{opp}} := V_{\text{dd}} - 400 \text{mV} \]

Total noise at output:

\[ N_{\text{ot}} := \frac{1}{2} \cdot \left( \frac{V_{\text{opp}}}{2 \cdot \text{DR}} \right)^2 \]

\[ \sqrt{N_{\text{ot}}} = 156.525 \cdot \mu \text{V} \]

Total noise at output:

\[ N_{\text{ot}} = \frac{k \cdot T}{\beta} \cdot \left( \frac{1}{C_f} + \frac{\alpha}{C_{\text{Ltot}}} \right) \]

OTA noise factor (topology & bias):

\[ \alpha := 4 \]

Sampling noise (phase 1):

\[ \sqrt{\frac{k_B \cdot T_r}{\beta} \cdot \frac{1}{C_f}} = 81.89 \cdot \mu \text{V} \quad \text{??} \quad \sqrt{N_{\text{ot}}} = 156.525 \cdot \mu \text{V} \]

Total load capacitance:

\[ C_{\text{Ltot}} := \frac{\alpha}{N_{\text{ot}}} \cdot \frac{\beta}{k_B \cdot T_r} - \frac{1}{C_f} \]

\[ C_{\text{Ltot}} = 3.015 \cdot \text{pF} \]

Total load capacitance:

\[ C_{\text{Ltot}} = C_L + (1 - \beta) \cdot C_f \]

Load capacitance:

\[ C_L := C_{\text{Ltot}} - (1 - \beta) \cdot C_f \]

\[ C_L = 1.612 \cdot \text{pF} \]
Design: Settling

Settling time (single pole, no slewing):

\[ t_s = -\tau \cdot \ln \left[ \varepsilon_d \cdot \left( 1 - \beta \cdot \frac{C_f}{C_f + C_L} \right) \right] \]

Settling time constant:

\[ \tau := \frac{-t_s}{\ln \left[ \varepsilon_d \cdot \left( 1 - \beta \cdot \frac{C_f}{C_f + C_L} \right) \right]} = 574.854 \cdot \text{ps} \]

Settling time constant:

\[ \tau = \frac{C_{L\text{tot}}}{\beta \cdot g_m} \]

\[ f_u := \frac{1}{2 \cdot \pi \cdot \tau} = 276.862 \text{ MHz} \]

Transconductance:

\[ g_m := \frac{C_{L\text{tot}}}{\beta \cdot \tau} = 17.57 \cdot \text{mS} \]
Design: Power Dissipation

Minimum cutoff frequency:

\[ f_T := \frac{1}{2 \cdot \pi} \cdot \frac{g_m}{C_x} = 3.995 \cdot \text{GHz} \]

Channel length:

\[ L_1 := 250\text{nm} \]

Current density:

\[ V_{\text{star}} := 120\text{mV} \]

Actual \( f_T \):

\[ f_{T_{\text{actual}}} := 6\text{GHz} \quad >? \quad f_T = 3.995 \text{GHz} \]

Actual \( C_x \) (update):

\[ C_{x_{\text{actual}}} := \frac{g_m}{2 \cdot \pi \cdot f_{T_{\text{actual}}}} = 0.466 \text{pF} \quad <? \quad C_x = 0.7 \text{pF} \]

Bias current:

\[ I_d := \frac{g_m \cdot V_{\text{star}}}{2} = 1.054 \cdot \text{mA} \]
Could we go Faster?
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Gain Boosting

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Openloop Gain

\[ a_{vo} = g_m R_o \]
Openloop Gain

\[ a_{vo} = g_m R_o \]
Gain Boosting
Gain Boosting

• Use feedback to boost low-frequency output resistance

• References
High Frequency Analysis
Overall Amplifier Response

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Pole-Zero Doublets

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Pole-Zero Doublets

Doublet Settling

- Amplifier model: replace $G_{mo}$ with

\[
G_m(s) = G_{mo} \frac{\omega_z}{1 + \frac{s}{\omega_p}}
\]

Doublet Settling

- Amplifier model: replace $G_m o$ with

$$G_m(s) = G_{mo} \frac{\omega_z}{1 + \frac{s}{\omega_p}}$$

with

$$\alpha = 1 + \varepsilon \quad \text{with} \quad |\varepsilon| << 1$$

$$\omega_p = \beta \omega_{-3dB}, \quad \omega_{-3dB} \quad \text{is bandwidth of } T(s)$$

- Closed-loop response

$$\frac{V_o}{V_{in}} = -c \frac{1}{1 + s C_{Leff}} \left( 1 + \frac{s}{\omega_{-3dB}} \right) \approx - \frac{c}{1 + \frac{s}{\omega_p}} \left( 1 + \frac{s}{\omega_z} \right)$$

with

$$\omega_{-3dB} = \frac{FG_{mo}}{C_{Leff}}$$

$$\omega_{pp} \cong \omega_p$$

Doublet Step Response

\[ v_{o,\text{step}}(t) = -cV_{\text{step}} \left( 1 + Ae^{-t \omega_{-3dB}} + Be^{-t \omega_{pp}} \right) \]

with

\[ A \approx -1 \]

\[ B \approx \varepsilon \frac{\beta}{1 - \beta^2} \]
Doublet Example

\[ \alpha = 1.5 \]
\[ \beta = 0.3 \]
Gain Boosting – Doublets