1. In the circuit below, $V_{ic}$ is adjusted such that $V_{ic} = 0$ V.

   a) Draw the low-frequency small-signal differential-mode half-circuit model. Calculate the values of all low-frequency small-signal parameters ($r_o$'s, etc.).
   b) Calculate the value of the low-frequency small-signal differential-mode gain, $A_{dm}$, of the circuit.
   c) Draw the low-frequency small-signal common-mode half-circuit model. Specify the values of all low-frequency small-signal parameters.
   d) Calculate the value of the low-frequency small-signal common-mode gain, $A_{cm}$, of the circuit.
   e) Because of manufacturing imperfections, the values of resistors $R_1$ and $R_2$ are slightly different: $R_1 = R_o + \Delta R/2$ and $R_1 = R_o + \Delta R/2$ with $R_o = 10$ kΩ and $\Delta R = 0.05R_o$. Calculate the low-frequency small-signal common-mode to differential gain, $A_{cdm}$, and the common-mode rejection ratio, CMRR.
   f) Describe a circuit modification resulting in at least a two-orders-of-magnitude improvement of the CMRR. You do not need to design the modification.

   ![Circuit Diagram]

   a) $I_D = \frac{5V}{10k\Omega} = 0.5 \text{ mA}$
   \[ g_{m1} = \sqrt{2\mu C_{ox} \frac{W}{L} I_D} = 3.79 \text{ mS} \]
   \[ r_o = \frac{1}{\lambda I_D} = 100 \text{ k}\Omega \]
   b) $A_{dm} = \frac{g_{m1}(R_1 || r_o)}{R_2} = 34.5$
   e) $A_{cdm} = \frac{\Delta R}{2R_3 + \frac{1}{g_{m1}}} = 0.117$
   \[ \text{CMRR} = \frac{A_{dm}}{A_{cdm}} = 323.6 \]
   c) $A_{cm} = \frac{g_{m1}R_1}{1 + 2g_{m1}R_3} = 2.345$
   d) $R_3$ with a current source, source resistance at least $100 \times R_3$. 
First try: current mirror $R_{out} = \frac{1}{2I_D} = 1.275 \text{ M}\Omega$  
$\lambda = 0.02 \text{ V}^{-1}$, $I_D = 39 \mu A$ 
doesn't meet spec = try cascode

Second try: cascode  
$R_{out} = \frac{g_m}{2} R_0^2$  \hspace{1cm} (1)

$g_m = \frac{2I_D}{V_{GS}-V_{TH}} = \frac{2I_D}{\sqrt{2I_D W \mu C_X \frac{W}{L}}} \hspace{1cm} (2)$

$R_0 = \frac{1}{2I_D} \hspace{1cm} (3)$

Combine equations (1), (2), (3):

$\lambda = 0.02 \text{ V}^{-1}$, $I_D = 39 \mu A$, $\mu m = 300 \text{ cm}^2/\text{Vs}$  
$C_X = 10 \text{ fF/um}^2$, $L = L_{min} = 180 \text{ nm}$, $W = 1 \text{ um}$

$R_{out} = 587 \text{ M}\Omega$ \hspace{1cm} meets spec

We could lower $R_{out}$ to 10 M\Omega and size W accordingly to minimize the transistor sizes.

Showing cascode biasing is important in schematic
Q3)

\[ \omega_{3dB} = \frac{1}{C_s R_{in}} \]

\[ R_{in} = \frac{1}{g_m} \]

\[ 2\pi \times 500\,MHz = \frac{1}{10pF \times R_{in}} \]

\[ R_{in} = 31.8\Omega \]

\[ R_{in} = \frac{1}{g_m} \Rightarrow g_m = 31.4mS \]

\[ g_m = \frac{I_C}{V_T} \Rightarrow I_C = I_{dc} = 785\mu A \]

\[ V_{0,\text{Low Frequency}} = i_s \times R_X \Rightarrow R_X = 1k\Omega \]

Q4)

We know from previous that the resistance looking down from the output node is:

\[ R_{0,\text{Down}} \approx g_m R_o^2 \]

This resistance is probably much larger than 2k so we can assume the low frequency gain is:

\[ a_v = -g_m R_L \Rightarrow g_m = \frac{10}{2k} = 5mS \]
The dominant pole is in input since the resistance is larger than output ($R_L$) and the cascade node ($1/g_m$). Also the capacitor is larger since there is miller effect and the gate-source capacitor.

\[
\omega_{3dB} = \frac{1}{R_s(C_{gs,1} + 2C_{ov,1})}
\]

\[
2\pi \times 800 MHz = \frac{1}{R_s\left(\frac{2}{3}WL_{ox} + WC_{ov} + 2WC_{ov}\right)}
\]

\[
2\pi \times 800 MHz = \frac{1}{10K\left(\frac{2}{3}W0.18 \times 10e-15 + 3W \times 0.2e-15\right)}
\]

\[W \approx 11.05 \mu m\]

\[g_m = \sqrt{\frac{2W}{L} \mu_n C_{ox} I_D}\]

\[I_D = 678 \mu A\]

\[V_{dsat} = \frac{g_m}{W/L \mu_n C_{ox}} = 270 mV\]

\[V_{GS} = V_{dsat} + V_{Th} = 670 mV\]

\[V_{Bias,min} = V_{dsat} + V_{GS} = 940 mV\]

\[r_0 = \frac{1}{\lambda I_D} = \frac{1}{0.02 * 678e-6} = 73.7 k\Omega \Rightarrow g_m r_0^2 = 27.2 M\Omega!\]
5. In the circuit below, $M_1$ and $M_2$ are used as switches to control current flow between nodes $V_1$ and $V_2$. The control voltages $V_{c1}$ and $V_{c2}$ are set to 0 V and 3 V to turn the switch on, and 3 V and 0 V to turn the switch off.

Determine the minimum width of $M_1$ and $M_2$ required such that the maximum resistance $R_{on}$ between nodes $V_1$ and $V_2$ is 10 Ω when the switch is on and $V_1$ varies between 0 V and 3 V for $V_2 \approx V_1$. Choose $L_1 = L_2 = 180$ nm. For which value of $V_1$ does $R_{on}$ reach its maximum?

Relevance: thousands of switches like this one tick along in the analog-to-digital and digital-to-analog converters used in audio and video cards, cameras, or radios.

\[ R_{on} = R_{on1} \| R_{on2} \]

$M_1$, $M_2$ in triode:

\[ R_{on1} = \frac{1}{M_n C_{ox} \left( \frac{W}{L} \right) \left( V_1 - V_{c1} - |V_{TH}| \right)} \quad V_1 - V_{c1} > |V_{TH}| \]

\[ R_{on2} = \frac{1}{M_n C_{ox} \left( \frac{W}{L} \right) \left( V_{c2} - V_1 - V_{TH} \right)} \quad V_{c2} - V_1 > V_{TH} \]

\[ R_{on} = \frac{1}{R_{on1}} + \frac{1}{R_{on2}} = \frac{1}{M_n C_{ox} \left( \frac{W}{L} \right) \left( V_1 - 0.4V \right) + M_n C_{ox} \left( \frac{W}{L} \right) \left( V_{2.6} \right)} \]

Set $W_1 = 2W_2$

\[ R_{on,\text{max}} = \frac{1}{2M_n C_{ox} \left( \frac{W_2}{L} \right) \left( 2.2V \right)} = 10.5 \quad |0.4 \leq V_1 \leq 2.6 \text{V}| \]

\[ \begin{align*} 
W_2 &= 2.73 \text{ mm} \quad W_1 = 54.5 \text{ mm} 
\end{align*} \]
\[ \frac{I_{D1}}{I_{D2}} = \frac{\frac{1}{2} M P \co (\frac{W}{L})_1 (V_{DD} - V_G - V_{TH})^2}{\frac{1}{2} M P \co (\frac{W}{L})_2 (V_{DD} - V_G - V_{TH})^2} = \frac{\omega_1}{\omega_2} = \frac{1}{10} \]

(b) \[ I_{D1} = I_{C1} = I_s e^{\frac{V_{BE1}}{V_T}} \]
\[ I_{D2} = I_{C2} = I_s e^{\frac{V_{BE2}}{V_T}} \]
\[ \frac{I_{D1}}{I_{D2}} = \frac{1}{10} = e^{-\frac{(V_{BE1} - V_{BE2})}{V_T}} \]

\[ V_{BE1} - V_{BE2} = (V_B - V_{E1}) - (V_B - V_{E2}) = V_{E1} = I_{D1} R_1 \]

Combine (1) & (2):
\[ I_{D1} = \frac{kT}{qR_1} ln(10) \]

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<tr>
<th>T</th>
<th>V_{DD}</th>
<th>I_{D1}</th>
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<tr>
<td>300K</td>
<td>3V</td>
<td>59.5 mA</td>
</tr>
<tr>
<td>200K</td>
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<tr>
<td>300K</td>
<td>5V</td>
<td>59.5 mA</td>
</tr>
</tbody>
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(c) The mobility dependence cancels for \( I_{D1} \). The transistors are also at the same temperature.