Effect of Frame Synchronization Errors on Pilot-aided Channel Estimation in OFDM: Analysis and Solution

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Abstract
Channel estimation in Orthogonal Frequency Division Multiplexing (OFDM [1]) systems requires transmission of pilot tones, especially in a high delay spread environment. The number of pilot tones needed exceeds the maximum length of the channel delay (normalized to the symbol period) by one. The best estimate of the channel values in between pilots can be acquired by an IFFT, zero padding and an FFT [2]. Such an estimator gives perfect estimate of the channel in the absence of noise and synchronization errors. However, in the presence of frame synchronization errors, its performance degrades, especially for timing offsets to the right. In this paper, we analyze the sensitivity of such channel estimator to frame synchronization errors, explaining how it is more affected by timing offsets to the right than those to the left. Then through utilizing this sensitivity, we propose a low complexity method to correct frame synchronization errors, which can improve the performance considerably without a need for added training overhead.

Keywords
OFDM, Pilot-aided channel estimation, Frame synchronizer

INTRODUCTION
Orthogonal Frequency Devision Multiplexing (OFDM) handles Inter-Symbol-Interference (ISI) resulting from delay spread by transmitting low rate data on narrow sub-channels in parallel [1]. This allows for an overall high speed transmission and has resulted in the recognition of OFDM as a standard for Digital Audio Broadcasting (DAB [3]), Digital Video Broadcasting (DVB [4]) and Wireless Local Area Networks. However, the performance of an OFDM system is affected by both channel estimator and frame synchronizer. Several methods have been proposed for timing synchronization. One is based on transmitting two identical frames [5]. Muller [6] has provided a good survey and comparison of such algorithms. The performance of these methods are good but there is a waste of bandwidth in transmitting a redundant frame. Another category is based on using the cyclic prefix [7]. Then the start of the frame is where the correlation of the start and end symbols is maximized. In the absence of delay spread, this would work fine. However, in the presence of delay spread, the cyclic prefix would be affected by the previous frame resulting in a degradation in the performance of the correlation-based frame synchronizer. There are other methods that use the cyclic prefix for a coarse frame synchronization followed by a fine tuning [8]. However, Yang in [8] makes the assumption that the first path of the channel is the strongest, which may not be true in environments with no line of sight path.

Similarly there has been a lot of work in channel estimation for OFDM systems (most of the work is done assuming perfect frame synchronization). For applications with considerable delay spread, there is a need for inserting pilot tones for channel estimation. If maximum channel delay spans ν symbol periods, Negi [2] has shown that only $L = \nu + 1$ pilots are needed for perfect channel estimation in the absence of AWGN and timing errors. To get the time domain channel from the pilots, an IFFT of length L should be performed. This is then followed by an FFT of length N, where N is the number of sub-carriers, to get an estimate of the channel in all the sub-carriers. Note that there are other less optimum ways of interpolating the channel in between pilot sub-carriers. As delay spread goes higher the performance of these sub-optimum methods degrade drastically while that of the optimum interpolator[1] is not affected [2]. Furthermore in the presence of AWGN, equally-spaced pilots will result in minimum channel estimation error.[3] The effect of frame synchronization errors on the optimum pilot-aided channel estimator was not studied previously. It is the goal of this paper to analyze the effect of frame synchronization errors on the performance of such channel estimator. We will show that the pilot-aided channel estimator [4] is quite sensitive to frame synchronization errors. After thorough analysis, we show how this sensitivity can be exploited to design a robust frame synchronizer. The performance of the proposed frame synchronizer is then evaluated through our simulations.

SYSTEM MODEL AND CHANNEL ESTIMATION
Consider an OFDM system with N sub-carriers. Fig. 1 shows a discrete baseband model of such a system. In the

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[1]Note that "optimum interpolator" refers to the aforementioned one which finds the estimated channel through an IFFT of length L, zero padding and FFT of length N [2]
[2]Provided that the length of the channel stays less than the maximum predicted length of ν
[3]Note that in the presence of AWGN, the channel estimate acquired with L pilots becomes noisy. Increasing the number of pilots beyond L would then reduce the effect of noise
[4]Note that from this point on in this paper, "pilot-aided channel estimator" refers to the optimum interpolator

absence of synchronization errors, at the ith sub-channel, we will have:

$$Y_i = H_i X_i + W_i \quad 0 \leq i \leq N-1 \qquad (1)$$

where Y_i, H_i, X_i and W_i represent received symbol, channel, transmitted symbol and noise at the ith sub-channel respectively. We assume that the channel is constant over

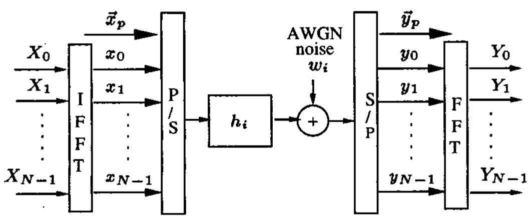


Fig. 1 Discrete baseband equivalent model

one OFDM frame but varies from one frame to the next due to mobility. Let ν be the maximum predicted normalized length of the channel[5]. Throughout this paper, we assume that the length of the channel is always smaller than or equal to ν. Therefore, only $L = \nu + 1$ equally-spaced pilot tones, P_{l_i} for $0 \leq i \leq L-1$, are needed to estimate the channel where $l_i = \frac{i \times N}{L}$. Then we will have,

$$\hat{H}_{l_i} = Y_{l_i}/P_{l_i} = H_{l_i} + W_{l_i}/P_{l_i} \quad 0 \leq i \leq L-1 \qquad (2)$$

Through an IFFT of length L, estimate of the channel in time domain would be $\hat{h}_k = \frac{1}{L}\sum_{i=0}^{L-1} \hat{H}_{l_i} e^{j2\pi ik/L}$ for $0 \leq k \leq L-1$. Then through an FFT of length N, the estimate of the channel in all the sub-carriers can be achieved as $\hat{H}_i = \sum_{k=0}^{L-1} \hat{h}_k e^{-j2\pi ik/N}$ for $0 \leq i \leq N-1$.

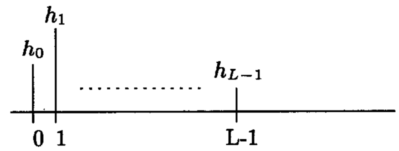


Fig. 2a Original channel of length $\nu = L-1$

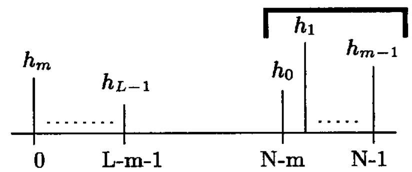


Fig. 2b Equivalent channel for $m \geq 1$

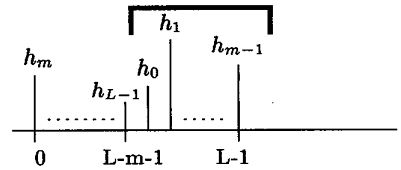


Fig. 2c Estimated channel for $m \geq 1$

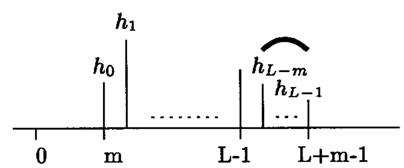


Fig. 2d Equivalent channel for $m \leq -1$

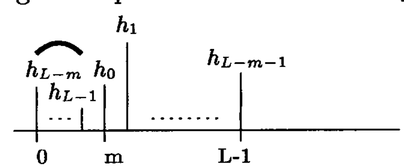


Fig. 2e Estimated channel for $m \leq -1$

EFFECT OF FRAME SYNCHRONIZATION ERRORS

Consider a scenario where the frame synchronizer made a timing error of m symbols. Throughout this paper, $m \geq 1$ and $m \leq -1$ denote an erroneous offset of m symbols to the right and left respectively. Such errors have two effects on an OFDM system:

Effect#1: Loss of Symbols

They introduce Inter-Carrier-Interference (ICI) due to the loss of m symbols. For the case of $m \geq 1$, the m erroneous symbols are from the next frame. In the case of $m \leq -1$, depending on the length of the channel delay spread, the loss may be less than m symbols. This is due to the redundancy of the cyclic prefix. For a cyclic prefix of length ν and a channel of length [6] C, the loss is $max(0, C - m - \nu)$ symbols for $m \leq -1$.

[5]Note that guard interval will span ν symbols as well

[6]Note that we assume that $C \leq \nu$

Effect#2: Rotation of the Channel

They ruin the order of the transmitted sequence. Consider $x_0, x_1, \ldots, x_{N-1}$, the data part of a time domain transmitted sequence, as is shown in Fig. 1. Then for $m \geq 1$ and $h_k = \delta_k$, $[x_m, x_{m+1}, \ldots, x_{N-1}, n_0, n_1, \ldots, n_{m-1}]$ is the received sequence, where the n_i are symbols from the next frame. To study the impact of timing errors on the order of transmitted symbols, lets assume for now that Effect#1 is negligible and the n_i are equal to the lost symbols. Then the received sequence is $[x_m, x_{m+1}, \ldots, x_{N-1}, x_0, x_1, \ldots, x_{m-1}]$. This shows how m timing errors ruin the order of the transmitted sequence. The same can be concluded for $m \leq -1$ as well. By introducing an equivalent channel, Effect#2 can be taken into the channel model. Therefore, it can be observed that the equivalent channel for $h_k = \delta_k$ is $h_{eq}(k) = \delta((k+m))_N$, where $((\ \ ))_N$ represents circular shift in the base N. The same effect can be observed for a non-impulse channel. It can be easily shown that in such a case m errors lead to the following equivalent channel model:

$$h_{eq}(k) = \delta((k+m))_N \otimes_N h_k = h_{((k+m))_N} \quad (3)$$

where $\otimes_N$ represents circular convolution of length N. Fig. 2b,d show the equivalent channel for the case of $m \geq 1$ and $m \leq -1$ for the channel of Fig. 2a respectively. As can be seen, even one error to the right results in an equivalent channel of length $N-1$. For $m \leq -1$, the equivalent length is $\nu - m$. In both cases, the length of the channel has increased beyond ν, the maximum predicted length. This length increase is more pronounced for errors to the right.

In general both Effect#1 and Effect#2 need to be considered. Then Y_k will be,

$$Y_k = H_{eq}(k)X_k + W_k + I_k \quad 0 \leq k \leq N-1 \quad (4)$$

$H_{eq}(k) = \beta e^{j2\pi km/N} H_k$ where H_k is the Fourier transform of h_k and β is a constant. I_k represents the ICI introduced by Effect#1. Note that Effect#1 introduces a constant β in the equivalent channel [9]. Then to make our model complete, we include it in the equivalent channel model of Eq. 3:

$$h_{eq}(k) = \beta h_{((k+m))_N} \quad (5)$$

IMPACT OF FRAME SYNCHRONIZATION ERRORS ON THE PILOT-AIDED CHANNEL ESTIMATOR

Consider a case of m timing errors. Utilizing Eq. 2 will result in the following estimate of the equivalent channel at pilots:

$$\hat{H}_{eq}(l_k) = \beta H_{l_k} e^{j2\pi l_k m/N} + U_{l_k} + V_{l_k} \quad 0 \leq k \leq L-1 \quad (6)$$

Where $U_{l_k} = W_{l_k}/P_{l_k}$ and $V_{l_k} = I_{l_k}/P_{l_k}$. Then an IFFT of length L will result in an estimate of the time domain channel:

$$\hat{h}_{eq}(k) = \beta h_k \otimes_L \delta((k+m))_L + u_k + v_k \quad (7)$$

$$\hat{h}_{eq}(k) = \beta h_{((k+m))_L} + u_k + v_k \quad 0 \leq k \leq L-1 \quad (8)$$

Where u_k and v_k are IFFTs of U_{l_k} and V_{l_k}.

In general, the estimated channel of Eq. 8 can be different from the equivalent channel of Eq. 5, resulting in a performance degradation. To see the impact of Effect#2, Fig. 2c,e show the estimated channel for the equivalent channel of Fig. 2b,d for the case of m errors (note that β is assumed one in Fig. 2, without loss of generality). As can be seen, for a channel of length ν, m errors result in the misplacement of the last m paths of the equivalent channel, which is h_0 to h_{m-1} for $m \geq 1$ and h_{L-m} to h_{L-1} for $m \leq -1$. Since h_0 to h_{m-1} are often stronger than h_{L-m} to h_{L-1}, the case of $m \geq 1$ degrades the performance more. In general the equivalent channel length for $m \leq -1$ varies depending on the length of h. For instance, for a channel of length $C \leq \nu$ the equivalent channel length will be $C-m$ for $m \leq -1$. Therefore for $C - \nu \leq m \leq -1$, the equivalent length would still be less than or equal to ν, which poses no problem for the channel estimator. However, the equivalent channel length will be $N-1$ for any $m \geq 1$, affecting the performance of the channel estimator which was designed for a channel of maximum length of ν.

Furthermore, Effect#1 and noise increase channel estimation error. However, Effect#2 has a considerably stronger effect on the performance. To see this, we calculate normalized channel estimation error for the case of $m \geq 0$. Define the average normalized channel estimation error at the kth sub-carrier as $E_n(k) = \overline{\frac{|H_{eq}(k) - \hat{H}_{eq}(k)|^2}{|H_{eq}(k)|^2}}$, where $\hat{H}_{eq}(k)$ is the FFT of $\hat{h}_{eq}(k)$ and $\overline{z}$ denotes the average of z. Then it can be shown that for $0 \leq k \leq N-1$,

$$E_n(k) = \underbrace{4P_{\%} sin^2(\pi \frac{k}{r})}_{\text{Effect\#2}} + \underbrace{\frac{1}{SNR_{rec}}}_{\text{noise \& Effect\#1}} + \underbrace{\frac{1}{SIR_{rec}}}_{\text{Effect\#1}} \quad (9)$$

Where $P_{\%} = \frac{\sum_{i \leq m-1} \overline{|h_i|^2}}{\sigma_h^2}$, $SNR_{rec} = \frac{(N-m)^2 \sigma_X^2 \sigma_h^2}{N^2 \sigma_W^2}$ and $SIR_{rec} = \frac{(N-m)^2 \sigma_X^2 \sigma_h^2}{N^2 \sigma_I^2}$ for $m \geq 0$. Furthermore, $\sigma_h^2 = \sum_{i \leq \nu} \overline{|h_i|^2}$ and $r = \frac{N}{L}$. σ_X^2, σ_I^2 and σ_W^2 are powers of X, I and W respectively. As can be seen from Eq. 9, Effect#2 does not affect those sub-channels carrying pilot tones. However, it results in a considerable increase of error for other sub-carriers particularly $k = o_{odd} \times ceil(r/2)$, where o_{odd} represents odd integers. In a reasonable SNR_{rec} environment, the overall impact of Effect#2 is considerably higher than that of Effect#1. Call the ratio of the first term to the sum of the last two terms on the right hand side of Eq. 9, R. Examining R for different values of $\alpha = \frac{m}{N}$ and $P_{\%}$ in a reasonable SNR environment shows how Effect#2 is the dominant factor with high probability. For instance, consider the case of $\frac{\sigma_X^2 \sigma_h^2}{\sigma_W^2} = 20dB$, $\alpha = .1$ and $P_{\%} = 50\%$. Utilizing the approximated formula for σ_I^2 from [9], it can be shown that R is at least 12dB for L of the sub-carriers with $k = o_{odd} \times ceil(r/2)$.

FRAME SYNCHRONIZATION ERROR CORRECTION

The coherent channel estimator is sensitive to the timing errors mainly due to Effect#2. If a sub-optimum interpolator were used instead, the sensitivity to the timing errors would have been less. Also, for a receiver with differential detection, only the last two terms on the right hand side of Eq. 9 would degrade the performance. However, neither the sub-optimum interpolators nor the differential detector can produce an acceptable performance in environments with considerable delay spread. Furthermore, the high sensitivity of the optimum channel estimator can be exploited to correct timing errors in the following manner.

After a coarse frame synchronizer has detected a start point for the frame, $\hat{H}_{eq}(k)$ can be obtained through using pilot tones. Due to timing errors, this channel estimate may be far from $H_{eq}(k)$. Call $\hat{X}_k = Y_k/\hat{H}_{eq}(k)$, the estimated input. Let $\tilde{X}_k = Dec(\hat{X}_k)$ represent the estimated input after passing through the decision device. Define a decision-directed measure function as $M = \sum_{k=0}^{N-1} |\hat{X}_k - \tilde{X}_k|^2$. In the presence of Effect#2, M becomes very large. Therefore, frame synchronization error correction can be obtained through minimizing M. Note that we detect frame errors solely through the large impact of Effect#2 on the performance. Therefore, as long as Effect#2 is the major cause of performance loss, which is the case with high probability, we can detect frame errors. Due to the presence of Effect#2, it is possible to perform all the updates neccesary to find the best timing correction solely in frequency domain without a need to go back and forth from frequency to time domain. Consider correcting errors to the right. As can be seen in Fig. 2c, the position of the m final paths of the estimated channel is different from that of the equivalent channel, where m is unknown. Therefore through an iterative process, we update the estimated channel, correcting for one mismatched path at a time. This means that at the first iteration, the last path in $\hat{h}_{eq}(k)$ should be transferred from position $L-1$ to position $N-1$. Following the same procedure, the update necessary in the ith iteration ($i \geq 1$) for the kth sub-channel for correcting errors to the right will be as follows (a path is moved from position $L-i$ to $N-i$):

$$\hat{H}^{(i+1)}_{right}(k) = \hat{H}^{(i)}_{right}(k) + c_1 \times \hat{h}_{eq}(L-i) \times e^{j2\pi ik/N} \quad (10)$$

Similarly for detecting errors to the left, we will have,

$$\hat{H}^{(i+1)}_{left}(k) = \hat{H}^{(i)}_{left}(k) - c_1 \times \hat{h}_{eq}(i-1) \times e^{-j2\pi(i-1)k/N} \quad (11)$$

Where $c_1 = 1 - e^{-j2\pi Lk/N}$ and $\hat{H}^{(1)}_{right}(k) = \hat{H}^{(1)}_{Left}(k) = \hat{H}_{eq}(k)$. In each iteration, the measure function, $M^{(i)}$, will be evaluated. Finally the iteration that results in the minimum M is chosen. Call this iteration $i_{corr,s}$. $s=1$ denotes that a timing error to the right have been detected. Similarly $s=-1$ indicates detection of an error to the left. Then it is possible to go back to the time domain and correct the start of the frame. For $s=1$, the initial start point of the frame should be moved to the left by $i_{corr,s}$. The opposite will be the case for $s=-1$.

IMPLEMENTATION ISSUES

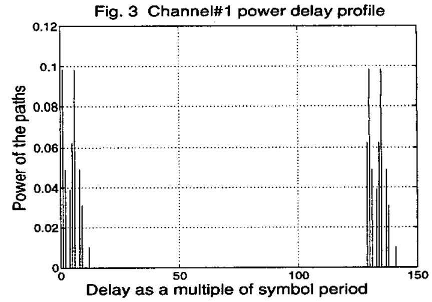
In practice, a coarse frame synchronizer should be used for the initial acquisition. A traditional choice for this would be a sliding correlator that correlates the start of the frame with its end, looking for a peak. After that, fine synchronization can be achieved as was previously described.

Note that the sensitivity of the channel estimator to Effect#2 allows us to design a frame synchronizer with a performance considerably better than that of the traditional one. If a sub-optimum channel estimator were to be used instead, the same improvement would not be achieved for two reasons. First, due to the lack of Error#2, the effect of coarse synchronization errors would not be seen in the estimated channel. Then to perform a decision-Directed correction, there would be a need to go back and forth between time and frequency domain, which will increase the complexity. Second, high sensitivity of the optimum channel estimator allows for detecting frame synchronization errors in the presence of noise and Doppler. However, such sensitivity does not exist for other sub-optimum estimators, resulting in a rather high probability of false correction in the fine synchronization stage. In general, we were able to keep the performance of the optimum channel estimator while improving the quality of the frame synchronizer.

SIMULATION RESULTS

We simulate an OFDM system in a high delay spread environment as is the case for Single Frequency Networks (SFN)[7]. We chose the following parameters for simulations[8]. Input modulation is 8PSK. Bit rate is 7.3Mbps. N=892 and L=223. The simulated channels have two main clusters each with 9 paths, to represent an SFN channel. We consider two power-delay profiles. For channel#1, the delay spread spans 64% of the guard interval. This channel is shown in Fig. 3. Channel#2 has the same specifications, but the delay between its two clusters has increased such that the total length of it spans 100% of the guard interval (worst case channel). In both channels two main clusters are equal power as is seen in Fig. 3. Each channel path is generated as a random variable with Rayleigh distributed amplitude and uniformly distributed phase. We simulate three methods. The first two methods only utilize the traditional correlation-based frame synchronizer. The first method picks the maximum correlation point. The second method picks the point from which sum of the next L correlation points are maximized, hence reducing the chance of an offset to the right (which is more serious) at the price of increasing offsets to the left. The third method utilizes method I for initial coarse frame synchronization followed by the proposed decision-directed time adjustment method of the previous sections. To evaluate the performance of these methods, we measure $P_{error}(m)$, the probability of making a timing error of m symbols. To measure the cost of making an erroneous offset, $P_b(m)$, the average BER in case of an offset of m symbols, is simulated for both channels.

Fig. 3 Channel#1 power delay profile

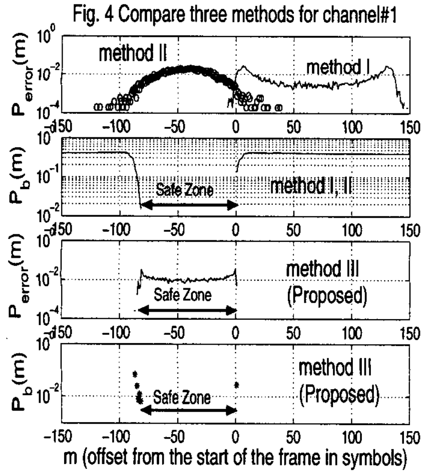


The first and third curves of Fig. 4 show $P_{error}(m)$ of the three methods, for channel#1 in the absence of noise. The second and fourth curves of Fig. 4 show $P_b(m)$ for the three

[7] Similar improvement is also obtained with non-SFN channels
[8] System parameters are chosen based on the Sirius Radio second generation system specification proposal

methods[9]. Note that the delay of channel#1 spans 64% of the guard interval. This allows for a timing offset of up to 36% of guard interval (which becomes 82 symbols) to the left without any loss of performance. We call this region the "safe zone" which can be seen in Fig. 4. The proposed method has a considerably lower frame error profile in the positions that BER is high and mainly has timing offsets in the safe zone. From $P_{error}(m)$ and $P_b(m)$, the average BER due to frame synchronization errors would be: $\bar{P}_{b,method\ I} = .398, \bar{P}_{b,method\ II} = .0079, \bar{P}_{b,proposed\ method} = 4.4{\times}10^{-4}$ for this channel. Fig. 5 shows similar curves for channel#2. In this case, there will be more sensitivity to $m \leq -1$, since the delay spread spans 100% of the guard interval. This can be seen from P_b curves in Fig. 5, where there is almost no safe zone. This affects the shape of $P_{error}(m)$ of method III as can be seen. However, method II still makes a considerable number of timing errors to the left which has a heavy cost for this channel. To see the effect of noise, Fig. 6 shows average bit error rate vs. received SNR for channel#1. The dashed line shows the BER of the perfect synchronizer for comparison. As can be seen, the proposed method has a performance very close to that of the perfect synchronizer. Overall, the proposed method behaves robustly in mitigating frame synchronization errors and can be a promising candidate to accompany a pilot-aided channel estimator in OFDM systems.

Fig. 4 Compare three methods for channel#1



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[9]Note that $P_b(m)$ for method III is different from that of method I and II. To obtain $P_b(m)$ for a specific frame synchronization offset, bit error rate should be averaged over different input, noise and channel (this is averaging over Rayleigh) realizations at that synchronization offset. For a given offset of m symbols, channel realizations that would lead to that offset are different for method III than for method I and II. This is due to the fact that method III uses channel estimation to correct frame errors.

Fig. 5 Compare three methods for channel#2

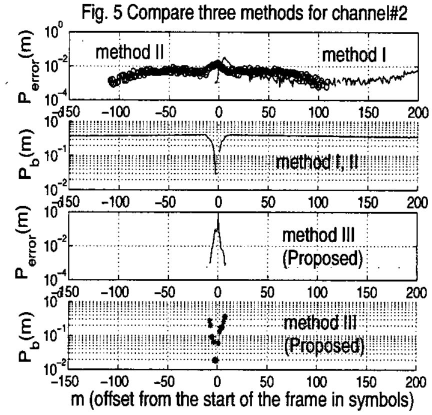
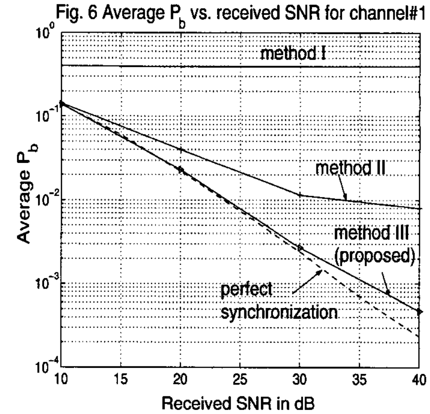


Fig. 6 Average P_b vs. received SNR for channel#1



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