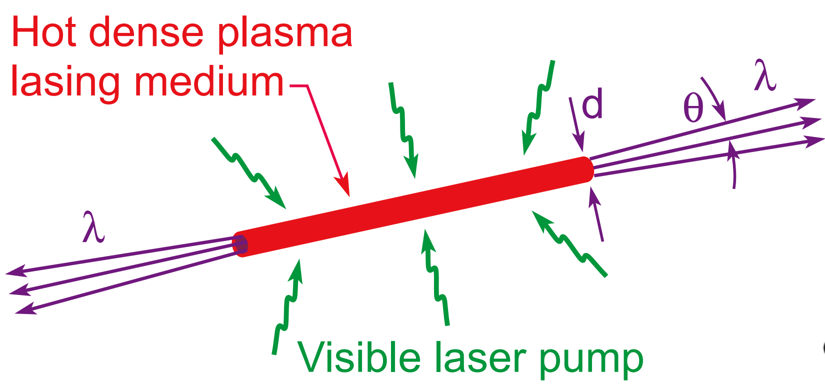
Chapter 7

EXTREME ULTRAVIOLET AND SOFT X-RAY LASERS



$$\frac{I}{I_0} = e^{GL} \qquad (7.2)$$

$$G = n_u \sigma_{\text{stim}} F \qquad (7.4)$$

$$\sigma_{\text{stim}} = \frac{\pi \lambda r_e}{(\Delta\lambda/\lambda)} \left(\frac{g_l}{g_u}\right) f_{lu} \qquad (7.18)$$

$$\frac{P}{A} = \frac{16\pi^2 c^2 \hbar (\Delta\lambda/\lambda) GL}{\lambda^4} \qquad (7.22)$$

Exponential Gain from Stimulated Emission

$$\frac{I}{I_0} = e^{GL} \tag{7.2}$$

where G is the gain per unit length and L is the propagation distance in the lasing media.

$$G = n_u \sigma_{\text{stim}} - n_l \sigma_{\text{abs}} \tag{7.3}$$

or

$$G \equiv n_u \sigma_{\text{stim}} F \tag{7.4}$$

where the density inversion factor is

$$F \equiv 1 - \frac{n_l \sigma_{\text{abs}}}{n_u \sigma_{\text{stim}}} = 1 - \frac{n_l g_u}{n_u g_l} \tag{7.5}$$

where g_u and g_ℓ are degeneracies (same energy, different quantum numbers) associated with the upper and lower states.

Transition Rates: Einstein A and B Coefficients

In equilibrium the number of transitions up equals the number down. On a per unit time and per unit volume basis

$$n_u A_{ul} + n_u B_{ul} U_{\Delta\omega} = n_l B_{lu} U_{\Delta\omega} \qquad (7.6)$$

spontaneous emission (down)	stimulated emission (down)	absorption (up)

where n_u and n_ℓ are the densities of ions in the upper and lower states, $A_{u\ell}$ is the spontaneous decay rate (inverse of natural life time), $B_{u\ell}$ is the stimulated transition rate from u to ℓ, $B_{\ell \mathrm{u}}$ is the absorption transition rate, and $U_{\Delta\omega}$ is the spectral energy density, in units of energy per unit volume per unit frequency interval.

Transition Rates: Einstein A and B Coefficients (continued)

In equilibrium

$$U_{\Delta\omega} = \frac{\hbar\omega^3}{\pi^2 c^3 (e^{\hbar\omega/\kappa T} - 1)} \qquad (7.7)$$

with equal density of states ($g_\ell = g_u$)

$$\frac{n_l}{n_u} = e^{(E_u - E_l)/\kappa T} = e^{\hbar\omega/\kappa T} \qquad (7.8)$$

The (transition) rate equation (7.6) can be written as

$$A_{u\ell} + B_{u\ell} U_{\Delta\omega} = \frac{n_\ell}{n_u} B_{u\ell} U_{\Delta\omega}$$

where $B_{u\ell} = B_{\ell u}$.

$$A_{u\ell} = B_{u\ell} \left[\frac{n_\ell}{n_u} - 1 \right] U_{\Delta\omega}$$

Transition Rates: Einstein A and B Coefficients (continued further)

$$\frac{A_{u\ell}}{B_{u\ell}} = \left[\frac{n_\ell}{n_u} - 1\right] U_{\Delta\omega}$$

Substituting for n_ℓ/n_u from eq.(7.8) and $U_{\Delta\omega}$ from eq.(7.7)

$$\frac{A_{ul}}{B_{ul}} = \cancel{(e^{\hbar\omega/\kappa T} - 1)}\left[\frac{\hbar\omega^3}{\pi^2 c^3 \cancel{(e^{\hbar\omega/\kappa T} - 1)}}\right]$$

or

$$\frac{A_{ul}}{B_{ul}} = \frac{\hbar\omega^3}{\pi^2 c^3} \qquad (7.9) \quad \text{(near equilibium)}$$

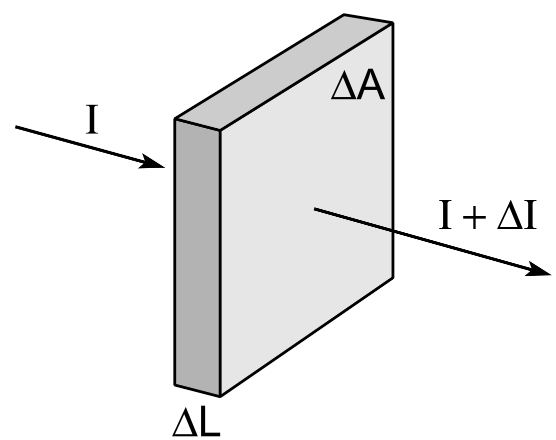
Radiated Power and Intensity

If stimulated emission dominates spontaneous emission, the radiated power is

$$\underbrace{\frac{\Delta E}{\Delta t}}_{\text{Radiated power}} = \underbrace{[n_u B_{ul} U_{\Delta\omega}}_{\substack{\text{\# Transitions}\\ \text{from } u \text{ to } \ell,\\ \text{per unit vol.,}\\ \text{per sec.}}} - \underbrace{n_l B_{lu} U_{\Delta\omega}]}_{\substack{\text{\# Transitions}\\ \text{from } \ell \text{ to } u,\\ \text{per unit vol.,}\\ \text{per time}}} \underbrace{\hbar\omega}_{\substack{\text{Photon}\\ \text{energy}}} \cdot \underbrace{\Delta A \cdot \Delta L}_{\substack{\text{Unit}\\ \text{volume}}} \qquad (7.10)$$

$$\frac{\Delta E}{\Delta t} = P = \Delta I \cdot \Delta A$$

$$\Delta I = n_u F B_{ul} U_{\Delta\omega} \hbar\omega \cdot \Delta L \qquad (7.11)$$



Radiated Power and Intensity (continued)

$$\Delta I = n_u F B_{ul} U_{\Delta\omega} \hbar\omega \cdot \Delta L \qquad (7.11)$$

In terms of the radiation energy density $U_{\Delta\omega}$

$$I = U_{\Delta\omega} \cdot \Delta\omega \cdot c$$

then

$$\frac{\Delta I}{I} = \frac{n_u F B_{ul} \hbar\omega \cdot \Delta L}{(\Delta\omega) c} \qquad (7.13)$$

Using the relationship between A and B coefficients

$$\frac{A_{ul}}{B_{ul}} = \frac{\hbar\omega^3}{\pi^2 c^3} \qquad (7.9)$$

$$\frac{\Delta I}{I} = \frac{\pi^2 c^2 n_u F A_{ul} \cdot \Delta L}{(\Delta\omega)\omega^2} \qquad (7.14a)$$

Radiated Power and Intensity (continued further)

$$\frac{\Delta I}{I} = \frac{\pi^2 c^2 n_u F A_{ul} \cdot \Delta L}{(\Delta\omega)\omega^2} \qquad (7.14a)$$

Noting that $(\Delta\omega)\omega^2 = (\Delta\omega/\omega)\omega^3 = (\Delta\lambda/\lambda)(2\pi)^3 c^3/\lambda^3$

$$\frac{\Delta I}{I} = \frac{\lambda^3 n_u F A_{ul} \cdot \Delta L}{8\pi c(\Delta\lambda/\lambda)} \qquad (7.14b)$$

Integrating from $\Delta L = 0$ to L

$$\frac{I}{I_0} = e^{GL} \quad ; \quad G = \frac{\lambda^3 n_u F A_{ul}}{8\pi c(\Delta\lambda/\lambda)} \qquad (7.15)$$

Stimulated Emission Cross-Section

$$\frac{I}{I_0} = e^{GL} \quad ; \quad G = \frac{\lambda^3 n_u F A_{ul}}{8\pi c(\Delta\lambda/\lambda)} \qquad (7.15)$$

Recalling our earlier definition that

$$G \equiv n_u \sigma_{\text{stim}} F \qquad (7.4)$$

We identify the cross-section as

$$\sigma_{\text{stim}} = \frac{\lambda^3 A_{ul}}{8\pi c(\Delta\lambda/\lambda)} \qquad (7.16)$$

The Einstein A coefficient is the inverse lifetime for a transition between two states, $A_{u\ell} = 1/\tau$, and can be calulated using quantum mechanics (Silfvast, Corney), to be

$$A_{ul} = \frac{e^2\omega^2}{2\pi\epsilon_0 m c^3}\left(\frac{g_l}{g_u}\right) f_{lu} \qquad (7.17)$$

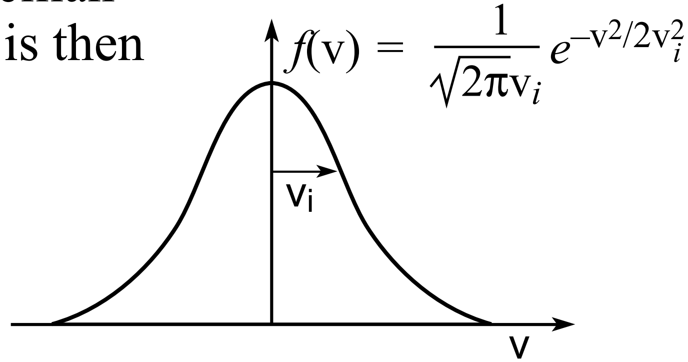
So that the <u>cross-section for stimulated emission</u> is

$$\boxed{\sigma_{\text{stim}} = \frac{\pi\lambda r_e}{\Delta\lambda/\lambda}\left(\frac{g_l}{g_u}\right) f_{lu}} \qquad (7.18)$$

where $f_{\ell u}$ is the oscillator strength of the transition and r_e is the classical radius. Now need $\Delta\lambda/\lambda$.

Doppler Broadened Line Width

For EUV and soft x-ray lasing, created in a hot plasma, the line width is typically dominated by Doppler broadening, rather than by natural lifetime. For a Maxwellian velocity distribution the full rms line width is then



$$\left.\frac{(\Delta\lambda)}{\lambda}\right|_{\mathrm{rms}} = \frac{2\mathrm{v}_i}{c}$$

or expressed as a FWHM quantity, is

$$\left.\frac{(\Delta\lambda)}{\lambda}\right|_{\mathrm{FWHM}} = \frac{2\sqrt{2\ln 2}}{c}\sqrt{\frac{\kappa T_i}{M}} \qquad (7.19a)$$

where $\mathrm{v}_i = \sqrt{\kappa T_i/M}$ is the rms ion velocity, κT_i is the ion temperature, and M is the ion mass. Taking $M \simeq 2m_pZ$ and κT_i in eV

$$\left.\frac{(\Delta\lambda)}{\lambda}\right|_{\mathrm{FWHM}} = 7.69 \times 10^{-5}\left(\frac{\kappa T_i}{2Z}\right)^{1/2} \qquad (7.19b)$$

or about 10^{-4} for argon ions at a temperature of about 60 eV.

Wavelength Scaling of EUV and Soft X-Ray Lasers

To maintain an inverted population density requires delivery of a power

$$P = \frac{\hbar\omega n_u F V}{\tau} \qquad (7.21)$$

With $\tau = 1/A_{u\ell}$

$$\frac{P}{V} = \hbar\omega n_u F A_{u\ell}$$

From the gain expression (7.15)

$$G = \frac{\lambda^3 n_u F A_{u\ell}}{8\pi c\,(\Delta\lambda/\lambda)} \;\;\Rightarrow\;\; n_u F A_{u\ell} = 8\pi c(\Delta\lambda/\lambda)G/\lambda^3$$

the required power per unit volume is

$$\boxed{\frac{P}{V} = \frac{16\pi^2 c^2 \hbar(\Delta\lambda/\lambda)G}{\lambda^4}}$$

With $V = AL$, the resultant lasing intensity would be

$$\boxed{\mathrm{I} = \frac{P}{A} = \frac{16\pi^2 c^2 \hbar(\Delta\lambda/\lambda)GL}{\lambda^4}} \qquad (7.22)$$

Thus both required power per unit volume and potential lasing intensity scale as $1/\lambda^4$.

Wavelength Scaling of EUV and Soft X-Ray Lasers (continued)

Lasing intensity scales as $1/\lambda^4$.

High $\kappa T_e \propto 1/\lambda$

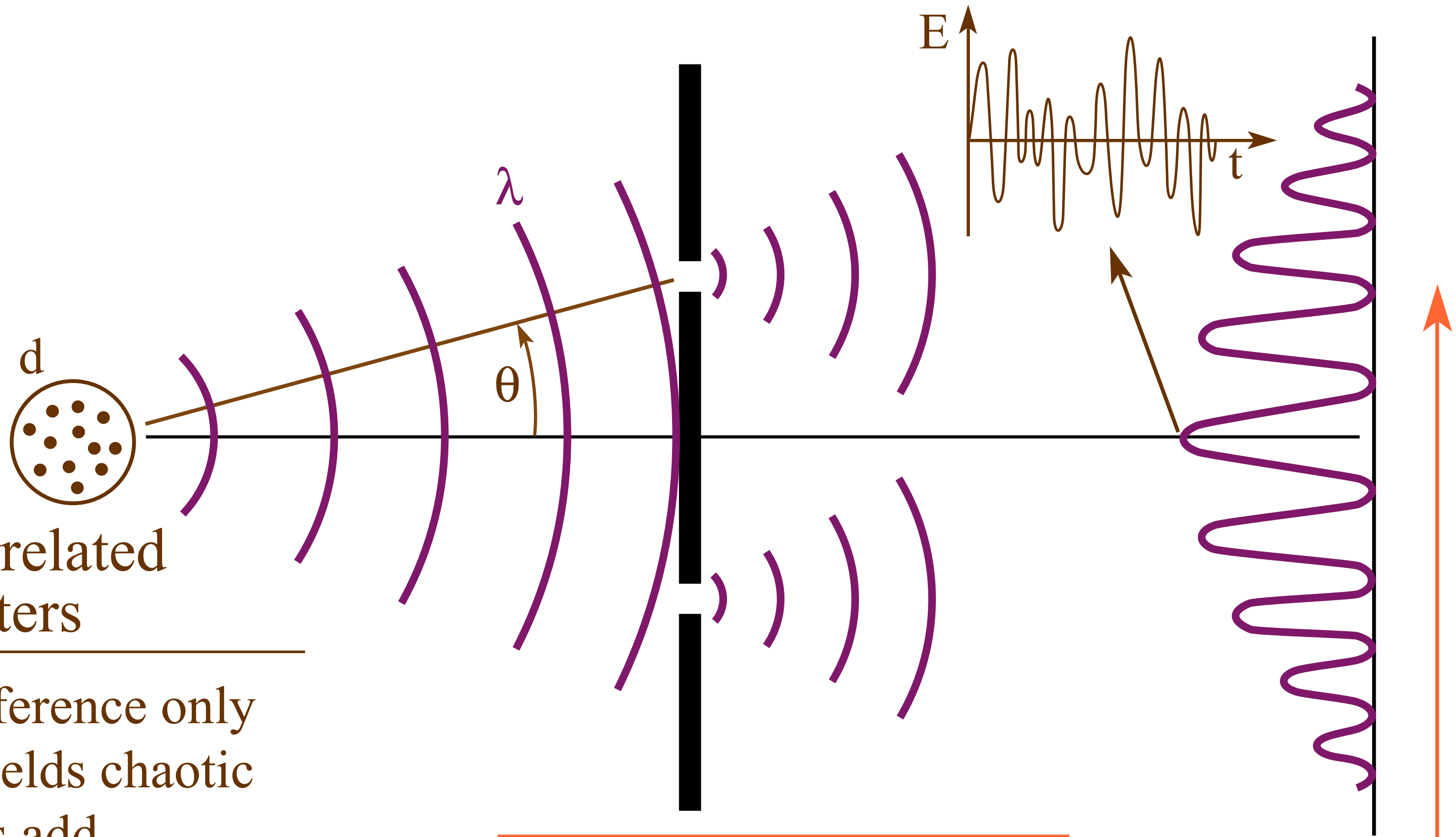
Cross-section $\propto 1/\lambda$

Plasma lifetime $\propto 1/\lambda^2$

Line width $\propto 1/\lambda^{1/2}$

Seeded EUV Lasers

Young's Double Slit Experiment with Random Emitters: Young did not have a laser



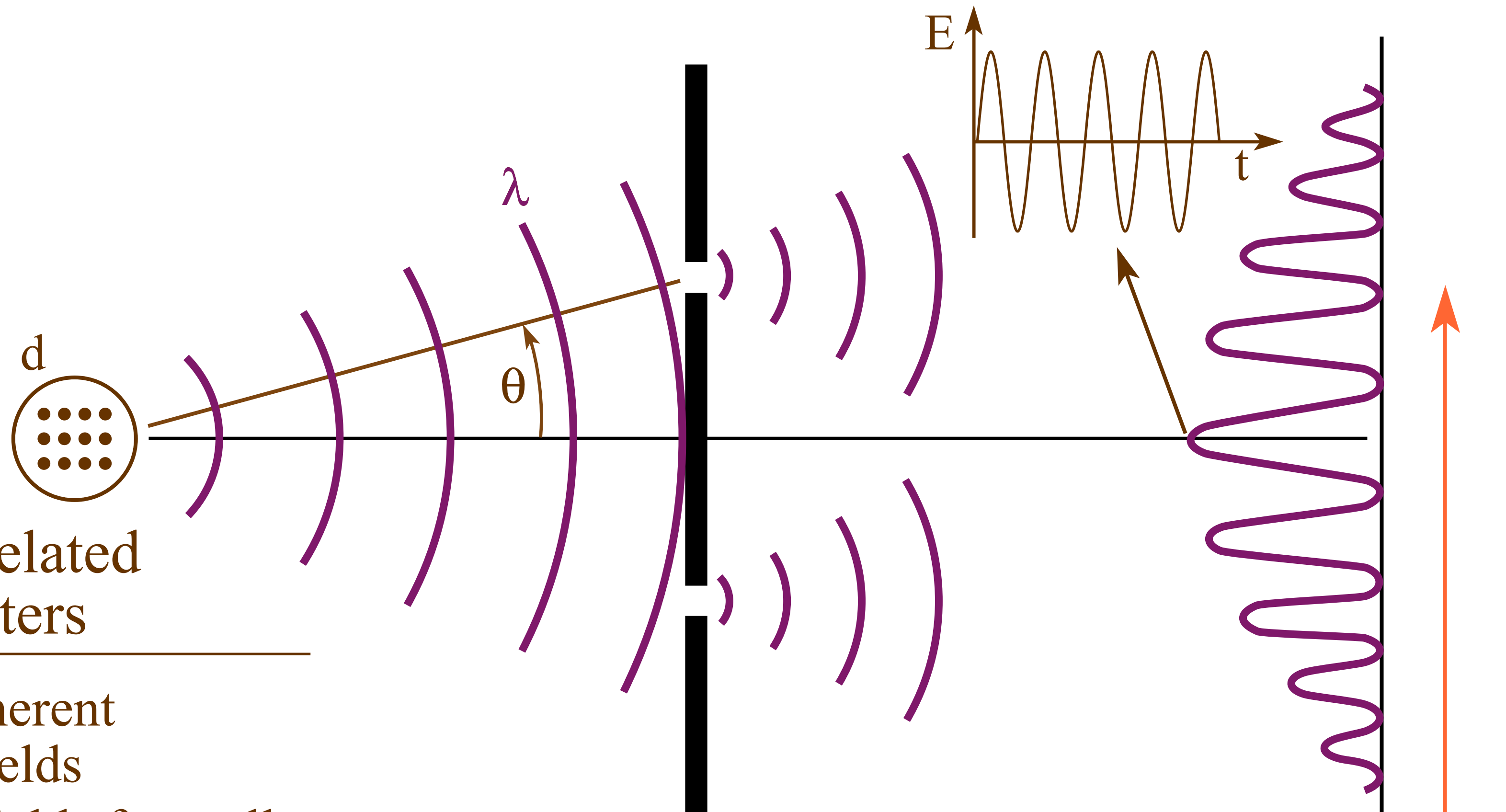
N uncorrelated emitters

- Self-interference only
- Electric fields chaotic
- Intensities add
- Radiated power ~ N

$$d \cdot 2\theta|_{FWHM} \simeq \lambda/2$$

$$\lambda_{coh} = \lambda^2/2\Delta\lambda = \frac{1}{2} N_{coh}\lambda$$

Young's Double Slit Experiment with Phase Coherent Emitters (some lasers, or properly seeded FELs)



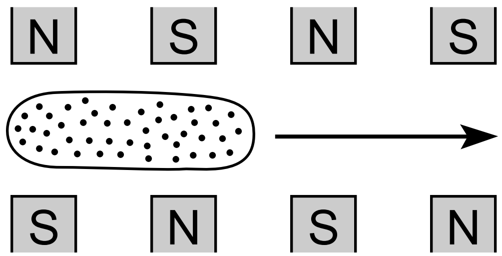
N correlated emitters

- Phase coherent electric fields
- Electric fields from all particles interfere constructively
- Radiated power ~ N^2
- New phase sensitive probing of matter possible

$$d \cdot 2\theta|_{FWHM} \simeq \lambda/2$$

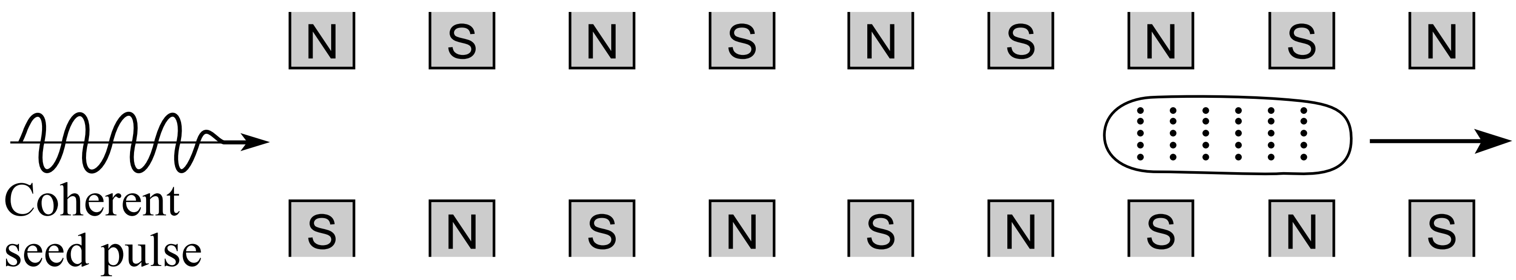
$$\lambda_{coh} = \lambda^2/2\Delta\lambda = \frac{1}{2} N_{coh}\lambda$$

Undulators and FELs



$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

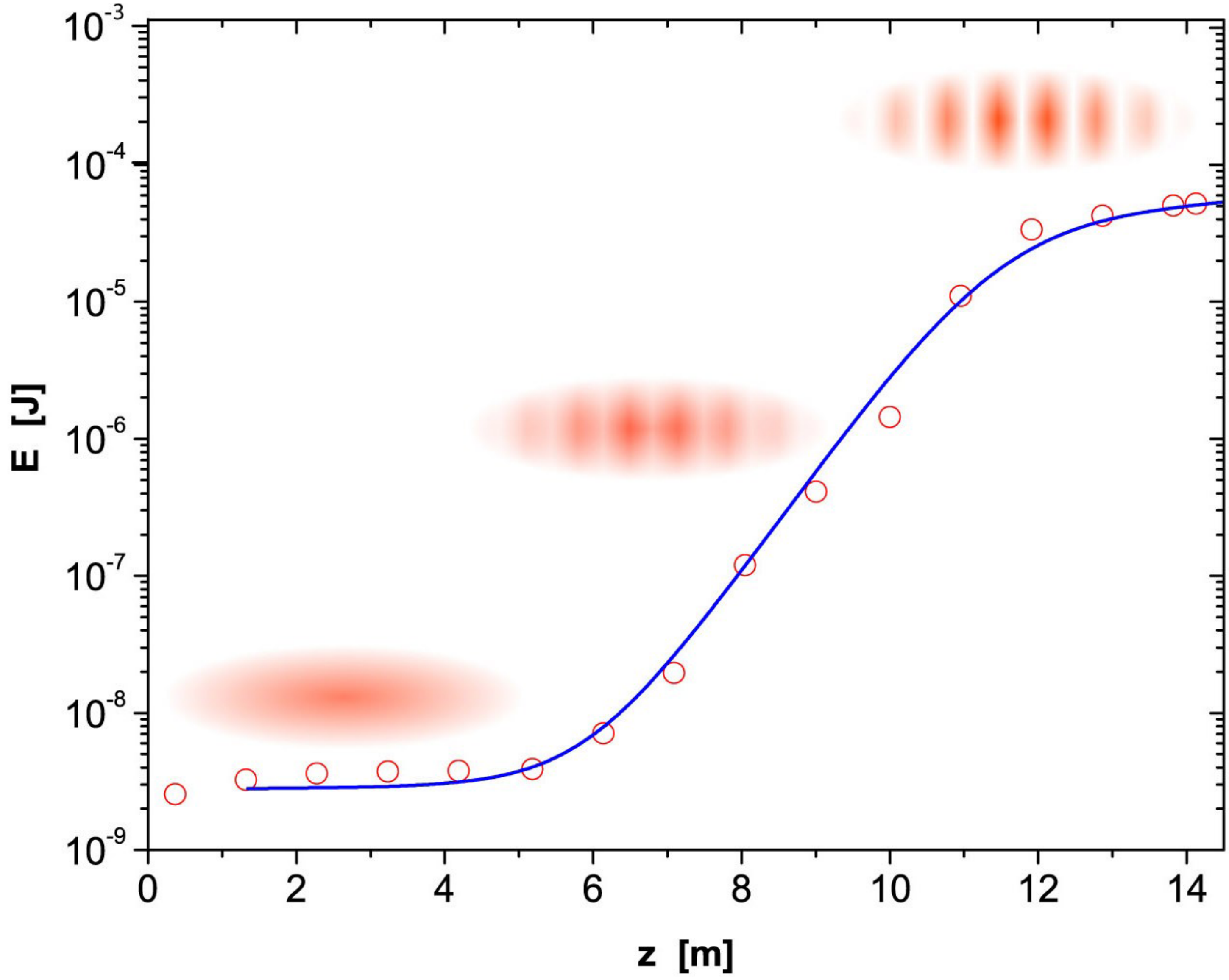
Undulator – uncorrelated electron positions, radiated fields uncorrelated, intensities add, limited coherence, power ~ N.



Free Electron Laser (FEL) – very long undulator, electrons are "microbunched" by their own radiated fields into strongly correlated waves of electrons, all radiated electric fields now add, spatially coherent, power ~ N^2

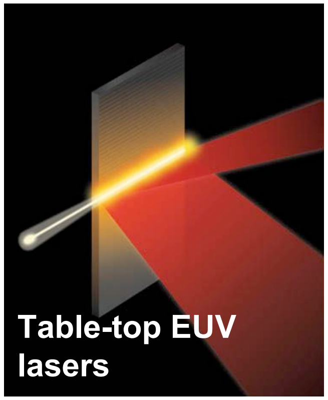
$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

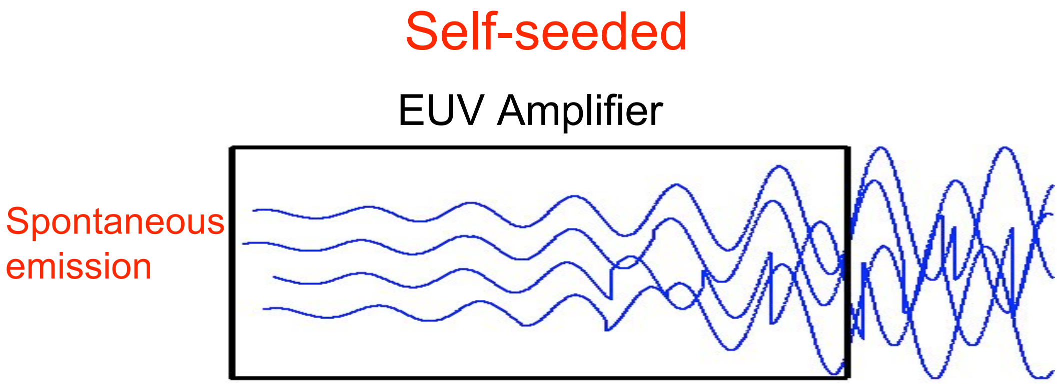
Gain and Saturation in an FEL

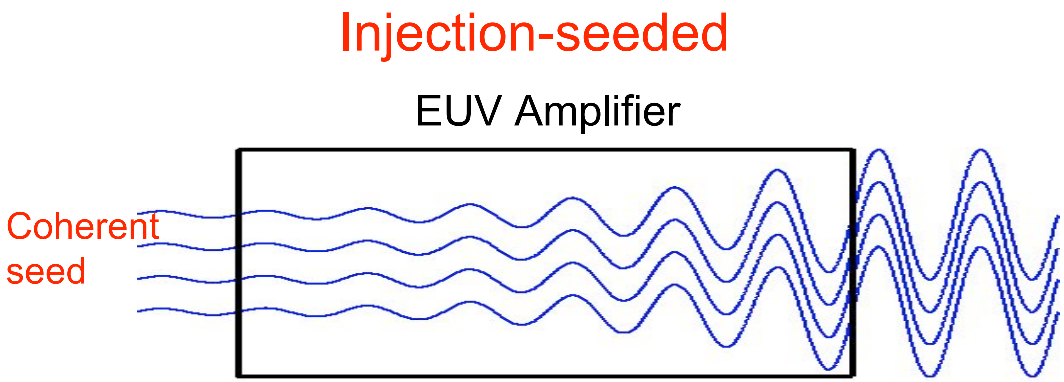


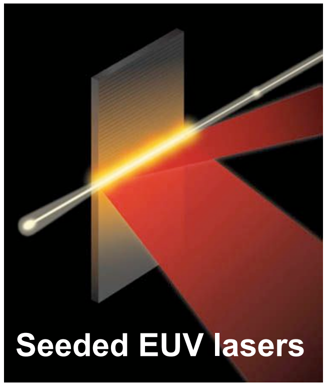
Courtesy of K-J. Kim

Lasers seeded by spontaneous emission have poor temporal coherence









Seed pulses can be greatly amplified preserving or even improving their properties

Courtesy of Jorge Rocca, CSU

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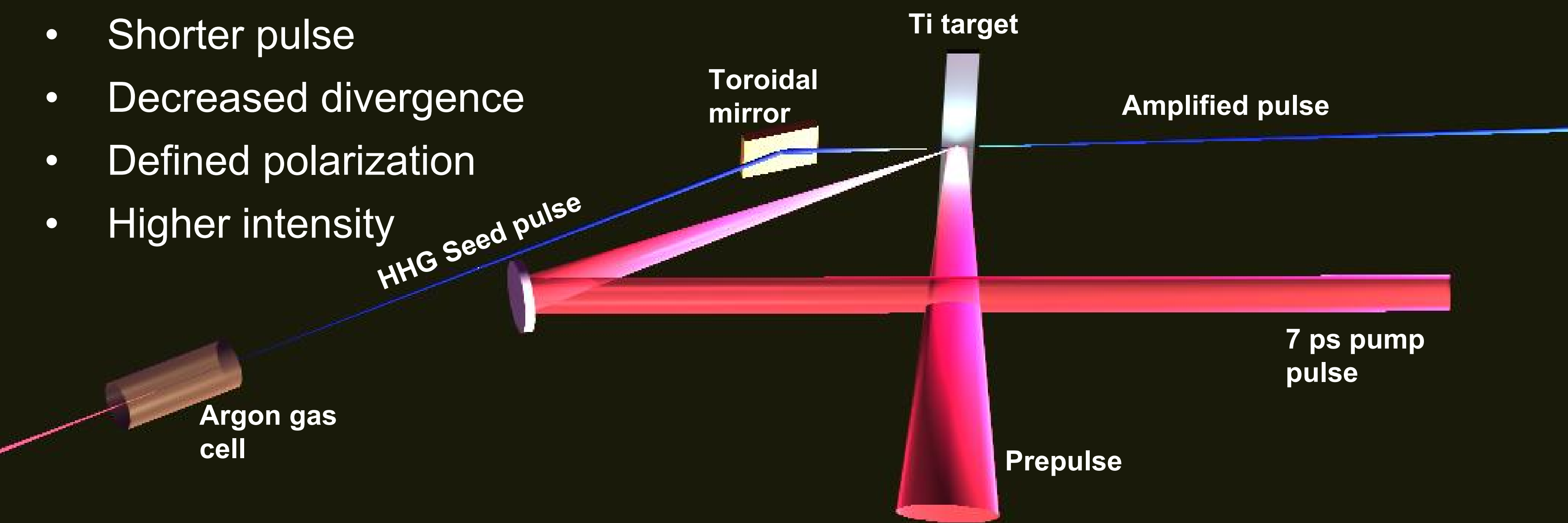
Seeding with high harmonic pulse can greatly increase spatial coherence and beam brightness

High harmonic seeding of solid target soft x-ray laser amplifier

- Shorter pulse
- Decreased divergence
- Defined polarization
- Higher intensity



- **Proof of principle experiment**: T. Ditmire et al. Phys. Rev. A. 51, R 4337, (1995): Amplification of HHG by ~ 3 X in the λ= 25.1 nm line of a Gallium laser amplifier
- **HHG seeding of OFI amplifier:** P. Zeitoun, G. Faivre, S. Sebban, T. Mocek *et al*, Nature ,**431**, 426, (2004).

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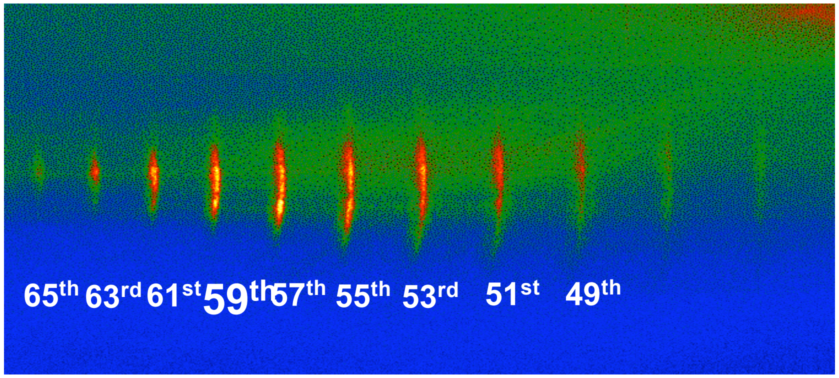
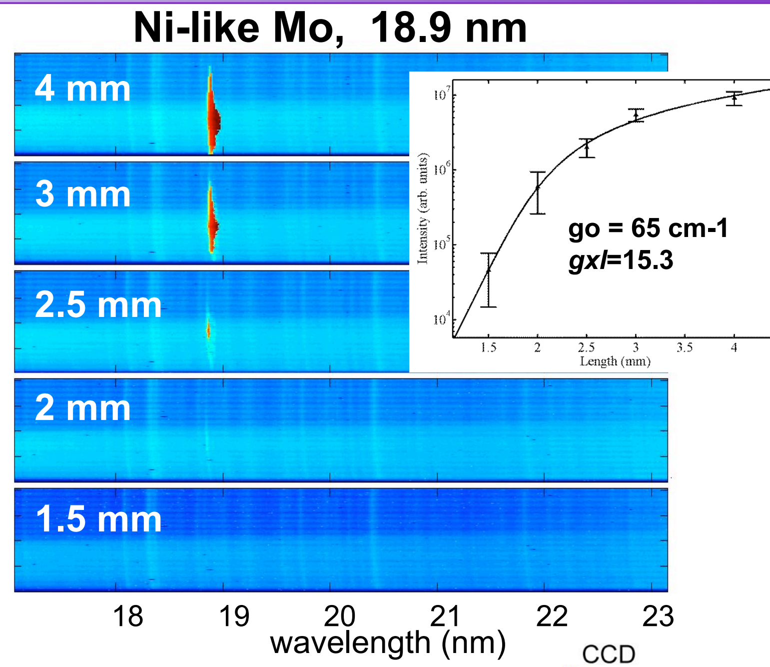
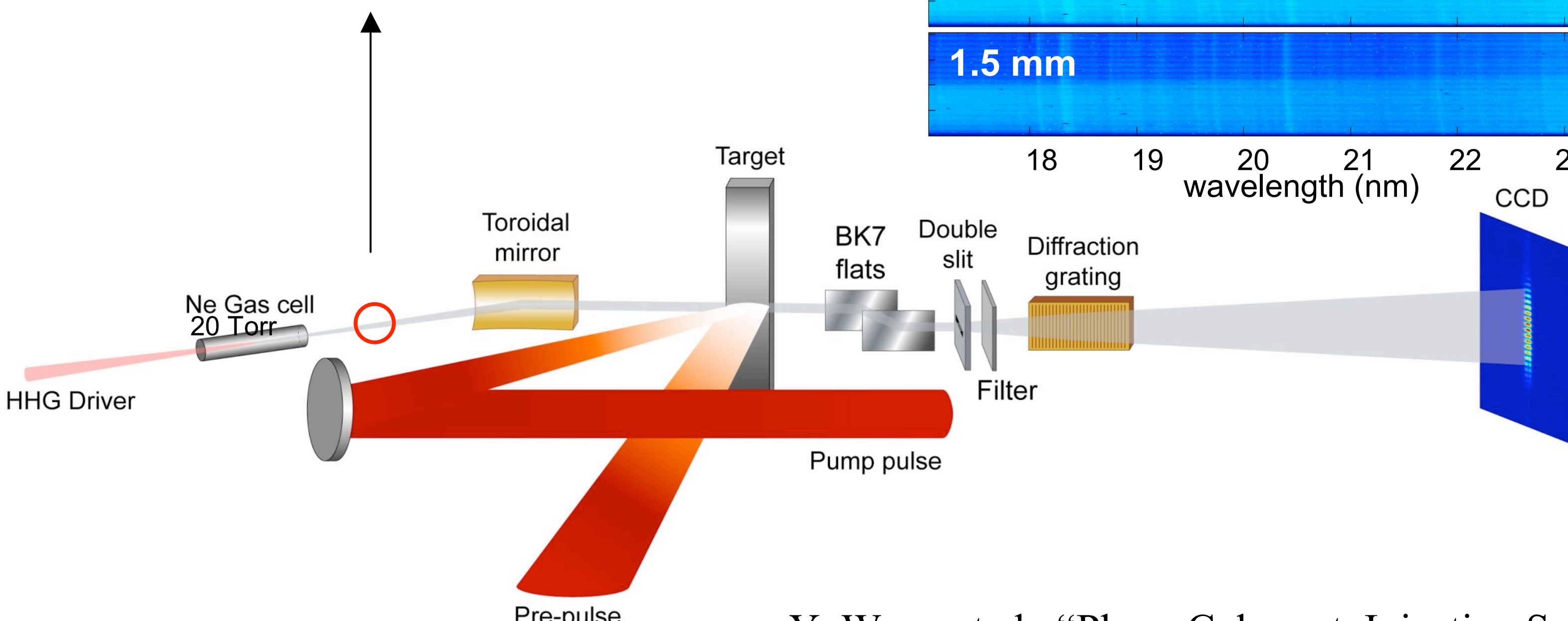
Injection-seeded λ=10-20 nm lasers

Colorado State University

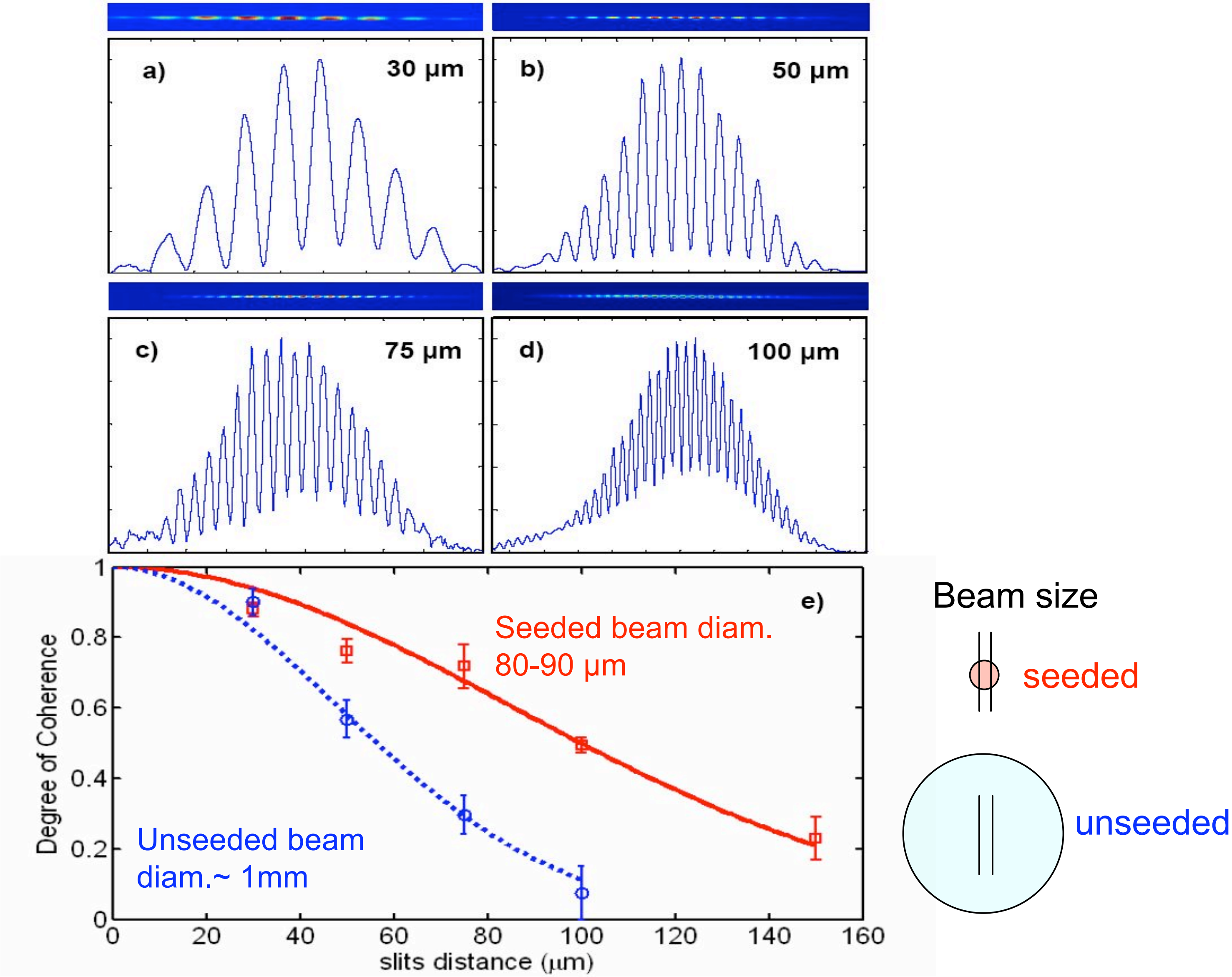






Y. Wang et al., "Phase Coherent, Injection Seeded, Table-top Soft X-ray Lasers at 18.9 nm and 13.9 nm", *Nature Photonics* **2**, 94 (2008).

Essentially fully coherent output Ni-like Mo 18.9 nm

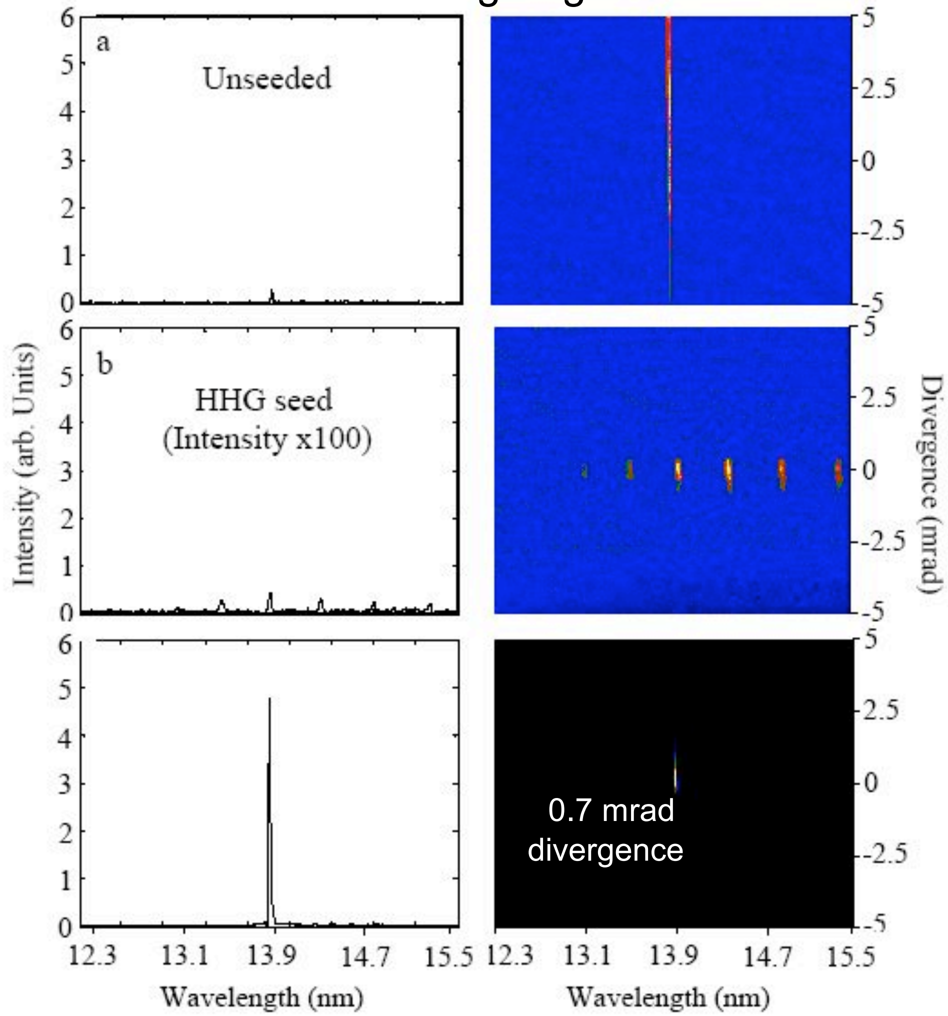


Courtesy of Jorge Rocca, CSU

Injection seeded Ni-like Ag at 13.9 nm

Ni-like Ag, 13.9 nm

3 mm long target



Seeding Ni-like Silver (3 mm target)
Harmonics x100

Y. Wang et al. Nature Photonics, 2, 94, (2008

Courtesy of Jorge Rocca, CSU

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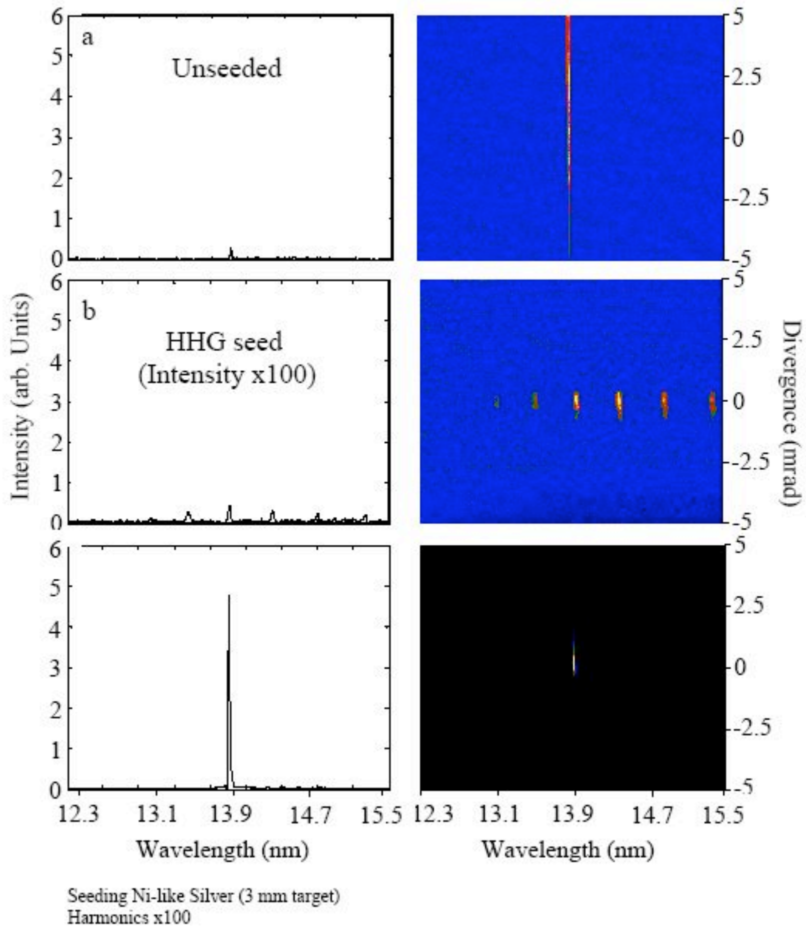
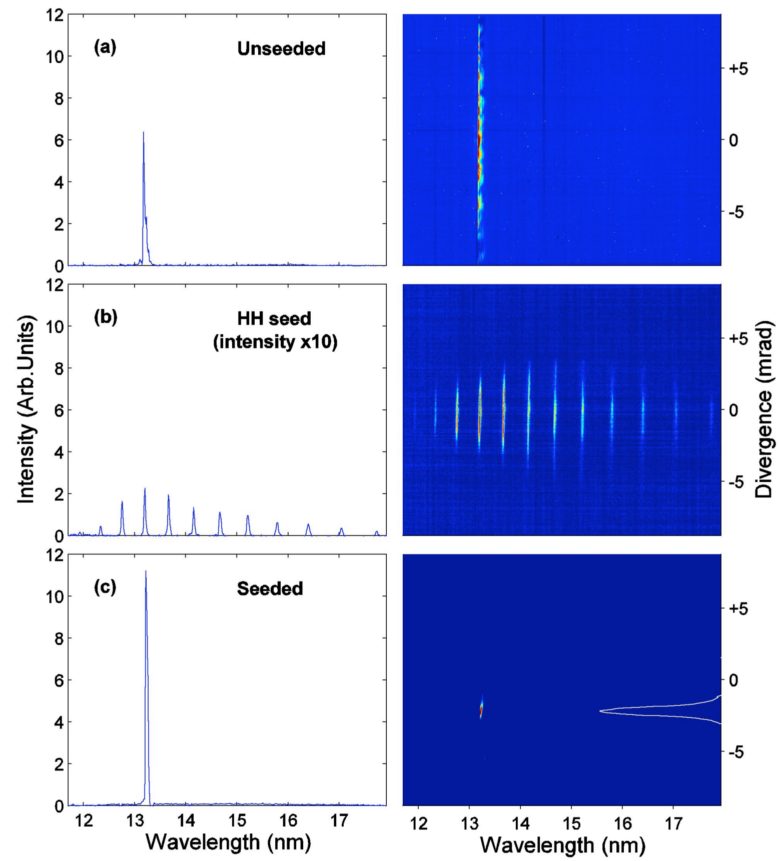
Phase coherent Ni-like Ag and Ni-like Cd lasers at 13.9 nm and 13.2 nm

Colorado State University

Ni-like Ag



Ni-like Cd



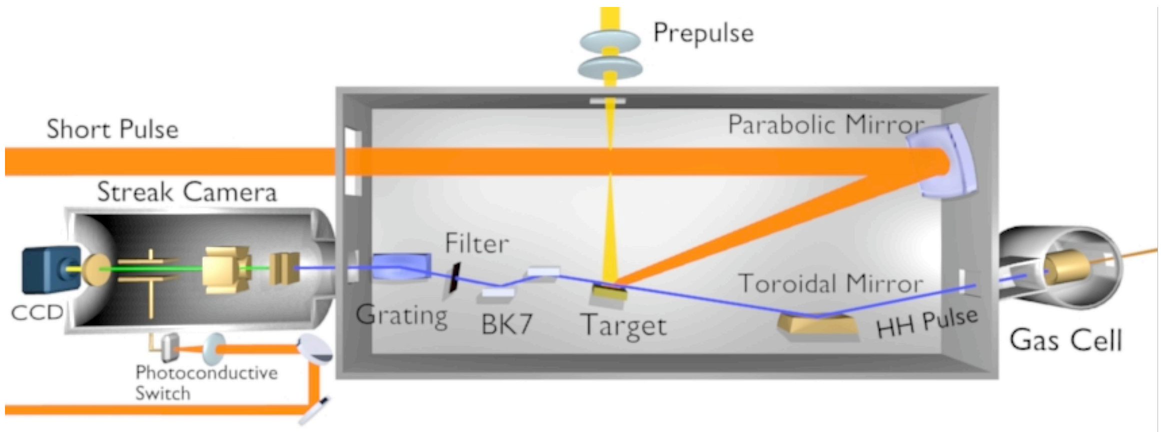
Y. Wang et al. Nature Photonics, 2, 94, (2008)

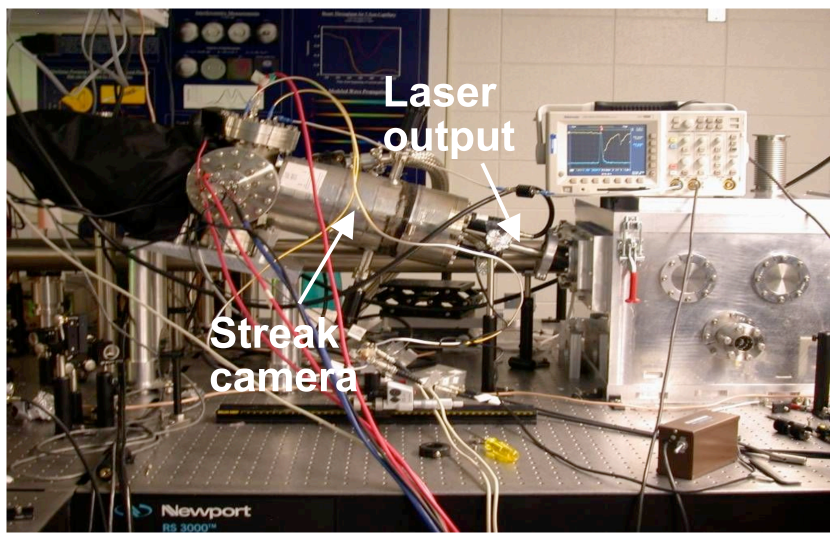
F. Pedaci et al. Optics Lett., 33, 491, (2008)

Courtesy of Jorge Rocca, CSU

Streak camera* measurement of injection-seeded laser pulsewidth

* Collaboration with Zenghu Chang et al. Kansas State University



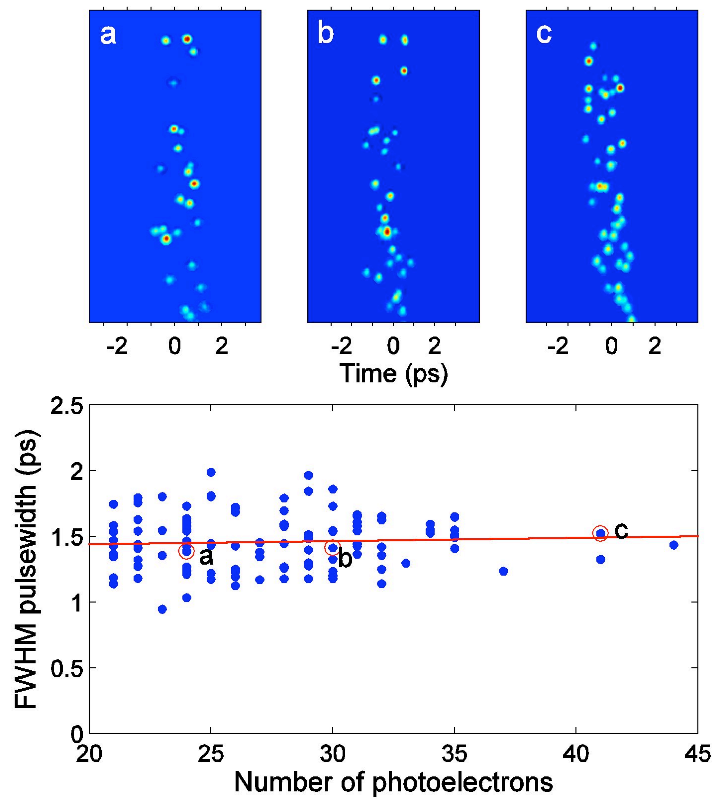


Y. Wang et. al. Phys. Rev. A. **79**, 023810, (2009).

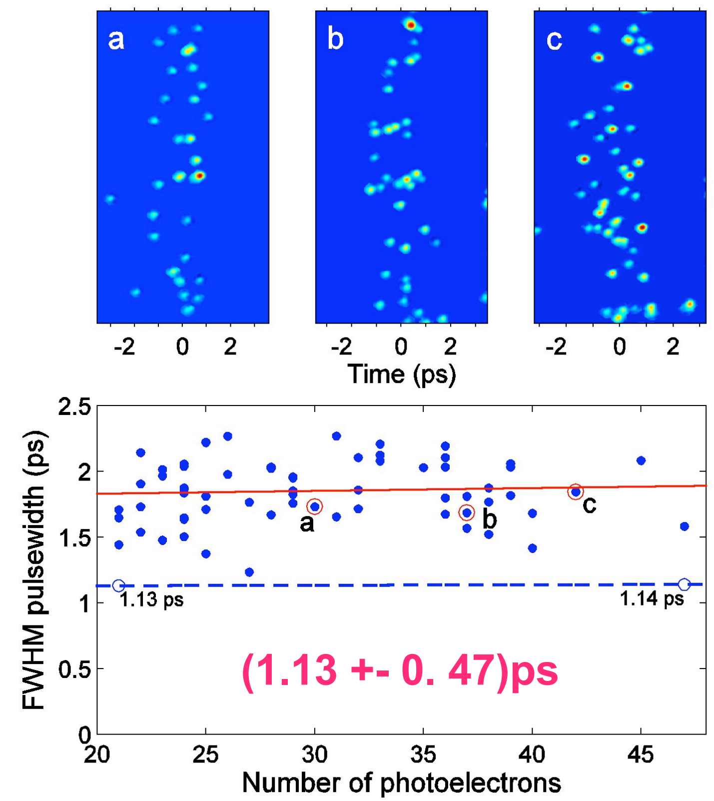
Courtesy of Jorge Rocca, CSU

Measurement of 1 ps soft x-ray laser pulses

Determination of Streak camera resolution with HH pulses



Seeded Ne-likeTi laser pulsewidth measurement



Y. Wang et. al. Phys. Rev. A. **79**, 023810, (2009).

Courtesy of Jorge Rocca, CSU