Chapter 7

EXTREME ULTRAVIOLET AND SOFT X-RAY LASERS

\[
\frac{I}{I_0} = e^{GL} \tag{7.2}
\]

\[
G = n_u \sigma_{stim} F \tag{7.4}
\]

\[
\sigma_{stim} = \frac{\pi \lambda r_e}{(\Delta \lambda / \lambda)} \left( \frac{g_l}{g_u} \right) f_{lu} \tag{7.18}
\]

\[
\frac{P}{A} = \frac{16\pi^2 c^2 \hbar (\Delta \lambda / \lambda) G L}{\lambda^4} \tag{7.22}
\]
where $G$ is the gain per unit length and $L$ is the propagation distance in the lasing media.

\[
\frac{I}{I_0} = e^{GL} \quad (7.2)
\]

or

\[
G = n_u \sigma_{\text{stim}} - n_l \sigma_{\text{abs}} \quad (7.3)
\]

where the density inversion factor is

\[
F \equiv 1 - \frac{n_l \sigma_{\text{abs}}}{n_u \sigma_{\text{stim}}} = 1 - \frac{n_l g_u}{n_u g_l} \quad (7.5)
\]

where $g_u$ and $g_l$ are degeneracies (same energy, different quantum numbers) associated with the upper and lower states.
In equilibrium the number of transitions up equals the number down. On a per unit time and per unit volume basis

\[ n_u A_{ul} + n_u B_{ul} U_{\Delta \omega} = n_\ell B_{\ell u} U_{\Delta \omega} \]  

(7.6)

where \( n_u \) and \( n_\ell \) are the densities of ions in the upper and lower states, \( A_{ul} \) is the spontaneous decay rate (inverse of natural life time), \( B_{ul} \) is the stimulated transition rate from \( u \) to \( \ell \), \( B_{\ell u} \) is the absorption transition rate, and \( U_{\Delta \omega} \) is the spectral energy density, in units of energy per unit volume per unit frequency interval.
In equilibrium

\[ U_{\Delta\omega} = \frac{\hbar \omega^3}{\pi^2 c^3 (e^{\hbar \omega / \kappa T} - 1)} \]  

(7.7)

with equal density of states \((g_\ell = g_u)\)

\[ \frac{n_\ell}{n_u} = e^{(E_u - E_i) / \kappa T} = e^{\hbar \omega / \kappa T} \]  

(7.8)

The (transition) rate equation (7.6) can be written as

\[ A_{\ell u} + B_{\ell u} U_{\Delta\omega} = \frac{n_\ell}{n_u} B_{\ell u} U_{\Delta\omega} \]

where \(B_{\ell u} = B_{u\ell}\).

\[ A_{\ell u} = B_{\ell u} \left[ \frac{n_\ell}{n_u} - 1 \right] U_{\Delta\omega} \]
Transition Rates: Einstein A and B Coefficients (continued further)

\[ \frac{A_{u\ell}}{B_{u\ell}} = \left[ \frac{n_\ell}{n_u} - 1 \right] U_{\Delta\omega} \]

Substituting for \( n_\ell/n_u \) from eq.(7.8) and \( U_{\Delta\omega} \) from eq.(7.7)

\[ \frac{A_{ul}}{B_{ul}} = \left( e^{\hbar\omega/\kappa T} - 1 \right) \left[ \frac{\hbar \omega^3}{\pi^2 c^3 \left( e^{\hbar\omega/\kappa T} - 1 \right)} \right] \]

or

\[ \frac{A_{ul}}{B_{ul}} = \frac{\hbar \omega^3}{\pi^2 c^3} \quad (7.9) \quad \text{(near equilibrium)} \]
If stimulated emission dominates spontaneous emission, the radiated power is

\[
\frac{\Delta E}{\Delta t} = \left[ n_u B_{ul} U_{\Delta\omega} - n_l B_{lu} U_{\Delta\omega} \right] \hat{\hbar}\omega \cdot \Delta A \cdot \Delta L \quad (7.10)
\]

Radiated power
\begin{align*}
\frac{\Delta E}{\Delta t} &= P = \Delta I \cdot \Delta A \\
\Delta I &= n_u F B_{ul} U_{\Delta\omega} \hat{\hbar}\omega \cdot \Delta L \quad (7.11)
\end{align*}
\[ \Delta I = n_u F B_{ul} U_{\Delta \omega} \hbar \omega \cdot \Delta L \quad (7.11) \]

In terms of the radiation energy density \( U_{\Delta \omega} \)

\[ I = U_{\Delta \omega} \cdot \Delta \omega \cdot c \]

then

\[ \frac{\Delta I}{I} = \frac{n_u F B_{ul} \hbar \omega \cdot \Delta L}{(\Delta \omega)c} \quad (7.13) \]

Using the relationship between A and B coefficients

\[ \frac{A_{ul}}{B_{ul}} = \frac{\hbar \omega^3}{\pi^2 c^3} \quad (7.9) \]

\[ \frac{\Delta I}{I} = \frac{\pi^2 c^2 n_u F A_{ul} \cdot \Delta L}{(\Delta \omega)\omega^2} \quad (7.14a) \]
Radiated Power and Intensity (continued further)

\[
\frac{\Delta I}{I} = \frac{\pi^2 c^2 n_u F A_{ul} \cdot \Delta L}{(\Delta \omega) \omega^2} 
\]  
(7.14a)

Noting that \((\Delta \omega) \omega^2 = (\Delta \omega/\omega) \omega^3 = (\Delta \lambda/\lambda) (2\pi)^3 c^3 / \lambda^3\)

\[
\frac{\Delta I}{I} = \frac{\lambda^3 n_u F A_{ul} \cdot \Delta L}{8\pi c (\Delta \lambda/\lambda)} 
\]  
(7.14b)

Integrating from \(\Delta L = 0\) to \(L\)

\[
\frac{I}{I_0} = e^{GL} ; \quad G = \frac{\lambda^3 n_u F A_{ul}}{8\pi c (\Delta \lambda/\lambda)} 
\]  
(7.15)
Stimulated Emission Cross-Section

\[
\frac{I}{I_0} = e^{GL} ; \quad G = \frac{\lambda^3 n_u FA_{ul}}{8\pi c(\Delta\lambda/\lambda)} \quad (7.15)
\]

Recalling our earlier definition that

\[
G \equiv n_u \sigma_{\text{stim}} F \quad (7.4)
\]

We identify the cross-section as

\[
\sigma_{\text{stim}} = \frac{\lambda^3 A_{ul}}{8\pi c(\Delta\lambda/\lambda)} \quad (7.16)
\]

The Einstein \( A \) coefficient is the inverse lifetime for a transition between two states, \( A_{ul} = 1/\tau \), and can be calculated using quantum mechanics (Silfvast, Corney), to be

\[
A_{ul} = \frac{e^2 \omega^2}{2\pi \epsilon_0 mc^3} \left( \frac{g_l}{g_u} \right) f_{lu} \quad (7.17)
\]

So that the cross-section for stimulated emission is

\[
\sigma_{\text{stim}} = \frac{\pi \lambda r_e}{\Delta\lambda/\lambda} \left( \frac{g_l}{g_u} \right) f_{lu} \quad (7.18)
\]

where \( f_{lu} \) is the oscillator strength of the transition and \( r_e \) is the classical radius. Now need \( \Delta\lambda/\lambda \).
Doppler Broadened Line Width

For EUV and soft x-ray lasing, created in a hot plasma, the line width is typically dominated by Doppler broadening, rather than by natural lifetime. For a Maxwellian velocity distribution the full rms line width is then

$$\frac{(\Delta \lambda)}{\lambda}_{\text{rms}} = \frac{2v_i}{c}$$

or expressed as a FWHM quantity, is

$$\frac{(\Delta \lambda)}{\lambda}_{\text{FWHM}} = \frac{2\sqrt{2 \ln 2}}{c} \sqrt{\frac{\kappa T_i}{M}}$$  \hspace{1cm} (7.19a)

where $v_i = \sqrt{\frac{\kappa T_i}{M}}$ is the rms ion velocity, $\kappa T_i$ is the ion temperature, and $M$ is the ion mass. Taking $M \approx 2m_pZ$ and $\kappa T_i$ in eV

$$\frac{(\Delta \lambda)}{\lambda}_{\text{FWHM}} = 7.69 \times 10^{-5} \left(\frac{\kappa T_i}{2Z}\right)^{1/2}$$  \hspace{1cm} (7.19b)

or about $10^{-4}$ for argon ions at a temperature of about 60 eV.
Wavelength Scaling of EUV and Soft X-Ray Lasers

To maintain an inverted population density requires delivery of a power

\[ P = \frac{\hbar \omega n_u FV}{\tau} \]  

With \( \tau = 1/A_{u\ell} \)

\[ \frac{P}{V} = \hbar \omega n_u FA_{u\ell} \]

From the gain expression (7.15)

\[ G = \frac{\lambda^3 n_u F A_{u\ell}}{8\pi c (\Delta\lambda/\lambda)} \rightarrow n_u F A_{u\ell} = \frac{8\pi c (\Delta\lambda/\lambda)G}{\lambda^3} \]

the required power per unit volume is

\[ \frac{P}{V} = \frac{16\pi^2 c^2 \hbar (\Delta\lambda/\lambda) G}{\lambda^4} \]

With \( V = AL \), the resultant lasing intensity would be

\[ I = \frac{P}{A} = \frac{16\pi^2 c^2 \hbar (\Delta\lambda/\lambda) G L}{\lambda^4} \quad (7.22) \]

Thus both required power per unit volume and potential lasing intensity scale as \( 1/\lambda^4 \).
Lasing intensity scales as $1/\lambda^4$.

High $\kappa T_e \propto 1/\lambda$

Cross-section $\propto 1/\lambda$

Plasma lifetime $\propto 1/\lambda^2$

Line width $\propto 1/\lambda^{1/2}$
Seeded EUV Lasers
Young's Double Slit Experiment with Random Emitters: Young did not have a laser

- N uncorrelated emitters
- Self-interference only
- Electric fields chaotic
- Intensities add
- Radiated power \( \sim N \)

\[
d \cdot 2\theta_{\text{FWHM}} \approx \frac{\lambda}{2}
\]

\[
\lambda_{\text{coh}} = \frac{\lambda^2}{2\Delta\lambda} = \frac{1}{2} N_{\text{coh}} \lambda
\]
Young's Double Slit Experiment with Phase Coherent Emitters (some lasers, or properly seeded FELs)

- Phase coherent electric fields
- Electric fields from all particles interfere constructively
- Radiated power $\sim N^2$
- New phase sensitive probing of matter possible

\[ d \cdot 2\theta|_{\text{FWHM}} \approx \frac{\lambda}{2} \]

\[ \lambda_{\text{coh}} = \frac{\lambda^2}{2\Delta\lambda} = \frac{1}{2} N_{\text{coh}} \lambda \]
Undulators and FELs

Undulator – uncorrelated electron positions, radiated fields uncorrelated, intensities add, limited coherence, power $\sim N$.

Free Electron Laser (FEL) – very long undulator, electrons are “microbunched” by their own radiated fields into strongly correlated waves of electrons, all radiated electric fields now add, spatially coherent, power $\sim N^2$. 

$$\frac{dp}{dt} = -e(E + v \times B)$$
Gain and Saturation in an FEL

![Graph showing gain and saturation in an FEL](Gain_Saturation_FEL_graph.ai)

Courtesy of K-J. Kim
Lasers seeded by spontaneous emission have poor temporal coherence.

Free Electron Laser
Table-top EUV lasers
Seeded EUV lasers

Self-seeded
EUV Amplifier
Spontaneous emission

Injection-seeded
EUV Amplifier
Coherent seed

Seed pulses can be greatly amplified preserving or even improving their properties

Courtesy of Jorge Rocca, CSU
Seeding with high harmonic pulse can greatly increase spatial coherence and beam brightness.

High harmonic seeding of solid target soft x-ray laser amplifier

- Shorter pulse
- Decreased divergence
- Defined polarization
- Higher intensity


Injection-seeded $\lambda=10$-20 nm lasers

Essentially fully coherent output Ni-like Mo 18.9 nm

Seeded beam diam. 80-90 μm
Unseeded beam diam. ~ 1 mm

Courtesy of Jorge Rocca, CSU
Injection seeded Ni-like Ag at 13.9 nm

Ni-like Ag, 13.9 nm
3 mm long target

Unseeded

HHG seed (Intensity x100)

0.7 mrad divergence


Courtesy of Jorge Rocca, CSU
Phase coherent Ni-like Ag and Ni-like Cd lasers at 13.9 nm and 13.2 nm

Ni-like Ag

Ni-like Cd


Courtesy of Jorge Rocca, CSU
Streak camera* measurement of injection-seeded laser pulsewidth

* Collaboration with Zenghu Chang et al. Kansas State University


Courtesy of Jorge Rocca, CSU
Measurement of 1 ps soft x-ray laser pulses

Determination of Streak camera resolution with HH pulses

Seeded Ne-like Ti laser pulsewidth measurement


Courtesy of Jorge Rocca, CSU