

## Soft X-Rays and Extreme Ultraviolet Radiation: Principles and Applications

### Chapter 6 Homework Problems

**6.1** Describe the basic features of microscopic, kinetic, and fluid descriptions of a plasma. Give examples of phenomena that require these different levels of description.

**6.2** For a plasma of electron density  $4 \times 10^{21}$  e/cm<sup>3</sup>, 120 eV electron temperature, and magnetic flux density 0.3 tesla, (a) what are the electron plasma frequency, (b) rms electron velocity, (c) electron Debye screening distance, (d) number of electrons per Debye sphere, and (f) electron cyclotron frequency? (e) What is the significance of each of these parameters? (f) What is the significance of  $\omega_c/\omega_p$ ? (g) For what wavelength of incident radiation is the electron density just equal to the critical electron density? (h) What is the significance of the critical electron density? (i) If the plasma consists of carbon atoms whose average charge is +5e, what is the ion density and ion plasma frequency? (j) If the ion temperature is 60 eV, what is the rms ion velocity? (k) Why might the ion temperature be less than the electron temperature?

**6.3** In the kinetic description of a plasma, (a) what is the significance of  $v_e$ ? (b) Is there a similar measure for ions? (c) How do these relate to the phase velocities of various plasma waves?

**6.4** (a) Explain the processes of Landau damping and Landau growth. (b) Why are these inherently kinetic phenomena? (c) What are the important parameters? (d) Briefly describe experiments in which one could observe these phenomena.

**6.5** (a) In what general circumstances is the fluid model of plasma processes advantageous? (b) What are the basic fluid equations? (c) When can the dyadic pressure be replaced by a scalar pressure? (d) What are the conditions of validity for the perfect gas relation? (e) Give an example where a scalar pressure would not be an accurate approximation.

**6.6** (a) What are the three modes of wave propagation in an isotropic electron-ion plasma? (b) What are their dispersion relations? (c) Give relations for the phase velocity and group velocity for the transverse wave. (d) How do these compare to the velocity of light in vacuum? (e) Does this conflict in any way with the theory of relativity? (f) What is the phase velocity of the electron acoustic (electron plasma) wave for  $\omega/\omega_p = 1, 1.5,$  and  $2$ ? (g) How do these compare to the electron thermal velocity? (h) What do the latter indicate about the potential for wave-particle interactions?

**6.7** As a laser produced plasma streams away from its point of origin after cessation of the laser pulse, (a) what expansion velocity do we expect? (b) Explain physically why this is so.

**6.8** (a) Why is the ion-acoustic wave sensitive to electron temperature, while the electron-acoustic wave is generally insensitive to the ion temperature? (b) For the ion-acoustic wave, what is the sound speed for  $T_e/T_i = 10$  and  $T_e/T_i = 1$ ?

**6.9** (a) What ionospheric electron density is required to reflect your favorite AM radio station to other geographic area during evening broadcasts? (b) Why would this not occur during the day? (c) Why might such distant transmissions be sporadic?

**6.10** (a) What is the significance of the electron-ion collision frequency, as given in Eq. (6.115)? (b) Should this have been included in Eq. (6.43)? (c) What approximation was implicit in its omission? (d) For a propagating transverse electromagnetic wave how is energy absorbed by the presence of electrons and ions? (e) To which particles is the absorbed energy transmitted? (f) Why does the absorption length for a transverse wave vary in proportion to  $(\kappa T_e)^{3/2}$ , and inversely proportional to  $n_e^2 Z$ , as described by Eq. (6.122)?

**6.11** How are the waves described in problem 6.6 affected by the presence of a strong magnetic field  $\omega_c \simeq \omega_p$ ? Consult Chapter 7 of the text by D. Nicholson, Ref. 4 of this chapter.

**6.12** (a) Write Newton's second law of motion for an electron moving at a right angle to an applied magnetic field. (b) Show that the solution is circular motion at the cyclotron frequency  $\omega_c = eB/m$ , as given in Eq. (6.8). (c) Show that the Larmor radius of this circular motion is  $r_L = mv/eB$ . (d) For an electron of 100 eV energy in a 9 tesla field, what is the cyclotron frequency and Larmor radius?

**6.13** Choose two of the non-linear terms in Eq. (6.123) and explain under what circumstances they lead to non-linear behavior, and how each might manifest itself in nature or in laboratory experiments.

**6.14** (a) What is the normalized electron oscillation energy  $|v_{os}/v_e|^2$  for incident laser radiation of 1.06  $\mu\text{m}$  wavelength, intensity of  $3 \times 10^{12}$  W/cm<sup>2</sup>, and electron temperature of 50 eV? (b) Why is this a reasonable first-order index of non-linear behavior in high intensity laser-plasma interactions? (c) What additional parameter, or modified form, has been determined to be of significance, as indicated by the recent literature?

**6.15** (a) What are the basic assumptions that lead to black body radiation? (b) What are the primary observables of black body radiation? (c) What experimental features might one observe to study departures from the idealized black body radiator? (d) Make a table showing values of peak photon energy, peak spectral brightness, and radiated intensity, all in appropriate units for temperatures of 10 eV, 30 eV, 50 eV, 100 eV, and 200 eV. (e) Explain why peak spectral brightness scales as  $(\kappa T)^3$  while radiated intensity ( $I$ ) scales as  $(\kappa T)^4$ .

**6.16** (a) Explain what is meant by an "ionization bottleneck" in the process of successive electron removal from an atom in an hot dense plasma. (b) Why do closed electron shells play an important role? (c) How might this effect be used to provide a spectrally intense source of narrow bandwidth radiation? (d) Quantify your answers with examples of neon-like and helium-like titanium. (e) What are the strong emission lines that result for He-like Ti? (f) Discuss the use of He-like N as a water window radiation source. (g) What elements might provide particularly intense, narrow band emissions from Ne-like ions in the 500 eV photon energy range? (h) What are the relevant ionization potentials for the ions you have selected and what strong emission lines do you expect to see? Consult Table 6.2 to narrow your choices, and consult R.L. Kelly's extensive tabulations as given in Ref. 54 of this chapter.

**6.17** (a) Discuss several techniques by which emission spectroscopy can be used to determine electron temperature of a hot plasma. (b) What techniques would additionally permit measures of density? Consult *Principles of Plasma Spectroscopy*, by H.R. Griem, Ref. 9 in this chapter, and the chapter on "X-Ray Radiation from Laser Plasma," by R. Kauffman, Ref. 49 in this chapter, and the article by C. De Michelis and M. Mattioli, "Soft X-ray Spectroscopic Diagnostics of Laboratory Plasmas", *Nuclear Fusion*, 21, 677-754 (1981).

6.18 The EUVL Engineering Test Stand (an “ $\alpha$ -like” stepper) at Sandia National Laboratories in Livermore, CA uses three Nd:YAG laser “chains” (three synchronized oscillators, each with its own amplifier) to create a Xenon laser produced plasma that radiates strong EUV emission at 13.5 nm. If each separate chain generates 0.25 J, 7 ns FWHM pulses at 1.06  $\mu\text{m}$  laser wavelength and 2kHz repetition rate (each), (a) what is the average output per chain, and what is the combined average power for the three? (b) If the output of a single train is brought to a 100  $\mu\text{m}$  diameter focus, what is the focal intensity in units of  $\text{W}/\text{cm}^2$  averaged over the duration of a single pulse? (c) If you assume that 80% of the incident light is absorbed by the plasma and 65% of that is re-radiated in a near-black-body continuum, what temperature in eV characterizes the emission? (d) What is the peak photon energy in eV for the near-black-body continuum? (e) What ionization states\* would you expect to observe through their radiation in a Xe plasma? (f) Calculate  $(v_{os}/v_e)^2$  from questions (b) and (c). Should the Nd laser be converted from 1.06 $\mu\text{m}$  wavelength to  $2\omega$  at 0.53 $\mu\text{m}$  or  $3\omega$  at 0.35 $\mu\text{m}$  to avoid non-thermal plasma processes (suprathermal electron generation)? (g) If we assume that on an average power basis 1% of the incident laser energy is converted to 13.5 nm radiation within a 2.5% relative spectral bandwidth and within a  $2\pi$  steradian solid angle (a hemisphere), what will be the “in-band” (2.5% bandpass) EUV radiation (in Watts) at 13.5 nm, within a collection angle of 1.8 steradians? This is the collection solid angle of the ETS condenser optics that illuminate the reflective mask. (h) How might the collected in-band EUV power be increased so as to provide a higher wafer throughput and thus decrease stepper cost of ownership?

\*Consult as necessary C.W. Allen, *Astrophysical Quantities* (Athlone Press, London, 1997), Third Edition, p. 39.

### Capillary Discharge Problem

At Sandia National Laboratories, the EUV electric capillary discharge source produces the following spectrum in Xenon (given as an MS-Excel file). A production EUV stepper will use a nine Moly-Silicon multilayer mirror system. (a) Determine an appropriate center wavelength and associated bilayer spacing required to maximize EUV throughput. Use  $\sigma=0.5$  nm rms, and  $\Gamma=0.44$ . Three single mirror reflectivities are provided in the spreadsheet available at <http://www.coe.berkeley.edu/AST/sxreuv/hw6/spectrum.xls>. (b) Plot the reflectivity curve for the nine mirror system and the emerging output spectrum. (c) The power output of EUV sources are often given for 2% bandwidth (that’s +/- 1%). Is this an adequate indicator of what intensity is available after nine mirrors?