

Soft X-Rays and Extreme Ultraviolet Radiation: Principles and Applications
Chapter 4 Homework Problems

4.1

Equation (4.1) describes the current density induced by an incident electric field in a collection of identical atoms of number density n_a . The contributions of all bound electrons are accounted for in $f^0(\omega)$. The superscript zero indicates that this formulation is appropriate for both forward scattering and long wavelength. Beginning with the general definition of current density

$$\mathbf{J} = qn\mathbf{v}$$

use equation (2.65) to show that in these regions of validity ($f \rightarrow f^0$), the product $n\mathbf{v}$ is replaced by

$$n\mathbf{v} \rightarrow n_a \sum_{s=1}^Z \mathbf{v}_s = n_a f^0(\omega) \mathbf{v}_e$$

where \mathbf{v}_e is the response of a single free electron to an incident electric field \mathbf{E}_i . Note that in the text \mathbf{v}_e is replaced by \mathbf{v} for simplicity in equation (4.1). Equations (4.1) and (4.3) then follow directly.

4.2

Using Snell's law, equation (3.38), derive the correction to Bragg's law

$$m\lambda = 2d \sin \theta \left[1 - \frac{4\bar{\delta}d^2}{m^2\lambda^2} \right]$$

where λ and θ are external to the multilayer, and $\bar{\delta}$ is a density profile averaged value for a bi-layer pair. Start with Bragg's law within the periodic structure as given by equation (4.6b),

$$m\lambda' = 2d \sin \theta'$$

where λ' and θ' are within the multilayer (or crystal). Use Snell's law to replace λ' and θ' , in the approximation $\beta/\bar{\delta} \ll 1$. For further discussion consult Compton and Allison, p. 674 therein, given as Ref. 6 in Chapter 3; Richtmyer, Kennard, and Lauritsen, p. 392 therein, given as Ref. 28 in Chapter 1 of this book; or Leighton, p. 456 therein, given as Ref. 27 in Chapter 1 of this book.

Note: There's a typo in the footnote on p. 101 of the book. The square root in the last expression is erroneous. The formula above is correct.

4.3

- (a) Describe some general considerations relevant to the choice of materials for a multilayer interference coating for use at a given photon energy.
- (b) Comment on the choice of angle of incidence, required d -spacing, choice of the low-Z "spacer" material, choice of the high-Z "scattering" material, optimum choice of Γ , required number of bi-layer pairs (N), and specification on substrate roughness. Be sure to consider materials compatibility, interface roughness, and chemical stability at the exposed surface.

4.4

You are designing an EUV telescope to observe the solar corona in the spectral vicinity of the Fe^{+10} (ten times ionized iron) emission line at 18.04 nm.

- What multilayer coating might you use for a normal incidence optic?
- Give the materials, d -spacing, Γ , σ , and N .
- What reflectivity do you anticipate? For information on spectral emission lines of highly ionized atoms of interest for x-ray astronomy see the paper by G.A. Doschek and R.D. Cowan, "A Solar Spectral Line List Between 10 and 200 Å Modified for Application to High Spectral Resolution X-Ray Astronomy," *Astro. Phys. J. Suppl.* 56, 67 (1984).

4.5

- Plot the reflectivity of a Mo/Si coating for wavelengths from 11 to 15 nm, at normal incidence ($\phi = 0$, $\theta = \pi/2$), assuming $d = 6.7$ nm, $\Gamma = 0.4$, and $N = 40$. Show results for a perfectly smooth, infinitely sharp interface ($\sigma = 0$), and for interfaces characterized by $\sigma = 0.1$ nm rms and 0.5 nm rms.
- What does this indicate regarding surface finish specifications for high mirror reflectivity?
- What throughput would be expected for a nine-mirror optical system with $\sigma = 0.5$ nm rms? Feel free to use the multilayer sub-routine at the CXRO website http://www.cxro.lbl.gov/optical_constants.

4.6

- Repeat problem 4.5 for a Mo/Be multilayer coating with $d = 5.7$ nm, $\Gamma = 0.40$, $N = 40$, and $\sigma = 0.1$ nm rms, for wavelengths from 11 to 15 nm.
- Compare the peak reflectivity and relative spectral bandpass to that of Mo/Si at $\sigma = 0.1$ nm rms.
- How does this change if one employs 70 bi-layer pairs of Mo/Be?
- What are the implications for an EUV optical stepper (lithographic optical reduction print camera) with seven multilayer coated surfaces?
- What role might the emission spectrum of the source play in the apparent advantage of Mo/Be?
- What disadvantages might there be?

4.7

You are interested in probing the magnetic properties of iron containing materials using polarized radiation near the iron L-edge.

- What photon energy range would be appropriate? You plan to use Brewster's angle optics to polarize the incident radiation and to measure Faraday rotation of the polarization vector as it traverses the material while exposed to an applied magnetic field.
- Describe the parameters of a W/C multilayer mirror to achieve the desired polarization, including the d -spacing, Γ , N , and σ .
- How can a polarimeter be constructed? Refer to the paper of J.B. Kortright *et al* in *Phys. Rev. B* 51, 10240 (1995). (Ref. 48 in the book)

4.8

An x-ray fluorescence microprobe uses multilayer coated Kirkpatrick-Baez (KB) optics to focus hard x-rays to a $1\ \mu\text{m}$ diameter spot size to study elemental compositions of samples for materials science, environmental sciences, and parts-per-billion (ppb) impurity analyses in which fluorescent emissions are detected from elements simultaneously. This is done with thick samples, in air, and in the presence of higher Z materials if necessary.

- (a) Plot the reflectivity of a single W/C multilayer mirror from zero to 12 keV photon energy, for $d = 4.0\ \text{nm}$, $\Gamma = 0.40$, $N = 50$ layer pairs, and a glancing incidence angle of 1.2° . Assume $\sigma = 0.1\ \text{nm rms}$.
- (b) What is the relative spectral bandpass?
- (c) How does this compare to $1/N$?
- (d) For what elements could multi-keV fluorescence emission from K-shells be observed?
- (e) From L-shells of which elements?

Homework problems involving the spectral bandwidth, throughput, and uniformity of multilayer coated optics and optical systems, are addressed in Chapter 10.